PUZZLER

Soft contact lenses are comfortable to wear because they attract the proteins in the wearer's tears, incorporating the complex molecules right into the lenses. They become, in a sense, part of the wearer. Some types of makeup exploit this same attractive force to adhere to the skin. What is the nature of this force? *(Charles D. Winters)*

chapter

23 Electric Fields

Chapter Outline

The electromagnetic force between charged particles is one of the fundamental forces of nature. We begin this chapter by describing some of the basic properties of electric forces. We then discuss Coulomb's law, which is the fundamental law governing the force between any two charged particles. Next, we introduce the concept of an electric field associated with a charge distribution and describe its effect on other charged particles. We then show how to use Coulomb's law to calculate the electric field for a given charge distribution. We conclude the chapter with a discussion of the motion of a charged particle in a uniform electric field.

23.1 PROPERTIES OF ELECTRIC CHARGES

11.2 A number of simple experiments demonstrate the existence of electric forces and charges. For example, after running a comb through your hair on a dry day, you will find that the comb attracts bits of paper. The attractive force is often strong enough to suspend the paper. The same effect occurs when materials such as glass or rubber are rubbed with silk or fur.

Another simple experiment is to rub an inflated balloon with wool. The balloon then adheres to a wall, often for hours. When materials behave in this way, they are said to be *electrified,* or to have become **electrically charged.** You can easily electrify your body by vigorously rubbing your shoes on a wool rug. The electric charge on your body can be felt and removed by lightly touching (and startling) a friend. Under the right conditions, you will see a spark when you touch, and both of you will feel a slight tingle. (Experiments such as these work best on a dry day because an excessive amount of moisture in the air can cause any charge you build up to "leak" from your body to the Earth.)

In a series of simple experiments, it is found that there are two kinds of electric charges, which were given the names **positive** and **negative** by Benjamin Franklin (1706–1790). To verify that this is true, consider a hard rubber rod that has been rubbed with fur and then suspended by a nonmetallic thread, as shown in Figure 23.1. When a glass rod that has been rubbed with silk is brought near the rubber rod, the two attract each other (Fig. 23.1a). On the other hand, if two charged rubber rods (or two charged glass rods) are brought near each other, as shown in Figure 23.1b, the two repel each other. This observation shows that the rubber and glass are in two different states of electrification. On the basis of these observations, we conclude that **like charges repel one another and unlike charges attract one another.**

QuickLab

Rub an inflated balloon against your hair and then hold the balloon near a thin stream of water running from a faucet. What happens? (A rubbed plastic pen or comb will also work.)

Using the convention suggested by Franklin, the electric charge on the glass rod is called positive and that on the rubber rod is called negative. Therefore, any charged object attracted to a charged rubber rod (or repelled by a charged glass rod) must have a positive charge, and any charged object repelled by a charged rubber rod (or attracted to a charged glass rod) must have a negative charge.

Attractive electric forces are responsible for the behavior of a wide variety of commercial products. For example, the plastic in many contact lenses, *etafilcon,* is made up of molecules that electrically attract the protein molecules in human tears. These protein molecules are absorbed and held by the plastic so that the lens ends up being primarily composed of the wearer's tears. Because of this, the wearer's eye does not treat the lens as a foreign object, and it can be worn comfortably. Many cosmetics also take advantage of electric forces by incorporating materials that are electrically attracted to skin or hair, causing the pigments or other chemicals to stay put once they are applied.

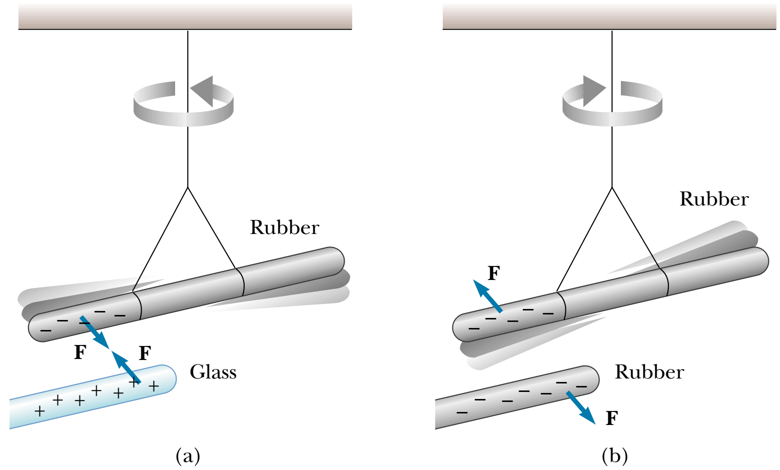


Figure 23.1 (a) A negatively charged rubber rod suspended by a thread is attracted to a positively charged glass rod. (b) A negatively charged rubber rod is repelled by another negatively charged rubber rod.

Charge is conserved

Another important aspect of Franklin's model of electricity is the implication that **electric charge is always conserved.** That is, when one object is rubbed against another, charge is not created in the process. The electrified state is due to a *transfer* of charge from one object to the other. One object gains some amount of negative charge while the other gains an equal amount of positive charge. For example, when a glass rod is rubbed with silk, the silk obtains a negative charge that is equal in magnitude to the positive charge on the glass rod. We now know from our understanding of atomic structure that negatively charged electrons are transferred from the glass to the silk in the rubbing process. Similarly, when rubber is rubbed with fur, electrons are transferred from the fur to the rubber, giving the rubber a net negative charge and the fur a net positive charge. This process is consistent with the fact that neutral, uncharged matter contains as many positive charges (protons within atomic nuclei) as negative charges (electrons).

Figure 23.2 Rubbing a balloon against your hair on a dry day causes the balloon and your hair to become charged.

Quick Quiz 23.1

If you rub an inflated balloon against your hair, the two materials attract each other, as shown in Figure 23.2. Is the amount of charge present in the balloon and your hair after rubbing (a) less than, (b) the same as, or (c) more than the amount of charge present before rubbing?

Charge is quantized

In 1909, Robert Millikan (1868–1953) discovered that electric charge always occurs as some integral multiple of a fundamental amount of charge e. In modern terms, the electric charge q is said to be **quantized,** where q is the standard symbol used for charge. That is, electric charge exists as discrete "packets," and we can write $q = Ne$, where N is some integer. Other experiments in the same period showed that the electron has a charge $-e$ and the proton has a charge of equal magnitude but opposite sign $+e$. Some particles, such as the neutron, have no charge. A neutral atom must contain as many protons as electrons.

Because charge is a conserved quantity, the net charge in a closed region remains the same. If charged particles are created in some process, they are always created in pairs whose members have equal-magnitude charges of opposite sign.

From our discussion thus far, we conclude that electric charge has the following important properties:

Properties of electric charge

- Two kinds of charges occur in nature, with the property that unlike charges attract one another and like charges repel one another.
- Charge is conserved.
- Charge is quantized.

23.2 INSULATORS AND CONDUCTORS

11.3 It is convenient to classify substances in terms of their ability to conduct electric charge:

Electrical **conductors** are materials in which electric charges move freely, whereas electrical **insulators** are materials in which electric charges cannot move freely.

Materials such as glass, rubber, and wood fall into the category of electrical insulators. When such materials are charged by rubbing, only the area rubbed becomes charged, and the charge is unable to move to other regions of the material.

Metals are good conductors

In contrast, materials such as copper, aluminum, and silver are good electrical conductors. When such materials are charged in some small region, the charge readily distributes itself over the entire surface of the material. If you hold a copper rod in your hand and rub it with wool or fur, it will not attract a small piece of paper. This might suggest that a metal cannot be charged. However, if you attach a wooden handle to the rod and then hold it by that handle as you rub the rod, the rod will remain charged and attract the piece of paper. The explanation for this is as follows: Without the insulating wood, the electric charges produced by rubbing readily move from the copper through your body and into the Earth. The insulating wooden handle prevents the flow of charge into your hand.

Semiconductors are a third class of materials, and their electrical properties are somewhere between those of insulators and those of conductors. Silicon and germanium are well-known examples of semiconductors commonly used in the fabrication of a variety of electronic devices, such as transistors and light-emitting diodes. The electrical properties of semiconductors can be changed over many orders of magnitude by the addition of controlled amounts of certain atoms to the materials.

When a conductor is connected to the Earth by means of a conducting wire or pipe, it is said to be **grounded.** The Earth can then be considered an infinite "sink" to which electric charges can easily migrate. With this in mind, we can understand how to charge a conductor by a process known as **induction.**

Charging by induction

To understand induction, consider a neutral (uncharged) conducting sphere insulated from ground, as shown in Figure 23.3a. When a negatively charged rubber rod is brought near the sphere, the region of the sphere nearest the rod obtains an excess of positive charge while the region farthest from the rod obtains an equal excess of negative charge, as shown in Figure 23.3b. (That is, electrons in the region nearest the rod migrate to the opposite side of the sphere. This occurs even if the rod never actually touches the sphere.) If the same experiment is performed with a conducting wire connected from the sphere to ground (Fig. 23.3c), some of the electrons in the conductor are so strongly repelled by the presence of

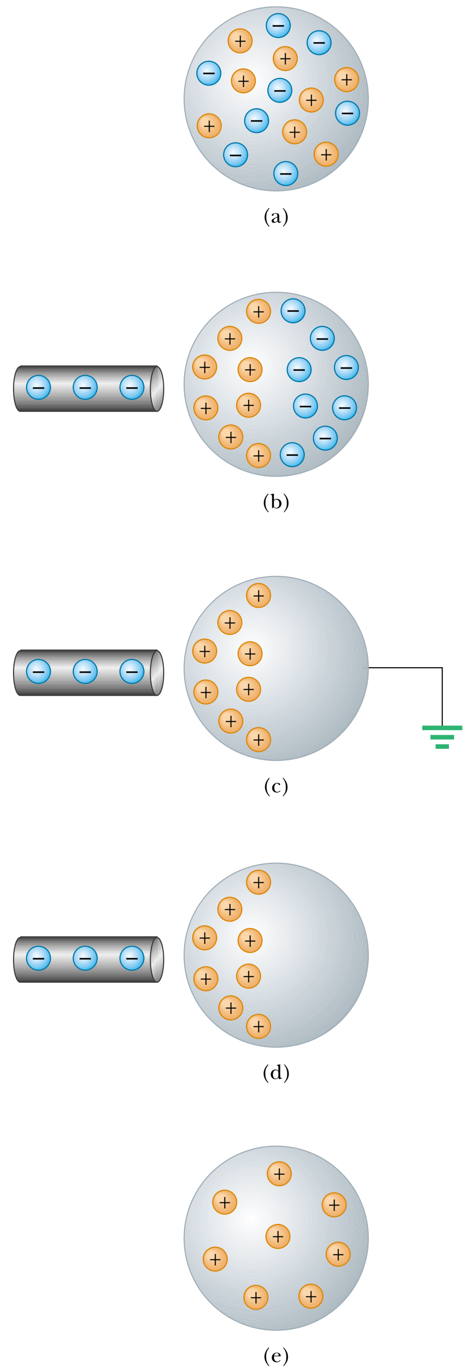
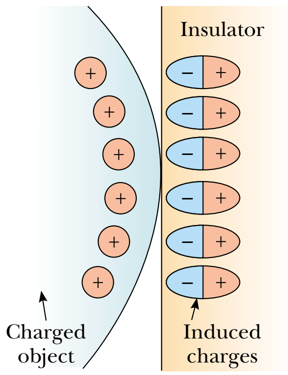


Figure 23.3 Charging a metallic object by *induction* (that is, the two objects never touch each other). (a) A neutral metallic sphere, with equal numbers of positive and negative charges. (b) The charge on the neutral sphere is redistributed when a charged rubber rod is placed near the sphere. (c) When the sphere is grounded, some of its electrons leave through the ground wire. (d) When the ground connection is removed, the sphere has excess positive charge that is nonuniformly distributed. (e) When the rod is removed, the excess positive charge becomes uniformly distributed over the surface of the sphere.



(a)

(b)

Figure 23.4 (a) The charged object on the left induces charges on the surface of an insulator. (b) A charged comb attracts bits of paper because charges are displaced in the paper.

the negative charge in the rod that they move out of the sphere through the ground wire and into the Earth. If the wire to ground is then removed (Fig. 23.3d), the conducting sphere contains an excess of *induced* positive charge. When the rubber rod is removed from the vicinity of the sphere (Fig. 23.3e), this induced positive charge remains on the ungrounded sphere. Note that the charge remaining on the sphere is uniformly distributed over its surface because of the repulsive forces among the like charges. Also note that the rubber rod loses none of its negative charge during this process.

Charging an object by induction requires no contact with the body inducing the charge. This is in contrast to charging an object by rubbing (that is, by *conduction*), which does require contact between the two objects.

A process similar to induction in conductors takes place in insulators. In most neutral molecules, the center of positive charge coincides with the center of negative charge. However, in the presence of a charged object, these centers inside each molecule in an insulator may shift slightly, resulting in more positive charge on one side of the molecule than on the other. This realignment of charge within individual molecules produces an induced charge on the surface of the insulator, as shown in Figure 23.4. Knowing about induction in insulators, you should be able to explain why a comb that has been rubbed through hair attracts bits of electrically neutral paper and why a balloon that has been rubbed against your clothing is able to stick to an electrically neutral wall.

QuickLab

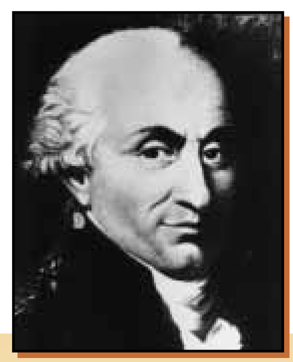
Tear some paper into very small pieces. Comb your hair and then bring the comb close to the paper pieces. Notice that they are accelerated toward the comb. How does the magnitude of the electric force compare with the magnitude of the gravitational force exerted on the paper? Keep watching and you might see a few pieces jump away from the comb. They don't just fall away; they are repelled. What causes this?

Quick Quiz 23.2

Object A is attracted to object B. If object B is known to be positively charged, what can we say about object A? (a) It is positively charged. (b) It is negatively charged. (c) It is electrically neutral. (d) Not enough information to answer.

23.3 COULOMB'S LAW

11.4 Charles Coulomb (1736–1806) measured the magnitudes of the electric forces between charged objects using the torsion balance, which he invented (Fig. 23.5).



Charles Coulomb **(1736–1806)**
Coulomb's major contribution to science was in the field of electrostatics and magnetism. During his lifetime, he also investigated the strengths of materials and determined the forces that affect objects on beams, thereby contributing to the field of structural mechanics. In the field of ergonomics, his research provided a fundamental understanding of the ways in which people and animals can best do work. *(Photo courtesy of AIP Niels Bohr Library/E. Scott Barr Collection)*

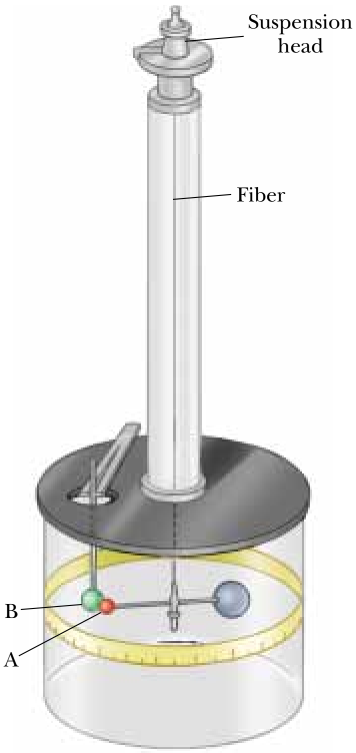


Figure 23.5 Coulomb's torsion balance, used to establish the inverse-square law for the electric force between two charges.

Coulomb confirmed that the electric force between two small charged spheres is proportional to the inverse square of their separation distance r—that is, $F_e \propto 1/r^2$. The operating principle of the torsion balance is the same as that of the apparatus used by Cavendish to measure the gravitational constant (see Section 14.2), with the electrically neutral spheres replaced by charged ones. The electric force between charged spheres A and B in Figure 23.5 causes the spheres to either attract or repel each other, and the resulting motion causes the suspended fiber to twist. Because the restoring torque of the twisted fiber is proportional to the angle through which the fiber rotates, a measurement of this angle provides a quantitative measure of the electric force of attraction or repulsion. Once the spheres are charged by rubbing, the electric force between them is very large compared with the gravitational attraction, and so the gravitational force can be neglected.

Coulomb's experiments showed that the **electric force** between two stationary charged particles

- is inversely proportional to the square of the separation r between the particles and directed along the line joining them;
- is proportional to the product of the charges q_1 and q_2 on the two particles;
- is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

From these observations, we can express **Coulomb's law** as an equation giving the magnitude of the electric force (sometimes called the *Coulomb force*) between two point charges:

$$F_e = k_e \frac{|q_1||q_2|}{r^2} \tag{23.1}$$

where k_e is a constant called the **Coulomb constant.** In his experiments, Coulomb was able to show that the value of the exponent of r was 2 to within an uncertainty of a few percent. Modern experiments have shown that the exponent is 2 to within an uncertainty of a few parts in 10^{16}.

The value of the Coulomb constant depends on the choice of units. The SI unit of charge is the **coulomb** (C). The Coulomb constant k_e in SI units has the value

Coulomb constant

$$k_e = 8.987\,5 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$$

This constant is also written in the form

$$k_e = \frac{1}{4\pi\epsilon_0}$$

where the constant ϵ_0 (lowercase Greek epsilon) is known as the *permittivity of free space* and has the value $8.854\,2 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$.

The smallest unit of charge known in nature is the charge on an electron or proton,[1] which has an absolute value of

Charge on an electron or proton

$$|e| = 1.602\,19 \times 10^{-19}\ \text{C}$$

Therefore, 1 C of charge is approximately equal to the charge of 6.24×10^{18} electrons or protons. This number is very small when compared with the number of

[1] No unit of charge smaller than e has been detected as a free charge; however, recent theories propose the existence of particles called *quarks* having charges $e/3$ and $2e/3$. Although there is considerable experimental evidence for such particles inside nuclear matter, *free* quarks have never been detected. We discuss other properties of quarks in Chapter 46 of the extended version of this text.

TABLE 23.1 Charge and Mass of the Electron, Proton, and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,191\,7 \times 10^{-19}$	$9.109\,5 \times 10^{-31}$
Proton (p)	$+1.602\,191\,7 \times 10^{-19}$	$1.672\,61 \times 10^{-27}$
Neutron (n)	0	$1.674\,92 \times 10^{-27}$

free electrons[2] in 1 cm^3 of copper, which is of the order of 10^{23}. Still, 1 C is a substantial amount of charge. In typical experiments in which a rubber or glass rod is charged by friction, a net charge of the order of 10^{-6} C is obtained. In other words, only a very small fraction of the total available charge is transferred between the rod and the rubbing material.

The charges and masses of the electron, proton, and neutron are given in Table 23.1.

EXAMPLE 23.1 The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Solution From Coulomb's law, we find that the attractive electric force has the magnitude

$$F_e = k_e \frac{|e|^2}{r^2} = \left(8.99 \times 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(1.60 \times 10^{-19}\ \text{C})^2}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= \boxed{8.2 \times 10^{-8}\ \text{N}}$$

Using Newton's law of gravitation and Table 23.1 for the particle masses, we find that the gravitational force has the magnitude

$$F_g = G\frac{m_e m_p}{r^2}$$

$$= \left(6.7 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)$$

$$= \times \frac{(9.11 \times 10^{-31}\ \text{kg})(1.67 \times 10^{-27}\ \text{kg})}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= \boxed{3.6 \times 10^{-47}\ \text{N}}$$

The ratio $F_e/F_g \approx 2 \times 10^{39}$. Thus, the gravitational force between charged atomic particles is negligible when compared with the electric force. Note the similarity of form of Newton's law of gravitation and Coulomb's law of electric forces. Other than magnitude, what is a fundamental difference between the two forces?

When dealing with Coulomb's law, you must remember that force is a vector quantity and must be treated accordingly. Thus, the law expressed in vector form for the electric force exerted by a charge q_1 on a second charge q_2, written $\mathbf{F}_{12}$, is

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2}\,\hat{\mathbf{r}} \qquad \textbf{(23.2)}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from q_1 to q_2, as shown in Figure 23.6a. Because the electric force obeys Newton's third law, the electric force exerted by q_2 on q_1 is

[2] A metal atom, such as copper, contains one or more outer electrons, which are weakly bound to the nucleus. When many atoms combine to form a metal, the so-called *free electrons* are these outer electrons, which are not bound to any one atom. These electrons move about the metal in a manner similar to that of gas molecules moving in a container.

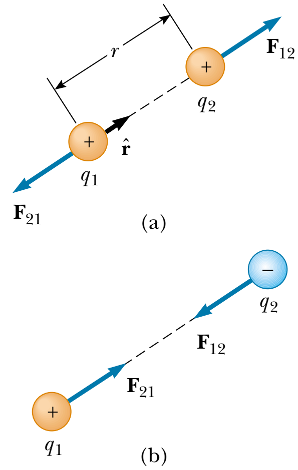


Figure 23.6 Two point charges separated by a distance r exert a force on each other that is given by Coulomb's law. The force $\mathbf{F}_{21}$ exerted by q_2 on q_1 is equal in magnitude and opposite in direction to the force $\mathbf{F}_{12}$ exerted by q_1 on q_2. (a) When the charges are of the same sign, the force is repulsive. (b) When the charges are of opposite signs, the force is attractive.

equal in magnitude to the force exerted by q_1 on q_2 and in the opposite direction; that is, $\mathbf{F}_{21} = -\mathbf{F}_{12}$. Finally, from Equation 23.2, we see that if q_1 and q_2 have the same sign, as in Figure 23.6a, the product q_1q_2 is positive and the force is repulsive. If q_1 and q_2 are of opposite sign, as shown in Figure 23.6b, the product q_1q_2 is negative and the force is attractive. Noting the sign of the product q_1q_2 is an easy way of determining the direction of forces acting on the charges.

Quick Quiz 23.3

Object A has a charge of $+2\ \mu$C, and object B has a charge of $+6\ \mu$C. Which statement is true?

(a) $\mathbf{F}_{AB} = -3\mathbf{F}_{BA}$. (b) $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$. (c) $3\mathbf{F}_{AB} = -\mathbf{F}_{BA}$.

When more than two charges are present, the force between any pair of them is given by Equation 23.2. Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the various individual charges. For example, if four charges are present, then the resultant force exerted by particles 2, 3, and 4 on particle 1 is

$$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}$$

EXAMPLE 23.2 Find the Resultant Force

Consider three point charges located at the corners of a right triangle as shown in Figure 23.7, where $q_1 = q_3 = 5.0\ \mu$C, $q_2 = -2.0\ \mu$C, and $a = 0.10$ m. Find the resultant force exerted on q_3.

Solution First, note the direction of the individual forces exerted by q_1 and q_2 on q_3. The force F_{23} exerted by q_2 on q_3 is attractive because q_2 and q_3 have opposite signs. The force F_{13} exerted by q_1 on q_3 is repulsive because both charges are positive.

The magnitude of $\mathbf{F}_{23}$ is

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(2.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{(0.10\ \text{m})^2}$$

$$= 9.0\ \text{N}$$

Note that because q_3 and q_2 have opposite signs, $\mathbf{F}_{23}$ is to the left, as shown in Figure 23.7.

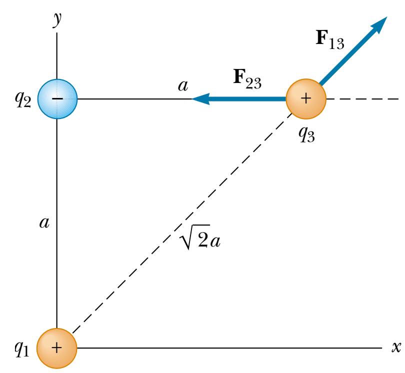


Figure 23.7 The force exerted by q_1 on q_3 is $\mathbf{F}_{13}$. The force exerted by q_2 on q_3 is $\mathbf{F}_{23}$. The resultant force $\mathbf{F}_3$ exerted on q_3 is the vector sum $\mathbf{F}_{13} + \mathbf{F}_{23}$.

The magnitude of the force exerted by q_1 on q_3 is

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

$$= \left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right) \frac{(5.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})}{2(0.10\ \mathrm{m})^2}$$

$$= 11\ \mathrm{N}$$

The force $\mathbf{F}_{13}$ is repulsive and makes an angle of 45° with the x axis. Therefore, the x and y components of $\mathbf{F}_{13}$ are equal, with magnitude given by $F_{13} \cos 45° = 7.9$ N.

The force $\mathbf{F}_{23}$ is in the negative x direction. Hence, the x and y components of the resultant force acting on q_3 are

$$F_{3x} = F_{13x} + F_{23} = 7.9\ \mathrm{N} - 9.0\ \mathrm{N} = -1.1\ \mathrm{N}$$

$$F_{3y} = F_{13y} = 7.9\ \mathrm{N}$$

We can also express the resultant force acting on q_3 in unit-vector form as

$$\mathbf{F}_3 = \boxed{(-1.1\mathbf{i} + 7.9\mathbf{j})\ \mathrm{N}}$$

Exercise Find the magnitude and direction of the resultant force $\mathbf{F}_3$.

Answer 8.0 N at an angle of 98° with the x axis.

EXAMPLE 23.3 Where Is the Resultant Force Zero?

Three point charges lie along the x axis as shown in Figure 23.8. The positive charge $q_1 = 15.0\ \mu$C is at $x = 2.00$ m, the positive charge $q_2 = 6.00\ \mu$C is at the origin, and the resultant force acting on q_3 is zero. What is the x coordinate of q_3?

Solution Because q_3 is negative and q_1 and q_2 are positive, the forces $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ are both attractive, as indicated in Figure 23.8. From Coulomb's law, $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ have magnitudes

$$F_{13} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2} \qquad F_{23} = k_e \frac{|q_2||q_3|}{x^2}$$

For the resultant force on q_3 to be zero, $\mathbf{F}_{23}$ must be equal in magnitude and opposite in direction to $\mathbf{F}_{13}$, or

$$k_e \frac{|q_2||q_3|}{x^2} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2}$$

Noting that k_e and q_3 are common to both sides and so can be dropped, we solve for x and find that

$$(2.00 - x)^2 |q_2| = x^2 |q_1|$$

$$(4.00 - 4.00x + x^2)(6.00 \times 10^{-6}\ \mathrm{C}) = x^2(15.0 \times 10^{-6}\ \mathrm{C})$$

Solving this quadratic equation for x, we find that $\boxed{x = 0.775\ \mathrm{m.}}$ Why is the negative root not acceptable?

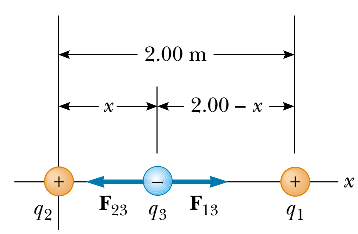


Figure 23.8 Three point charges are placed along the x axis. If the net force acting on q_3 is zero, then the force $\mathbf{F}_{13}$ exerted by q_1 on q_3 must be equal in magnitude and opposite in direction to the force $\mathbf{F}_{23}$ exerted by q_2 on q_3.

EXAMPLE 23.4 Find the Charge on the Spheres

Two identical small charged spheres, each having a mass of 3.0×10^{-2} kg, hang in equilibrium as shown in Figure 23.9a. The length of each string is 0.15 m, and the angle θ is 5.0°. Find the magnitude of the charge on each sphere.

Solution From the right triangle shown in Figure 23.9a, we see that $\sin\theta = a/L$. Therefore,

$$a = L \sin\theta = (0.15\ \mathrm{m}) \sin 5.0° = 0.013\ \mathrm{m}$$

The separation of the spheres is $2a = 0.026$ m.

The forces acting on the left sphere are shown in Figure 23.9b. Because the sphere is in equilibrium, the forces in the horizontal and vertical directions must separately add up to zero:

$$(1) \quad \sum F_x = T\sin\theta - F_e = 0$$

$$(2) \quad \sum F_y = T\cos\theta - mg = 0$$

From Equation (2), we see that $T = mg/\cos\theta$; thus, T can be eliminated from Equation (1) if we make this substitution. This gives a value for the magnitude of the electric force F_e:

$$(3) \quad F_e = mg\tan\theta$$
$$= (3.0 \times 10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)\tan 5.0°$$
$$= 2.6 \times 10^{-2}\ \text{N}$$

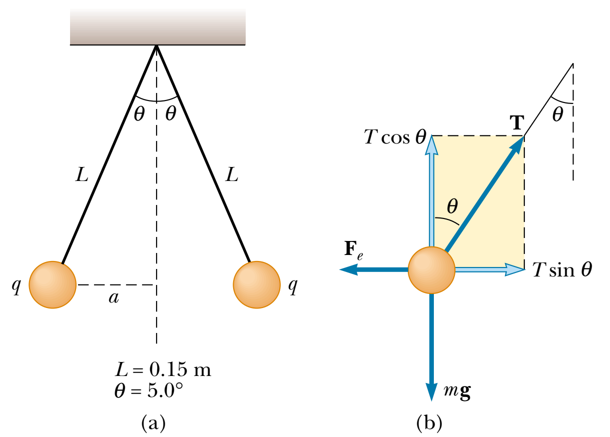


Figure 23.9 (a) Two identical spheres, each carrying the same charge q, suspended in equilibrium. (b) The free-body diagram for the sphere on the left.

From Coulomb's law (Eq. 23.1), the magnitude of the electric force is

$$F_e = k_e \frac{|q|^2}{r^2}$$

where $r = 2a = 0.026$ m and $|q|$ is the magnitude of the charge on each sphere. (Note that the term $|q|^2$ arises here because the charge is the same on both spheres.) This equation can be solved for $|q|^2$ to give

$$|q|^2 = \frac{F_e r^2}{k_e} = \frac{(2.6 \times 10^{-2}\ \text{N})(0.026\ \text{m})^2}{8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}$$

$$|q| = \boxed{4.4 \times 10^{-8}\ \text{C}}$$

Exercise If the charge on the spheres were negative, how many electrons would have to be added to them to yield a net charge of -4.4×10^{-8} C?

Answer 2.7×10^{11} electrons.

QuickLab

For this experiment you need two 20-cm strips of transparent tape (mass of each ≈ 65 mg). Fold about 1 cm of tape over at one end of each strip to create a handle. Press both pieces of tape side by side onto a table top, rubbing your finger back and forth across the strips. Quickly pull the strips off the surface so that they become charged. Hold the tape handles together and the strips will repel each other, forming an inverted "V" shape. Measure the angle between the pieces, and estimate the excess charge on each strip. Assume that the charges act as if they were located at the center of mass of each strip.

23.4 THE ELECTRIC FIELD

11.5 Two field forces have been introduced into our discussions so far—the gravitational force and the electric force. As pointed out earlier, field forces can act through space, producing an effect even when no physical contact between the objects occurs. The gravitational field $\mathbf{g}$ at a point in space was defined in Section 14.6 to be equal to the gravitational force $\mathbf{F}_g$ acting on a test particle of mass m divided by that mass: $\mathbf{g} \equiv \mathbf{F}_g/m$. A similar approach to electric forces was developed by Michael Faraday and is of such practical value that we shall devote much attention to it in the next several chapters. In this approach, an **electric field** is said to exist in the region of space around a charged object. When another charged object enters this electric field, an electric force acts on it. As an example, consider Figure 23.10, which shows a small positive test charge q_0 placed near a second object carrying a much greater positive charge Q. We define the strength (in other words, the magnitude) of the electric field at the location of the test charge to be the electric force *per unit charge*, or to be more specific

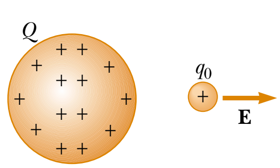


Figure 23.10 A small positive test charge q_0 placed near an object carrying a much larger positive charge Q experiences an electric field $\mathbf{E}$ directed as shown.

the electric field E at a point in space is defined as the electric force $\mathbf{F}_e$ acting on a positive test charge q_0 placed at that point divided by the magnitude of the test charge:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0} \tag{23.3}$$

Definition of electric field

Note that **E** is the field produced by some charge *external* to the test charge—it is not the field produced by the test charge itself. Also, note that the existence of an electric field is a property of its source. For example, every electron comes with its own electric field.

The vector **E** has the SI units of newtons per coulomb (N/C), and, as Figure 23.10 shows, its direction is the direction of the force a positive test charge experiences when placed in the field. We say that **an electric field exists at a point if a test charge at rest at that point experiences an electric force.** Once the magnitude and direction of the electric field are known at some point, the electric force exerted on *any* charged particle placed at that point can be calculated from

This dramatic photograph captures a lightning bolt striking a tree near some rural homes.

TABLE 23.2 Typical Electric Field Values

Source	E (N/C)
Fluorescent lighting tube	10
Atmosphere (fair weather)	100
Balloon rubbed on hair	1 000
Atmosphere (under thundercloud)	10 000
Photocopier	100 000
Spark in air	$>$3 000 000
Near electron in hydrogen atom	5×10^{11}

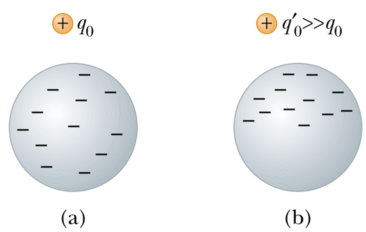


Figure 23.11 (a) For a small enough test charge q_0, the charge distribution on the sphere is undisturbed. (b) When the test charge q_0' is greater, the charge distribution on the sphere is disturbed as the result of the proximity of q_0'.

Equation 23.3. Furthermore, the electric field is said to exist at some point (even empty space) **regardless of whether a test charge is located at that point.** (This is analogous to the gravitational field set up by any object, which is said to exist at a given point regardless of whether some other object is present at that point to "feel" the field.) The electric field magnitudes for various field sources are given in Table 23.2.

When using Equation 23.3, we must assume that the test charge q_0 is small enough that it does not disturb the charge distribution responsible for the electric field. If a vanishingly small test charge q_0 is placed near a uniformly charged metallic sphere, as shown in Figure 23.11a, the charge on the metallic sphere, which produces the electric field, remains uniformly distributed. If the test charge is great enough ($q_0' \gg q_0$), as shown in Figure 23.11b, the charge on the metallic sphere is redistributed and the ratio of the force to the test charge is different: ($F_e'/q_0' \neq F_e/q_0$). That is, because of this redistribution of charge on the metallic sphere, the electric field it sets up is different from the field it sets up in the presence of the much smaller q_0.

To determine the direction of an electric field, consider a point charge q located a distance r from a test charge q_0 located at a point P, as shown in Figure 23.12. According to Coulomb's law, the force exerted by q on the test charge is

$$\mathbf{F}_e = k_e \frac{q q_0}{r^2}\, \hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from q toward q_0. Because the electric field at P, the position of the test charge, is defined by $\mathbf{E} = \mathbf{F}_e/q_0$, we find that at P, the electric field created by q is

$$\mathbf{E} = k_e \frac{q}{r^2}\, \hat{\mathbf{r}} \qquad \textbf{(23.4)}$$

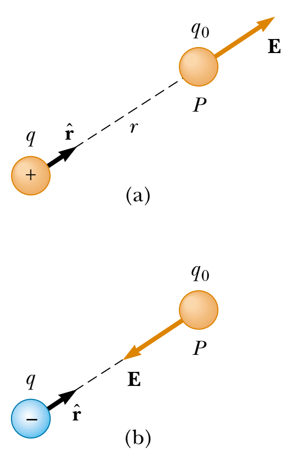


Figure 23.12 A test charge q_0 at point P is a distance r from a point charge q. (a) If q is positive, then the electric field at P points radially outward from q. (b) If q is negative, then the electric field at P points radially inward toward q.

If q is positive, as it is in Figure 23.12a, the electric field is directed radially outward from it. If q is negative, as it is in Figure 23.12b, the field is directed toward it.

To calculate the electric field at a point P due to a group of point charges, we first calculate the electric field vectors at P individually using Equation 23.4 and then add them vectorially. In other words,

at any point P, the total electric field due to a group of charges equals the vector sum of the electric fields of the individual charges.

This superposition principle applied to fields follows directly from the superposition property of electric forces. Thus, the electric field of a group of charges can

This metallic sphere is charged by a generator so that it carries a net electric charge. The high concentration of charge on the sphere creates a strong electric field around the sphere. The charges then leak through the gas surrounding the sphere, producing a pink glow.

be expressed as

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i \tag{23.5}$$

where r_i is the distance from the ith charge q_i to the point P (the location of the test charge) and $\hat{\mathbf{r}}_i$ is a unit vector directed from q_i toward P.

Quick Quiz 23.4

A charge of $+3\ \mu$C is at a point P where the electric field is directed to the right and has a magnitude of 4×10^6 N/C. If the charge is replaced with a -3-μC charge, what happens to the electric field at P?

EXAMPLE 23.5 Electric Field Due to Two Charges

A charge $q_1 = 7.0\ \mu$C is located at the origin, and a second charge $q_2 = -5.0\ \mu$C is located on the x axis, 0.30 m from the origin (Fig. 23.13). Find the electric field at the point P, which has coordinates (0, 0.40) m.

Solution First, let us find the magnitude of the electric field at P due to each charge. The fields $\mathbf{E}_1$ due to the 7.0-μC charge and $\mathbf{E}_2$ due to the -5.0-μC charge are shown in Figure 23.13. Their magnitudes are

$$E_1 = k_e\frac{|q_1|}{r_1^2} = \left(8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(7.0 \times 10^{-6}\ \text{C})}{(0.40\ \text{m})^2}$$
$$= 3.9 \times 10^5\ \text{N/C}$$

$$E_2 = k_e\frac{|q_2|}{r_2^2} = \left(8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(5.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2}$$
$$= 1.8 \times 10^5\ \text{N/C}$$

The vector $\mathbf{E}_1$ has only a y component. The vector $\mathbf{E}_2$ has an x component given by $E_2 \cos\theta = \frac{3}{5}E_2$ and a negative y component given by $-E_2 \sin\theta = -\frac{4}{5}E_2$. Hence, we can express the vectors as

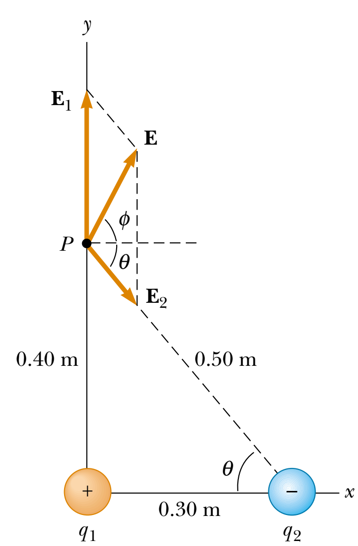


Figure 23.13 The total electric field $\mathbf{E}$ at P equals the vector sum $\mathbf{E}_1 + \mathbf{E}_2$, where $\mathbf{E}_1$ is the field due to the positive charge q_1 and $\mathbf{E}_2$ is the field due to the negative charge q_2.

$$\mathbf{E}_1 = 3.9 \times 10^5 \mathbf{j} \text{ N/C}$$

$$\mathbf{E}_2 = (1.1 \times 10^5 \mathbf{i} - 1.4 \times 10^5 \mathbf{j}) \text{ N/C}$$

The resultant field $\mathbf{E}$ at P is the superposition of $\mathbf{E}_1$ and $\mathbf{E}_2$:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \boxed{(1.1 \times 10^5 \mathbf{i} + 2.5 \times 10^5 \mathbf{j}) \text{ N/C}}$$

From this result, we find that $\mathbf{E}$ has a magnitude of 2.7×10^5 N/C and makes an angle ϕ of 66° with the positive x axis.

Exercise Find the electric force exerted on a charge of 2.0×10^{-8} C located at P.

Answer 5.4×10^{-3} N in the same direction as $\mathbf{E}$.

EXAMPLE 23.6 Electric Field of a Dipole

An **electric dipole** is defined as a positive charge q and a negative charge $-q$ separated by some distance. For the dipole shown in Figure 23.14, find the electric field $\mathbf{E}$ at P due to the charges, where P is a distance $y \gg a$ from the origin.

Solution At P, the fields $\mathbf{E}_1$ and $\mathbf{E}_2$ due to the two charges are equal in magnitude because P is equidistant from the charges. The total field is $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$, where

$$E_1 = E_2 = k_e \frac{q}{r^2} = k_e \frac{q}{y^2 + a^2}$$

The y components of $\mathbf{E}_1$ and $\mathbf{E}_2$ cancel each other, and the x components add because they are both in the positive x direction. Therefore, $\mathbf{E}$ is parallel to the x axis and has a magnitude equal to $2E_1 \cos\theta$. From Figure 23.14 we see that $\cos\theta = a/r = a/(y^2 + a^2)^{1/2}$. Therefore,

$$E = 2E_1 \cos\theta = 2k_e \frac{q}{(y^2 + a^2)} \frac{a}{(y^2 + a^2)^{1/2}}$$

$$= k_e \frac{2qa}{(y^2 + a^2)^{3/2}}$$

Because $y \gg a$, we can neglect a^2 and write

$$\boxed{E \approx k_e \frac{2qa}{y^3}}$$

Thus, we see that, at distances far from a dipole but along the perpendicular bisector of the line joining the two charges, the magnitude of the electric field created by the dipole varies as $1/r^3$, whereas the more slowly varying field of a point charge varies as $1/r^2$ (see Eq. 23.4). This is because at distant points, the fields of the two charges of equal magnitude and opposite sign almost cancel each other. The $1/r^3$ variation in E for the dipole also is obtained for a distant point along the x axis (see Problem 21) and for any general distant point.

The electric dipole is a good model of many molecules, such as hydrochloric acid (HCl). As we shall see in later chapters, neutral atoms and molecules behave as dipoles when placed in an external electric field. Furthermore, many molecules, such as HCl, are permanent dipoles. The effect of such dipoles on the behavior of materials subjected to electric fields is discussed in Chapter 26.

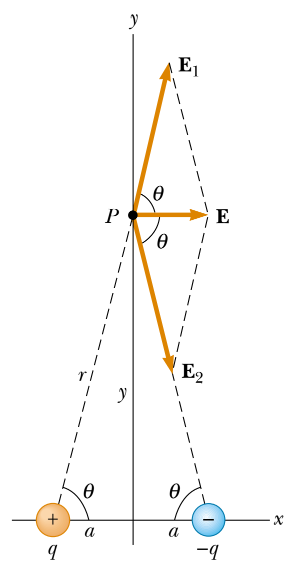


Figure 23.14 The total electric field $\mathbf{E}$ at P due to two charges of equal magnitude and opposite sign (an electric dipole) equals the vector sum $\mathbf{E}_1 + \mathbf{E}_2$. The field $\mathbf{E}_1$ is due to the positive charge q, and $\mathbf{E}_2$ is the field due to the negative charge $-q$.

23.5 ELECTRIC FIELD OF A CONTINUOUS CHARGE DISTRIBUTION

Very often the distances between charges in a group of charges are much smaller than the distance from the group to some point of interest (for example, a point where the electric field is to be calculated). In such situations, the system of charges is smeared out, or *continuous.* That is, the system of closely spaced charges is equivalent to a total charge that is continuously distributed along some line, over some surface, or throughout some volume.

A continuous charge distribution

To evaluate the electric field created by a continuous charge distribution, we use the following procedure: First, we divide the charge distribution into small elements, each of which contains a small charge Δq, as shown in Figure 23.15. Next, we use Equation 23.4 to calculate the electric field due to one of these elements at a point P. Finally, we evaluate the total field at P due to the charge distribution by summing the contributions of all the charge elements (that is, by applying the superposition principle).

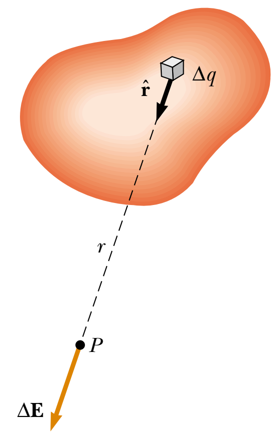


Figure 23.15 The electric field at P due to a continuous charge distribution is the vector sum of the fields $\Delta\mathbf{E}$ due to all the elements Δq of the charge distribution.

The electric field at P due to one element carrying charge Δq is

$$\Delta\mathbf{E} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

where r is the distance from the element to point P and $\hat{\mathbf{r}}$ is a unit vector directed from the charge element toward P. The total electric field at P due to all elements in the charge distribution is approximately

$$\mathbf{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i$$

where the index i refers to the ith element in the distribution. Because the charge distribution is approximately continuous, the total field at P in the limit $\Delta q_i \rightarrow 0$ is

$$\mathbf{E} = k_e \lim_{\Delta q_i \rightarrow 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}} \qquad \textbf{(23.6)}$$

Electric field of a continuous charge distribution

where the integration is over the entire charge distribution. This is a vector operation and must be treated appropriately.

We illustrate this type of calculation with several examples, in which we assume the charge is uniformly distributed on a line, on a surface, or throughout a volume. When performing such calculations, it is convenient to use the concept of a charge density along with the following notations:

- If a charge Q is uniformly distributed throughout a volume V, the **volume charge density** ρ is defined by

  Volume charge density

  $$\rho \equiv \frac{Q}{V}$$

  where ρ has units of coulombs per cubic meter (C/m^3).

- If a charge Q is uniformly distributed on a surface of area A, the **surface charge density** σ (lowercase Greek sigma) is defined by

  Surface charge density

  $$\sigma \equiv \frac{Q}{A}$$

  where σ has units of coulombs per square meter (C/m^2).

- If a charge Q is uniformly distributed along a line of length ℓ, the **linear charge density** λ is defined by

  Linear charge density

  $$\lambda \equiv \frac{Q}{\ell}$$

  where λ has units of coulombs per meter (C/m).

- If the charge is nonuniformly distributed over a volume, surface, or line, we have to express the charge densities as

$$\rho = \frac{dQ}{dV} \qquad \sigma = \frac{dQ}{dA} \qquad \lambda = \frac{dQ}{d\ell}$$

where dQ is the amount of charge in a small volume, surface, or length element.

EXAMPLE 23.7 The Electric Field Due to a Charged Rod

A rod of length ℓ has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end (Fig. 23.16).

Solution Let us assume that the rod is lying along the x axis, that dx is the length of one small segment, and that dq is the charge on that segment. Because the rod has a charge per unit length λ, the charge dq on the small segment is $dq = \lambda\, dx$.

The field $d\mathbf{E}$ due to this segment at P is in the negative x direction (because the source of the field carries a positive charge Q), and its magnitude is

$$dE = k_e \frac{dq}{x^2} = k_e\lambda \frac{dx}{x^2}$$

Because every other element also produces a field in the negative x direction, the problem of summing their contributions is particularly simple in this case. The total field at P due to all segments of the rod, which are at different distances from P, is given by Equation 23.6, which in this case becomes[3]

$$E = \int_a^{\ell+a} k_e\lambda \frac{dx}{x^2}$$

where the limits on the integral extend from one end of the rod ($x = a$) to the other ($x = \ell + a$). The constants k_e and λ can be removed from the integral to yield

$$E = k_e\lambda \int_a^{\ell+a} \frac{dx}{x^2} = k_e\lambda\left[-\frac{1}{x}\right]_a^{\ell+a}$$

$$= k_e\lambda\left(\frac{1}{a} - \frac{1}{\ell + a}\right) = \boxed{\frac{k_eQ}{a(\ell + a)}}$$

where we have used the fact that the total charge $Q = \lambda\ell$.

If P is far from the rod ($a \gg \ell$), then the ℓ in the denominator can be neglected, and $E \approx k_eQ/a^2$. This is just the form you would expect for a point charge. Therefore, at large values of a/ℓ, the charge distribution appears to be a point charge of magnitude Q. The use of the limiting technique ($a/\ell \rightarrow \infty$) often is a good method for checking a theoretical formula.

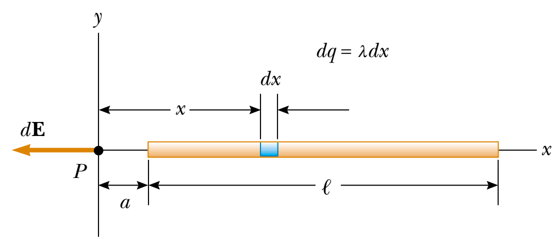


Figure 23.16 The electric field at P due to a uniformly charged rod lying along the x axis. The magnitude of the field at P due to the segment of charge dq is $k_e dq/x^2$. The total field at P is the vector sum over all segments of the rod.

EXAMPLE 23.8 The Electric Field of a Uniform Ring of Charge

A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring (Fig. 23.17a).

Solution The magnitude of the electric field at P due to the segment of charge dq is

$$dE = k_e \frac{dq}{r^2}$$

This field has an x component $dE_x = dE \cos\theta$ along the axis and a component $dE_\perp$ perpendicular to the axis. As we see in Figure 23.17b, however, the resultant field at P must lie along the x axis because the perpendicular components of all the

[3] It is important that you understand how to carry out integrations such as this. First, express the charge element dq in terms of the other variables in the integral (in this example, there is one variable, x, and so we made the change $dq = \lambda\, dx$). The integral must be over scalar quantities; therefore, you must express the electric field in terms of components, if necessary. (In this example the field has only an x component, so we do not bother with this detail.) Then, reduce your expression to an integral over a single variable (or to multiple integrals, each over a single variable). In examples that have spherical or cylindrical symmetry, the single variable will be a radial coordinate.

various charge segments sum to zero. That is, the perpendicular component of the field created by any charge element is canceled by the perpendicular component created by an element on the opposite side of the ring. Because $r = (x^2 + a^2)^{1/2}$ and $\cos\theta = x/r$, we find that

$$dE_x = dE\cos\theta = \left(k_e\frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2 + a^2)^{3/2}}\,dq$$

All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Thus, we can integrate to obtain the total field at P:

$$E_x = \int \frac{k_e x}{(x^2 + a^2)^{3/2}}\,dq = \frac{k_e x}{(x^2 + a^2)^{3/2}}\int dq$$

$$= \boxed{\frac{k_e x}{(x^2 + a^2)^{3/2}}\,Q}$$

This result shows that the field is zero at $x = 0$. Does this finding surprise you?

Exercise Show that at great distances from the ring ($x \gg a$) the electric field along the axis shown in Figure 23.17 approaches that of a point charge of magnitude Q.

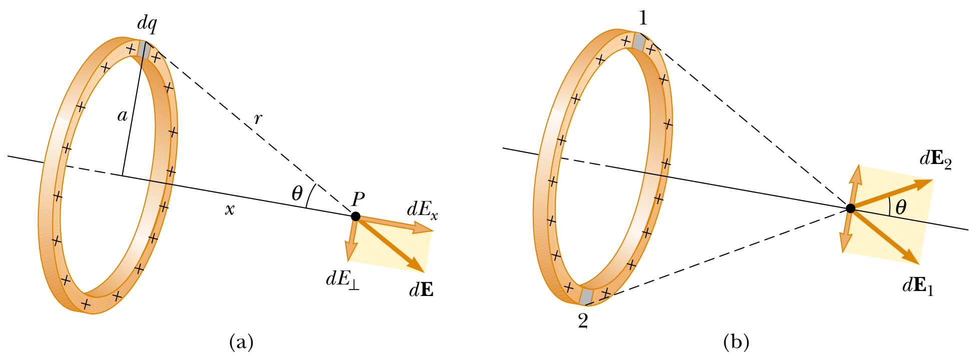


Figure 23.17 A uniformly charged ring of radius a. (a) The field at P on the x axis due to an element of charge dq. (b) The total electric field at P is along the x axis. The perpendicular component of the field at P due to segment 1 is canceled by the perpendicular component due to segment 2.

EXAMPLE 23.9 The Electric Field of a Uniformly Charged Disk

A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk (Fig. 23.18).

Solution If we consider the disk as a set of concentric rings, we can use our result from Example 23.8—which gives the field created by a ring of radius a—and sum the contributions of all rings making up the disk. By symmetry, the field at an axial point must be along the central axis.

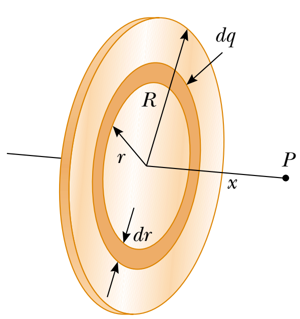


Figure 23.18 A uniformly charged disk of radius R. The electric field at an axial point P is directed along the central axis, perpendicular to the plane of the disk.

The ring of radius r and width dr shown in Figure 23.18 has a surface area equal to $2\pi r\,dr$. The charge dq on this ring is equal to the area of the ring multiplied by the surface charge density: $dq = 2\pi\sigma r\,dr$. Using this result in the equation given for E_x in Example 23.8 (with a replaced by r), we have for the field due to the ring

$$dE = \frac{k_e x}{(x^2 + r^2)^{3/2}}\,(2\pi\sigma r\,dr)$$

To obtain the total field at P, we integrate this expression over the limits $r = 0$ to $r = R$, noting that x is a constant. This gives

$$E = k_e x\pi\sigma\int_0^R \frac{2r\,dr}{(x^2 + r^2)^{3/2}}$$

$$= k_e x\pi\sigma\int_0^R (x^2 + r^2)^{-3/2}\,d(r^2)$$

$$= k_e x\pi\sigma\left[\frac{(x^2 + r^2)^{-1/2}}{-1/2}\right]_0^R$$

$$= 2\pi k_e\sigma\left(\frac{x}{|x|} - \frac{x}{(x^2 + R^2)^{1/2}}\right)$$

This result is valid for all values of x. We can calculate the field close to the disk along the axis by assuming that $R \gg x$; thus, the expression in parentheses reduces to unity:

$$E \approx 2\pi k_e \sigma = \boxed{\frac{\sigma}{2\epsilon_0}}$$

where $\epsilon_0 = 1/(4\pi k_e)$ is the permittivity of free space. As we shall find in the next chapter, we obtain the same result for the field created by a uniformly charged infinite sheet.

23.6 ELECTRIC FIELD LINES

11.5 A convenient way of visualizing electric field patterns is to draw lines that follow the same direction as the electric field vector at any point. These lines, called **electric field lines,** are related to the electric field in any region of space in the following manner:

- The electric field vector **E** is tangent to the electric field line at each point.
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, E is great when the field lines are close together and small when they are far apart.

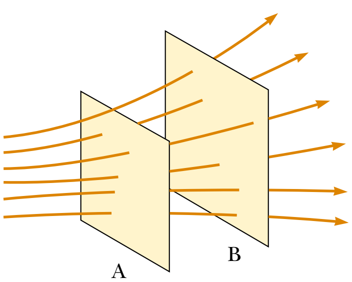


Figure 23.19 Electric field lines penetrating two surfaces. The magnitude of the field is greater on surface A than on surface B.

These properties are illustrated in Figure 23.19. The density of lines through surface A is greater than the density of lines through surface B. Therefore, the electric field is more intense on surface A than on surface B. Furthermore, the fact that the lines at different locations point in different directions indicates that the field is nonuniform.

Representative electric field lines for the field due to a single positive point charge are shown in Figure 23.20a. Note that in this two-dimensional drawing we show only the field lines that lie in the plane containing the point charge. The lines are actually directed radially outward from the charge in all directions; thus, instead of the flat "wheel" of lines shown, you should picture an entire sphere of lines. Because a positive test charge placed in this field would be repelled by the positive point charge, the lines are directed radially away from the positive point

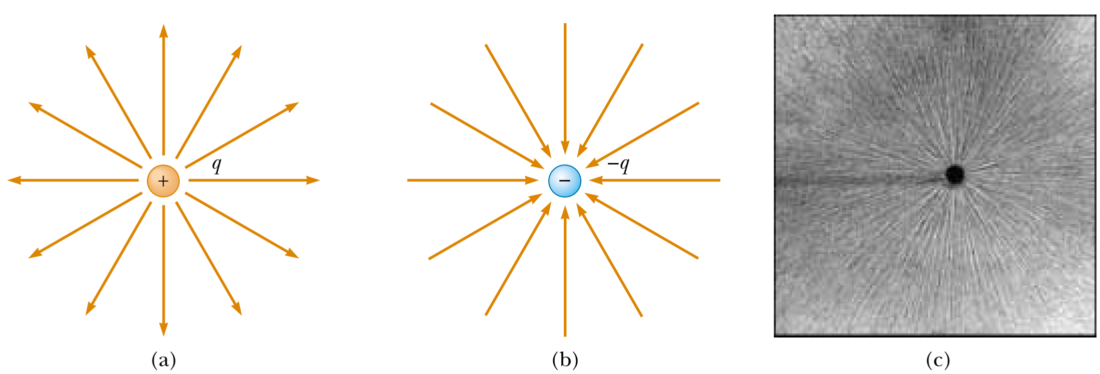


Figure 23.20 The electric field lines for a point charge. (a) For a positive point charge, the lines are directed radially outward. (b) For a negative point charge, the lines are directed radially inward. Note that the figures show only those field lines that lie in the plane containing the charge. (c) The dark areas are small pieces of thread suspended in oil, which align with the electric field produced by a small charged conductor at the center.

charge. The electric field lines representing the field due to a single negative point charge are directed toward the charge (Fig. 23.20b). In either case, the lines are along the radial direction and extend all the way to infinity. Note that the lines become closer together as they approach the charge; this indicates that the strength of the field increases as we move toward the source charge.

The rules for drawing electric field lines are as follows:

- The lines must begin on a positive charge and terminate on a negative charge.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.

Rules for drawing electric field lines

Is this visualization of the electric field in terms of field lines consistent with Equation 23.4, the expression we obtained for E using Coulomb's law? To answer this question, consider an imaginary spherical surface of radius r concentric with a point charge. From symmetry, we see that the magnitude of the electric field is the same everywhere on the surface of the sphere. The number of lines N that emerge from the charge is equal to the number that penetrate the spherical surface. Hence, the number of lines per unit area on the sphere is $N/4\pi r^2$ (where the surface area of the sphere is $4\pi r^2$). Because E is proportional to the number of lines per unit area, we see that E varies as $1/r^2$; this finding is consistent with Equation 23.4.

As we have seen, we use electric field lines to qualitatively describe the electric field. One problem with this model is that we always draw a finite number of lines from (or to) each charge. Thus, it appears as if the field acts only in certain directions; this is not true. Instead, the field is *continuous*—that is, it exists at every point. Another problem associated with this model is the danger of gaining the wrong impression from a two-dimensional drawing of field lines being used to describe a three-dimensional situation. Be aware of these shortcomings every time you either draw or look at a diagram showing electric field lines.

We choose the number of field lines starting from any positively charged object to be $C'q$ and the number of lines ending on any negatively charged object to be $C'|q|$, where C' is an arbitrary proportionality constant. Once C' is chosen, the number of lines is fixed. For example, if object 1 has charge Q_1 and object 2 has charge Q_2, then the ratio of number of lines is $N_2/N_1 = Q_2/Q_1$.

The electric field lines for two point charges of equal magnitude but opposite signs (an electric dipole) are shown in Figure 23.21. Because the charges are of equal magnitude, the number of lines that begin at the positive charge must equal the number that terminate at the negative charge. At points very near the charges, the lines are nearly radial. The high density of lines between the charges indicates a region of strong electric field.

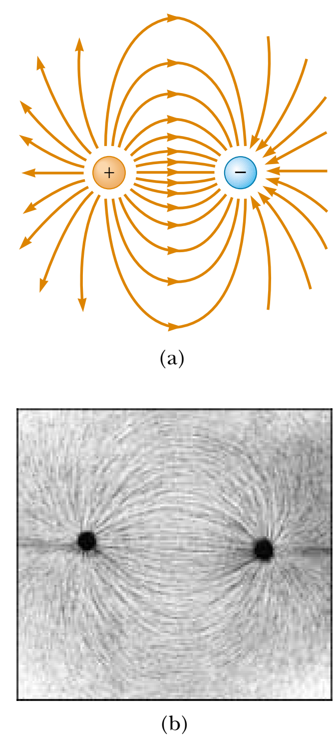


Figure 23.21 (a) The electric field lines for two point charges of equal magnitude and opposite sign (an electric dipole). The number of lines leaving the positive charge equals the number terminating at the negative charge. (b) The dark lines are small pieces of thread suspended in oil, which align with the electric field of a dipole.

Figure 23.22 shows the electric field lines in the vicinity of two equal positive point charges. Again, the lines are nearly radial at points close to either charge, and the same number of lines emerge from each charge because the charges are equal in magnitude. At great distances from the charges, the field is approximately equal to that of a single point charge of magnitude $2q$.

Finally, in Figure 23.23 we sketch the electric field lines associated with a positive charge $+2q$ and a negative charge $-q$. In this case, the number of lines leaving $+2q$ is twice the number terminating at $-q$. Hence, only half of the lines that leave the positive charge reach the negative charge. The remaining half terminate

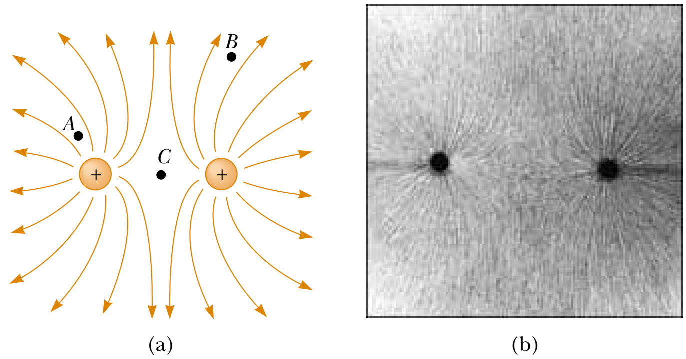


Figure 23.22 (a) The electric field lines for two positive point charges. (The locations A, B, and C are discussed in Quick Quiz 23.5.) (b) Pieces of thread suspended in oil, which align with the electric field created by two equal-magnitude positive charges.

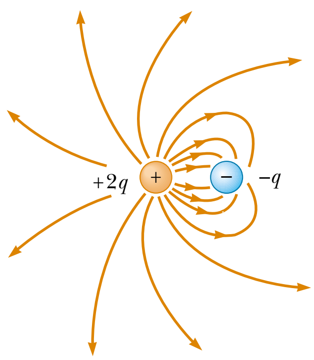


Figure 23.23 The electric field lines for a point charge $+2q$ and a second point charge $-q$. Note that two lines leave $+2q$ for every one that terminates on $-q$.

on a negative charge we assume to be at infinity. At distances that are much greater than the charge separation, the electric field lines are equivalent to those of a single charge $+q$.

Quick Quiz 23.5

Rank the magnitude of the electric field at points A, B, and C shown in Figure 23.22a (greatest magnitude first).

23.7 MOTION OF CHARGED PARTICLES IN A UNIFORM ELECTRIC FIELD

When a particle of charge q and mass m is placed in an electric field $\mathbf{E}$, the electric force exerted on the charge is $q\mathbf{E}$. If this is the only force exerted on the particle, it must be the net force and so must cause the particle to accelerate. In this case, Newton's second law applied to the particle gives

$$\mathbf{F}_e = q\mathbf{E} = m\mathbf{a}$$

The acceleration of the particle is therefore

$$\mathbf{a} = \frac{q\mathbf{E}}{m} \tag{23.7}$$

If $\mathbf{E}$ is uniform (that is, constant in magnitude and direction), then the acceleration is constant. If the particle has a positive charge, then its acceleration is in the direction of the electric field. If the particle has a negative charge, then its acceleration is in the direction opposite the electric field.

EXAMPLE 23.10 An Accelerating Positive Charge

A positive point charge q of mass m is released from rest in a uniform electric field $\mathbf{E}$ directed along the x axis, as shown in Figure 23.24. Describe its motion.

Solution The acceleration is constant and is given by $q\mathbf{E}/m$. The motion is simple linear motion along the x axis. Therefore, we can apply the equations of kinematics in one dimension (see Chapter 2):

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2$$
$$v_{xf} = v_{xi} + a_x t$$
$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Taking $x_i = 0$ and $v_{xi} = 0$, we have

$$x_f = \tfrac{1}{2}a_x t^2 = \frac{qE}{2m}t^2$$
$$v_{xf} = a_x t = \frac{qE}{m}t$$
$$v_{xf}^2 = 2a_x x_f = \left(\frac{2qE}{m}\right)x_f$$

The kinetic energy of the charge after it has moved a distance $x = x_f - x_i$ is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{2qE}{m}\right)x = qEx$$

We can also obtain this result from the work–kinetic energy theorem because the work done by the electric force is $F_e x = qEx$ and $W = \Delta K$.

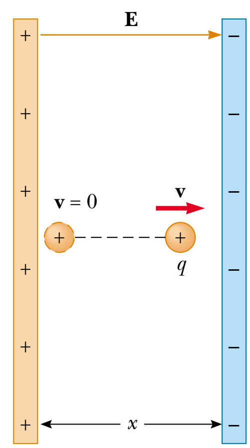


Figure 23.24 A positive point charge q in a uniform electric field **E** undergoes constant acceleration in the direction of the field.

The electric field in the region between two oppositely charged flat metallic plates is approximately uniform (Fig. 23.25). Suppose an electron of charge $-e$ is projected horizontally into this field with an initial velocity $v_i\mathbf{i}$. Because the electric field **E** in Figure 23.25 is in the positive y direction, the acceleration of the electron is in the negative y direction. That is,

$$\mathbf{a} = -\frac{eE}{m}\mathbf{j} \qquad \textbf{(23.8)}$$

Because the acceleration is constant, we can apply the equations of kinematics in two dimensions (see Chapter 4) with $v_{xi} = v_i$ and $v_{yi} = 0$. After the electron has been in the electric field for a time t, the components of its velocity are

$$v_x = v_i = \text{constant} \qquad \textbf{(23.9)}$$

$$v_y = a_y t = -\frac{eE}{m}t \qquad \textbf{(23.10)}$$

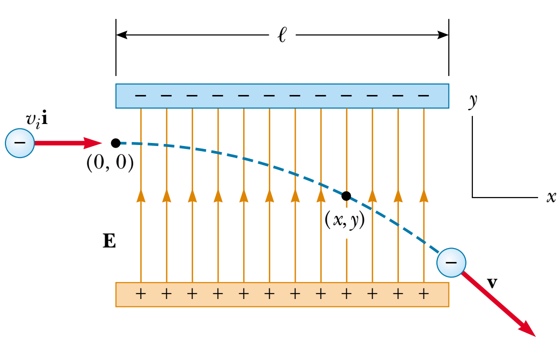


Figure 23.25 An electron is projected horizontally into a uniform electric field produced by two charged plates. The electron undergoes a downward acceleration (opposite **E**), and its motion is parabolic while it is between the plates.

Its coordinates after a time t in the field are

$$x = v_i t \tag{23.11}$$

$$y = \tfrac{1}{2} a_y t^2 = -\tfrac{1}{2} \frac{eE}{m} t^2 \tag{23.12}$$

Substituting the value $t = x/v_i$ from Equation 23.11 into Equation 23.12, we see that y is proportional to x^2. Hence, the trajectory is a parabola. After the electron leaves the field, it continues to move in a straight line in the direction of **v** in Figure 23.25, obeying Newton's first law, with a speed $v > v_i$.

Note that we have neglected the gravitational force acting on the electron. This is a good approximation when we are dealing with atomic particles. For an electric field of 10^4 N/C, the ratio of the magnitude of the electric force eE to the magnitude of the gravitational force mg is of the order of 10^{14} for an electron and of the order of 10^{11} for a proton.

EXAMPLE 23.11 An Accelerated Electron

An electron enters the region of a uniform electric field as shown in Figure 23.25, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m. (a) Find the acceleration of the electron while it is in the electric field.

Solution The charge on the electron has an absolute value of 1.60×10^{-19} C, and $m = 9.11 \times 10^{-31}$ kg. Therefore, Equation 23.8 gives

$$\mathbf{a} = -\frac{eE}{m}\mathbf{j} = -\frac{(1.60 \times 10^{-19}\ \text{C})(200\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}}\mathbf{j}$$

$$= \boxed{-3.51 \times 10^{13}\,\mathbf{j}\ \text{m/s}^2}$$

(b) Find the time it takes the electron to travel through the field.

Solution The horizontal distance across the field is $\ell = 0.100$ m. Using Equation 23.11 with $x = \ell$, we find that the time spent in the electric field is

$$t = \frac{\ell}{v_i} = \frac{0.100\ \text{m}}{3.00 \times 10^6\ \text{m/s}} = \boxed{3.33 \times 10^{-8}\ \text{s}}$$

(c) What is the vertical displacement y of the electron while it is in the field?

Solution Using Equation 23.12 and the results from parts (a) and (b), we find that

$$y = \tfrac{1}{2} a_y t^2 = \tfrac{1}{2}(-3.51 \times 10^{13}\ \text{m/s}^2)(3.33 \times 10^{-8}\ \text{s})^2$$

$$= -0.019\,5\ \text{m} = \boxed{-1.95\ \text{cm}}$$

If the separation between the plates is less than this, the electron will strike the positive plate.

Exercise Find the speed of the electron as it emerges from the field.

Answer 3.22×10^6 m/s.

The Cathode Ray Tube

The example we just worked describes a portion of a cathode ray tube (CRT). This tube, illustrated in Figure 23.26, is commonly used to obtain a visual display of electronic information in oscilloscopes, radar systems, television receivers, and computer monitors. The CRT is a vacuum tube in which a beam of electrons is accelerated and deflected under the influence of electric or magnetic fields. The electron beam is produced by an assembly called an *electron gun* located in the neck of the tube. These electrons, if left undisturbed, travel in a straight-line path until they strike the front of the CRT, the "screen," which is coated with a material that emits visible light when bombarded with electrons.

In an oscilloscope, the electrons are deflected in various directions by two sets of plates placed at right angles to each other in the neck of the tube. (A television

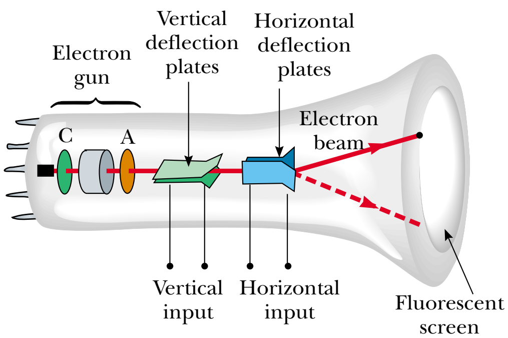


Figure 23.26 Schematic diagram of a cathode ray tube. Electrons leaving the hot cathode C are accelerated to the anode A. In addition to accelerating electrons, the electron gun is also used to focus the beam of electrons, and the plates deflect the beam.

CRT steers the beam with a magnetic field, as discussed in Chapter 29.) An external electric circuit is used to control the amount of charge present on the plates. The placing of positive charge on one horizontal plate and negative charge on the other creates an electric field between the plates and allows the beam to be steered from side to side. The vertical deflection plates act in the same way, except that changing the charge on them deflects the beam vertically.

SUMMARY

Electric charges have the following important properties:

- Unlike charges attract one another, and like charges repel one another.
- Charge is conserved.
- Charge is quantized—that is, it exists in discrete packets that are some integral multiple of the electronic charge.

Conductors are materials in which charges move freely. **Insulators** are materials in which charges do not move freely.

Coulomb's law states that the electric force exerted by a charge q_1 on a second charge q_2 is

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2}\,\hat{\mathbf{r}} \quad \textbf{(23.2)}$$

where r is the distance between the two charges and $\hat{\mathbf{r}}$ is a unit vector directed from q_1 to q_2. The constant k_e, called the Coulomb constant, has the value $k_e = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

The smallest unit of charge known to exist in nature is the charge on an electron or proton, $|e| = 1.602\,19 \times 10^{-19}$ C.

The electric field $\mathbf{E}$ at some point in space is defined as the electric force $\mathbf{F}_e$ that acts on a small positive test charge placed at that point divided by the magnitude of the test charge q_0:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0} \quad \textbf{(23.3)}$$

At a distance r from a point charge q, the electric field due to the charge is given by

$$\mathbf{E} = k_e \frac{q}{r^2}\,\hat{\mathbf{r}} \quad \textbf{(23.4)}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from the charge to the point in question. The electric field is directed radially outward from a positive charge and radially inward toward a negative charge.

The electric field due to a group of point charges can be obtained by using the superposition principle. That is, the total electric field at some point equals the vector sum of the electric fields of all the charges:

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i \qquad \textbf{(23.5)}$$

The electric field at some point of a continuous charge distribution is

$$\mathbf{E} = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}} \qquad \textbf{(23.6)}$$

where dq is the charge on one element of the charge distribution and r is the distance from the element to the point in question.

Electric field lines describe an electric field in any region of space. The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of $\mathbf{E}$ in that region.

A charged particle of mass m and charge q moving in an electric field $\mathbf{E}$ has an acceleration

$$\mathbf{a} = \frac{q\mathbf{E}}{m} \qquad \textbf{(23.7)}$$

Problem-Solving Hints

Finding the Electric Field

- **Units:** In calculations using the Coulomb constant $k_e\ (= 1/4\pi\epsilon_0)$, charges must be expressed in coulombs and distances in meters.
- **Calculating the electric field of point charges:** To find the total electric field at a given point, first calculate the electric field at the point due to each individual charge. The resultant field at the point is the vector sum of the fields due to the individual charges.
- **Continuous charge distributions:** When you are confronted with problems that involve a continuous distribution of charge, the vector sums for evaluating the total electric field at some point must be replaced by vector integrals. Divide the charge distribution into infinitesimal pieces, and calculate the vector sum by integrating over the entire charge distribution. You should review Examples 23.7 through 23.9.
- **Symmetry:** With both distributions of point charges and continuous charge distributions, take advantage of any symmetry in the system to simplify your calculations.

QUESTIONS

1. Sparks are often observed (or heard) on a dry day when clothes are removed in the dark. Explain.
2. Explain from an atomic viewpoint why charge is usually transferred by electrons.
3. A balloon is negatively charged by rubbing and then clings to a wall. Does this mean that the wall is positively charged? Why does the balloon eventually fall?
4. A light, uncharged metallic sphere suspended from a thread is attracted to a charged rubber rod. After touching the rod, the sphere is repelled by the rod. Explain.

5. Explain what is meant by the term "a neutral atom."
6. Why do some clothes cling together and to your body after they are removed from a dryer?
7. A large metallic sphere insulated from ground is charged with an electrostatic generator while a person standing on an insulating stool holds the sphere. Why is it safe to do this? Why wouldn't it be safe for another person to touch the sphere after it has been charged?
8. What are the similarities and differences between Newton's law of gravitation, $F_g = Gm_1m_2/r^2$, and Coulomb's law, $F_e = k_e q_1 q_2/r^2$?
9. Assume that someone proposes a theory that states that people are bound to the Earth by electric forces rather than by gravity. How could you prove this theory wrong?
10. How would you experimentally distinguish an electric field from a gravitational field?
11. Would life be different if the electron were positively charged and the proton were negatively charged? Does the choice of signs have any bearing on physical and chemical interactions? Explain.
12. When defining the electric field, why is it necessary to specify that the magnitude of the test charge be very small (that is, why is it necessary to take the limit of $\mathbf{F}_e/q$ as $q \rightarrow 0$)?
13. Two charged conducting spheres, each of radius a, are separated by a distance $r > 2a$. Is the force on either sphere given by Coulomb's law? Explain. (*Hint:* Refer to Chapter 14 on gravitation.)
14. When is it valid to approximate a charge distribution by a point charge?
15. Is it possible for an electric field to exist in empty space? Explain.
16. Explain why electric field lines never cross. (*Hint:* $\mathbf{E}$ must have a unique direction at all points.)
17. A free electron and free proton are placed in an identical electric field. Compare the electric forces on each particle. Compare their accelerations.
18. Explain what happens to the magnitude of the electric field of a point charge as r approaches zero.
19. A negative charge is placed in a region of space where the electric field is directed vertically upward. What is the direction of the electric force experienced by this charge?
20. A charge $4q$ is a distance r from a charge $-q$. Compare the number of electric field lines leaving the charge $4q$ with the number entering the charge $-q$.
21. In Figure 23.23, where do the extra lines leaving the charge $+2q$ end?
22. Consider two equal point charges separated by some distance d. At what point (other than ∞) would a third test charge experience no net force?
23. A negative point charge $-q$ is placed at the point P near the positively charged ring shown in Figure 23.17. If $x \ll a$, describe the motion of the point charge if it is released from rest.
24. Explain the differences between linear, surface, and volume charge densities, and give examples of when each would be used.
25. If the electron in Figure 23.25 is projected into the electric field with an arbitrary velocity $\mathbf{v}_i$ (at an angle to $\mathbf{E}$), will its trajectory still be parabolic? Explain.
26. It has been reported that in some instances people near where a lightning bolt strikes the Earth have had their clothes thrown off. Explain why this might happen.
27. Why should a ground wire be connected to the metallic support rod for a television antenna?
28. A light strip of aluminum foil is draped over a wooden rod. When a rod carrying a positive charge is brought close to the foil, the two parts of the foil stand apart. Why? What kind of charge is on the foil?
29. Why is it more difficult to charge an object by rubbing on a humid day than on a dry day?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
☐ = paired numerical/symbolic problems

Section 23.1 Properties of Electric Charges
Section 23.2 Insulators and Conductors
Section 23.3 Coulomb's Law

1. (a) Calculate the number of electrons in a small, electrically neutral silver pin that has a mass of 10.0 g. Silver has 47 electrons per atom, and its molar mass is 107.87 g/mol. (b) Electrons are added to the pin until the net negative charge is 1.00 mC. How many electrons are added for every 10^9 electrons already present?
2. (a) Two protons in a molecule are separated by a distance of 3.80×10^{-10} m. Find the electric force exerted by one proton on the other. (b) How does the magnitude of this force compare with the magnitude of the gravitational force between the two protons? (c) What must be the charge-to-mass ratio of a particle if the magnitude of the gravitational force between two of these particles equals the magnitude of the electric force between them?

WEB 3. Richard Feynman once said that if two persons stood at arm's length from each other and each person had 1% more electrons than protons, the force of repulsion between them would be enough to lift a "weight" equal to that of the entire Earth. Carry out an order-of-magnitude calculation to substantiate this assertion.

4. Two small silver spheres, each with a mass of 10.0 g, are separated by 1.00 m. Calculate the fraction of the elec-

trons in one sphere that must be transferred to the other to produce an attractive force of 1.00×10^4 N (about 1 ton) between the spheres. (The number of electrons per atom of silver is 47, and the number of atoms per gram is Avogadro's number divided by the molar mass of silver, 107.87 g/mol.)

5. Suppose that 1.00 g of hydrogen is separated into electrons and protons. Suppose also that the protons are placed at the Earth's north pole and the electrons are placed at the south pole. What is the resulting compressional force on the Earth?

6. Two identical conducting small spheres are placed with their centers 0.300 m apart. One is given a charge of 12.0 nC, and the other is given a charge of -18.0 nC. (a) Find the electric force exerted on one sphere by the other. (b) The spheres are connected by a conducting wire. Find the electric force between the two after equilibrium has occurred.

7. Three point charges are located at the corners of an equilateral triangle, as shown in Figure P23.7. Calculate the net electric force on the 7.00-μC charge.

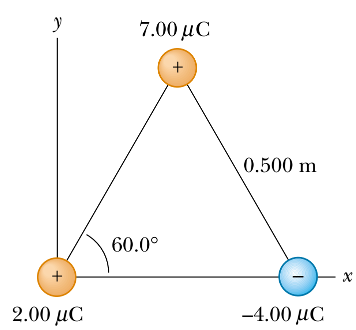


Figure P23.7 Problems 7 and 15.

8. Two small beads having positive charges $3q$ and q are fixed at the opposite ends of a horizontal insulating rod extending from the origin to the point $x = d$. As shown in Figure P23.8, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?

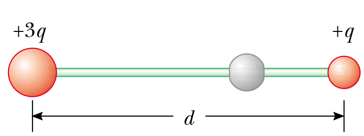


Figure P23.8

9. Review Problem. In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is 0.529×10^{-10} m. (a) Find the electric force between the two. (b) If this force causes the centripetal acceleration of the electron, what is the speed of the electron?

10. Review Problem. Two identical point charges each having charge $+q$ are fixed in space and separated by a distance d. A third point charge $-Q$ of mass m is free to move and lies initially at rest on a perpendicular bisector of the two fixed charges a distance x from the midpoint of the two fixed charges (Fig. P23.10). (a) Show that if x is small compared with d, the motion of $-Q$ is simple harmonic along the perpendicular bisector. Determine the period of that motion. (b) How fast will the charge $-Q$ be moving when it is at the midpoint between the two fixed charges, if initially it is released at a distance $x = a \ll d$ from the midpoint?

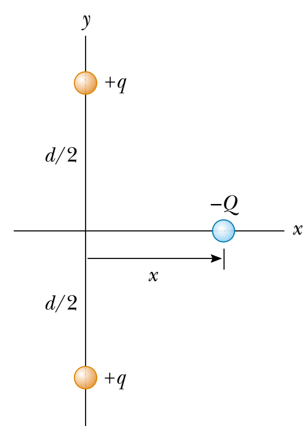


Figure P23.10

Section 23.4 The Electric Field

11. What are the magnitude and direction of the electric field that will balance the weight of (a) an electron and (b) a proton? (Use the data in Table 23.1.)

12. An object having a net charge of 24.0 μC is placed in a uniform electric field of 610 N/C that is directed vertically. What is the mass of this object if it "floats" in the field?

13. In Figure P23.13, determine the point (other than infinity) at which the electric field is zero.

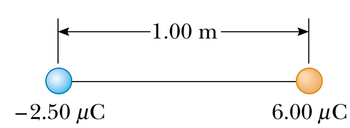


Figure P23.13

14. An airplane is flying through a thundercloud at a height of 2 000 m. (This is a very dangerous thing to do because of updrafts, turbulence, and the possibility of electric discharge.) If there are charge concentrations of $+40.0$ C at a height of 3 000 m within the cloud and of -40.0 C at a height of 1 000 m, what is the electric field E at the aircraft?

15. Three charges are at the corners of an equilateral triangle, as shown in Figure P23.7. (a) Calculate the electric field at the position of the 2.00-μC charge due to the 7.00-μC and -4.00-μC charges. (b) Use your answer to part (a) to determine the force on the 2.00-μC charge.

16. Three point charges are arranged as shown in Figure P23.16. (a) Find the vector electric field that the 6.00-nC and -3.00-nC charges together create at the origin. (b) Find the vector force on the 5.00-nC charge.

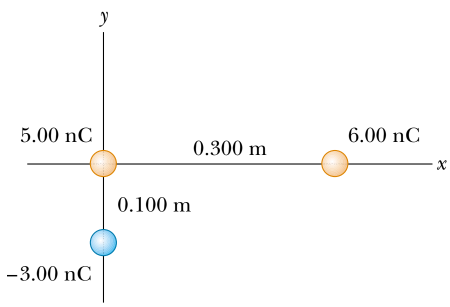


Figure P23.16

WEB **17.** Three equal positive charges q are at the corners of an equilateral triangle of side a, as shown in Figure P23.17. (a) Assume that the three charges together create an electric field. Find the location of a point (other than ∞) where the electric field is zero. (*Hint:* Sketch the field lines in the plane of the charges.) (b) What are the magnitude and direction of the electric field at P due to the two charges at the base?

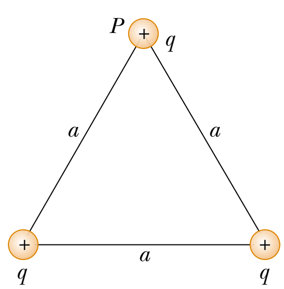


Figure P23.17

18. Two 2.00-μC point charges are located on the x axis. One is at $x = 1.00$ m, and the other is at $x = -1.00$ m. (a) Determine the electric field on the y axis at $y = 0.500$ m. (b) Calculate the electric force on a -3.00-μC charge placed on the y axis at $y = 0.500$ m.

19. Four point charges are at the corners of a square of side a, as shown in Figure P23.19. (a) Determine the magnitude and direction of the electric field at the location of charge q. (b) What is the resultant force on q?

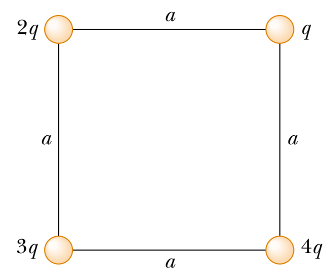


Figure P23.19

20. A point particle having charge q is located at point (x_0, y_0) in the xy plane. Show that the x and y components of the electric field at point (x, y) due to this charge q are

$$E_x = \frac{k_e q(x - x_0)}{[(x - x_0)^2 + (y - y_0)^2]^{3/2}}$$

$$E_y = \frac{k_e q(y - y_0)}{[(x - x_0)^2 + (y - y_0)^2]^{3/2}}$$

21. Consider the electric dipole shown in Figure P23.21. Show that the electric field at a *distant* point along the x axis is $E_x \cong 4k_e qa/x^3$.

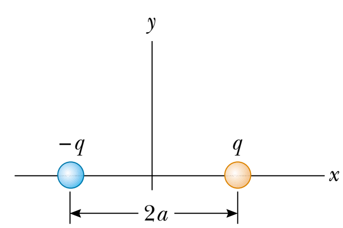


Figure P23.21

22. Consider n equal positive point charges each of magnitude Q/n placed symmetrically around a circle of radius R. (a) Calculate the magnitude of the electric field E at a point a distance x on the line passing through the center of the circle and perpendicular to the plane of the circle. (b) Explain why this result is identical to the one obtained in Example 23.8.

23. Consider an infinite number of identical charges (each of charge q) placed along the x axis at distances a, $2a$, $3a$, $4a$, . . . from the origin. What is the electric field at the origin due to this distribution? *Hint:* Use the fact that

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$

Section 23.5 Electric Field of a Continuous Charge Distribution

24. A rod 14.0 cm long is uniformly charged and has a total charge of -22.0 μC. Determine the magnitude and direction of the electric field along the axis of the rod at a point 36.0 cm from its center.

25. A continuous line of charge lies along the x axis, extending from $x = +x_0$ to positive infinity. The line carries a uniform linear charge density λ_0. What are the magnitude and direction of the electric field at the origin?

26. A line of charge starts at $x = +x_0$ and extends to positive infinity. If the linear charge density is $\lambda = \lambda_0 x_0/x$, determine the electric field at the origin.

27. A uniformly charged ring of radius 10.0 cm has a total charge of 75.0 μC. Find the electric field on the axis of the ring at (a) 1.00 cm, (b) 5.00 cm, (c) 30.0 cm, and (d) 100 cm from the center of the ring.

28. Show that the maximum field strength E_{max} along the axis of a uniformly charged ring occurs at $x = a/\sqrt{2}$ (see Fig. 23.17) and has the value $Q/(6\sqrt{3}\pi\epsilon_0 a^2)$.

29. A uniformly charged disk of radius 35.0 cm carries a charge density of 7.90×10^{-3} C/m^2. Calculate the electric field on the axis of the disk at (a) 5.00 cm, (b) 10.0 cm, (c) 50.0 cm, and (d) 200 cm from the center of the disk.

30. Example 23.9 derives the exact expression for the electric field at a point on the axis of a uniformly charged disk. Consider a disk of radius $R = 3.00$ cm having a uniformly distributed charge of $+5.20$ μC. (a) Using the result of Example 23.9, compute the electric field at a point on the axis and 3.00 mm from the center. Compare this answer with the field computed from the near-field approximation $E = \sigma/2\epsilon_0$. (b) Using the result of Example 23.9, compute the electric field at a point on the axis and 30.0 cm from the center of the disk. Compare this result with the electric field obtained by treating the disk as a $+5.20$-μC point charge at a distance of 30.0 cm.

31. The electric field along the axis of a uniformly charged disk of radius R and total charge Q was calculated in Example 23.9. Show that the electric field at distances x that are great compared with R approaches that of a point charge $Q = \sigma\pi R^2$. (*Hint:* First show that $x/(x^2 + R^2)^{1/2} = (1 + R^2/x^2)^{-1/2}$, and use the binomial expansion $(1 + \delta)^n \approx 1 + n\delta$ when $\delta \ll 1$.)

32. A piece of Styrofoam having a mass m carries a net charge of $-q$ and floats above the center of a very large horizontal sheet of plastic that has a uniform charge density on its surface. What is the charge per unit area on the plastic sheet?

WEB **33.** A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle, as shown in Figure P23.33. The rod has a total charge of -7.50 μC. Find the magnitude and direction of the electric field at O, the center of the semicircle.

34. (a) Consider a uniformly charged right circular cylindrical shell having total charge Q, radius R, and height h. Determine the electric field at a point a distance d from the right side of the cylinder, as shown in Figure P23.34. (*Hint:* Use the result of Example 23.8 and treat the cylinder as a collection of ring charges.) (b) Consider now a solid cylinder with the same dimensions and

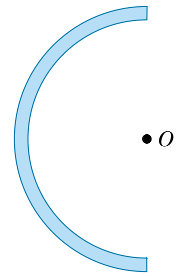


Figure P23.33

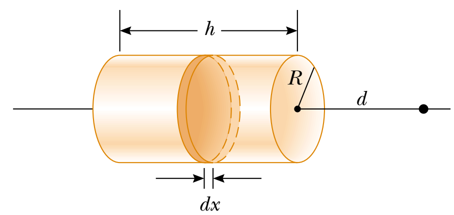


Figure P23.34

carrying the same charge, which is uniformly distributed through its volume. Use the result of Example 23.9 to find the field it creates at the same point.

35. A thin rod of length ℓ and uniform charge per unit length λ lies along the x axis, as shown in Figure P23.35. (a) Show that the electric field at P, a distance y from the rod, along the perpendicular bisector has no x component and is given by $E = 2k_e\lambda \sin\theta_0/y$. (b) Using your result to part (a), show that the field of a rod of infinite length is $E = 2k_e\lambda/y$. (*Hint:* First calculate the field at P due to an element of length dx, which has a charge $\lambda\, dx$. Then change variables from x to θ, using the facts that $x = y \tan\theta$ and $dx = y \sec^2\theta\, d\theta$, and integrate over θ.)

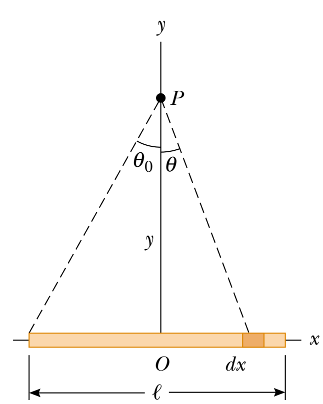
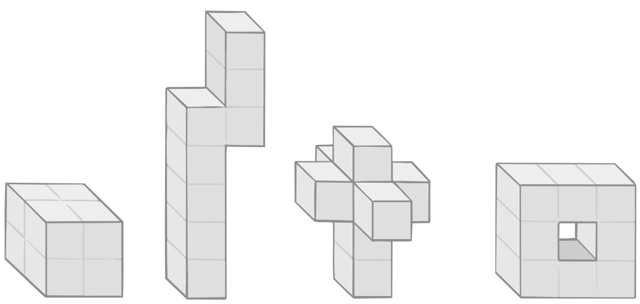


Figure P23.35

36. Three solid plastic cylinders all have a radius of 2.50 cm and a length of 6.00 cm. One (a) carries charge with uniform density 15.0 nC/m^2 everywhere on its surface. Another (b) carries charge with the same uniform density on its curved lateral surface only. The third (c) carries charge with uniform density 500 nC/m^3 throughout the plastic. Find the charge of each cylinder.

37. Eight solid plastic cubes, each 3.00 cm on each edge, are glued together to form each one of the objects (i, ii, iii, and iv) shown in Figure P23.37. (a) If each object carries charge with a uniform density of 400 nC/m^3 throughout its volume, what is the charge of each object? (b) If each object is given charge with a uniform density of 15.0 nC/m^2 everywhere on its exposed surface, what is the charge on each object? (c) If charge is placed only on the edges where perpendicular surfaces meet, with a uniform density of 80.0 pC/m, what is the charge of each object?



(i) (ii) (iii) (iv)

Figure P23.37

Section 23.6 Electric Field Lines

38. A positively charged disk has a uniform charge per unit area as described in Example 23.9. Sketch the electric field lines in a plane perpendicular to the plane of the disk passing through its center.

39. A negatively charged rod of finite length has a uniform charge per unit length. Sketch the electric field lines in a plane containing the rod.

40. Figure P23.40 shows the electric field lines for two point charges separated by a small distance. (a) Determine the ratio q_1/q_2. (b) What are the signs of q_1 and q_2?

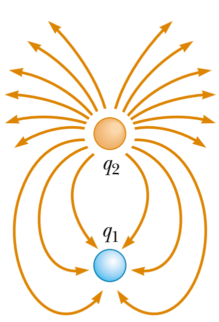


Figure P23.40

Section 23.7 Motion of Charged Particles in a Uniform Electric Field

41. An electron and a proton are each placed at rest in an electric field of 520 N/C. Calculate the speed of each particle 48.0 ns after being released.

42. A proton is projected in the positive x direction into a region of uniform electric field $\mathbf{E} = -6.00 \times 10^5\,\mathbf{i}$ N/C. The proton travels 7.00 cm before coming to rest. Determine (a) the acceleration of the proton, (b) its initial speed, and (c) the time it takes the proton to come to rest.

43. A proton accelerates from rest in a uniform electric field of 640 N/C. At some later time, its speed has reached 1.20×10^6 m/s (nonrelativistic, since v is much less than the speed of light). (a) Find the acceleration of the proton. (b) How long does it take the proton to reach this speed? (c) How far has it moved in this time? (d) What is its kinetic energy at this time?

44. The electrons in a particle beam each have a kinetic energy of 1.60×10^{-17} J. What are the magnitude and direction of the electric field that stops these electrons in a distance of 10.0 cm?

WEB **45.** The electrons in a particle beam each have a kinetic energy K. What are the magnitude and direction of the electric field that stops these electrons in a distance d?

46. A positively charged bead having a mass of 1.00 g falls from rest in a vacuum from a height of 5.00 m in a uniform vertical electric field with a magnitude of 1.00×10^4 N/C. The bead hits the ground at a speed of 21.0 m/s. Determine (a) the direction of the electric field (up or down) and (b) the charge on the bead.

47. A proton moves at 4.50×10^5 m/s in the horizontal direction. It enters a uniform vertical electric field with a magnitude of 9.60×10^3 N/C. Ignoring any gravitational effects, find (a) the time it takes the proton to travel 5.00 cm horizontally, (b) its vertical displacement after it has traveled 5.00 cm horizontally, and (c) the horizontal and vertical components of its velocity after it has traveled 5.00 cm horizontally.

48. An electron is projected at an angle of 30.0° above the horizontal at a speed of 8.20×10^5 m/s in a region where the electric field is $\mathbf{E} = 390\mathbf{j}$ N/C. Neglecting the effects of gravity, find (a) the time it takes the electron to return to its initial height, (b) the maximum height it reaches, and (c) its horizontal displacement when it reaches its maximum height.

49. Protons are projected with an initial speed $v_i = 9.55 \times 10^3$ m/s into a region where a uniform electric field $\mathbf{E} = (-720\mathbf{j})$ N/C is present, as shown in Figure P23.49. The protons are to hit a target that lies at a horizontal distance of 1.27 mm from the point where the protons are launched. Find (a) the two projection angles θ that result in a hit and (b) the total time of flight for each trajectory.

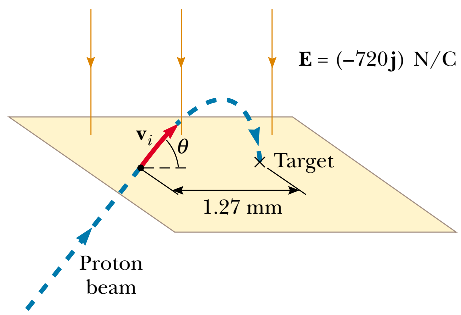


Figure P23.49

ADDITIONAL PROBLEMS

50. Three point charges are aligned along the x axis as shown in Figure P23.50. Find the electric field at (a) the position (2.00, 0) and (b) the position (0, 2.00).

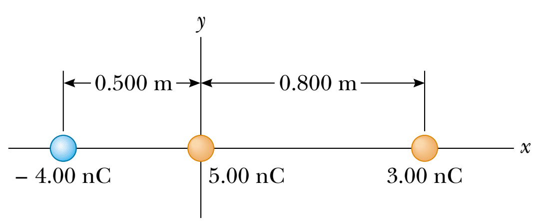


Figure P23.50

51. A uniform electric field of magnitude 640 N/C exists between two parallel plates that are 4.00 cm apart. A proton is released from the positive plate at the same instant that an electron is released from the negative plate. (a) Determine the distance from the positive plate at which the two pass each other. (Ignore the electrical attraction between the proton and electron.) (b) Repeat part (a) for a sodium ion (Na^+) and a chlorine ion (Cl^-).

52. A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field, as shown in Figure P23.52. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?

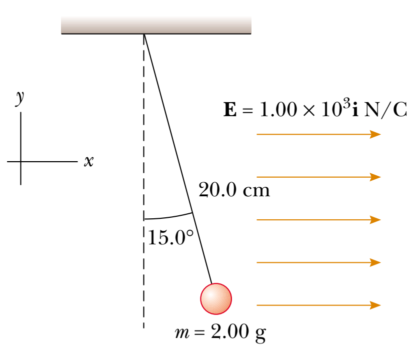


Figure P23.52

WEB **53.** A charged cork ball of mass 1.00 g is suspended on a light string in the presence of a uniform electric field, as shown in Figure P23.53. When $\mathbf{E} = (3.00\mathbf{i} + 5.00\mathbf{j}) \times 10^5$ N/C, the ball is in equilibrium at $\theta = 37.0°$. Find (a) the charge on the ball and (b) the tension in the string.

54. A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field, as shown in Figure P23.53. When $\mathbf{E} = (A\mathbf{i} + B\mathbf{j})$ N/C, where A and B are positive numbers, the ball is in equilibrium at the angle θ. Find (a) the charge on the ball and (b) the tension in the string.

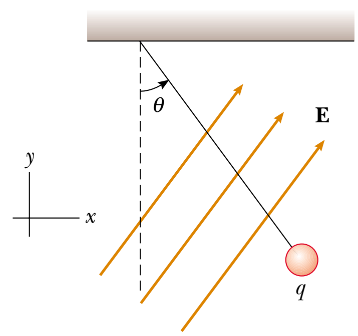


Figure P23.53 Problems 53 and 54.

55. Four identical point charges ($q = +10.0\ \mu$C) are located on the corners of a rectangle, as shown in Figure P23.55. The dimensions of the rectangle are $L = 60.0$ cm and $W = 15.0$ cm. Calculate the magnitude and direction of the net electric force exerted on the charge at the lower left corner by the other three charges.

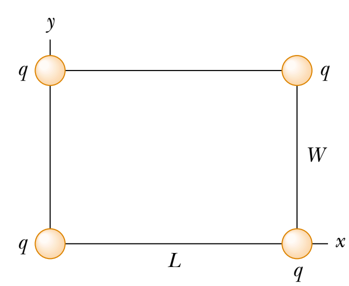


Figure P23.55

56. Three identical small Styrofoam balls ($m = 2.00$ g) are suspended from a fixed point by three nonconducting threads, each with a length of 50.0 cm and with negligible mass. At equilibrium the three balls form an equilateral triangle with sides of 30.0 cm. What is the common charge q carried by each ball?

57. Two identical metallic blocks resting on a frictionless horizontal surface are connected by a light metallic spring having the spring constant $k = 100$ N/m and an unstretched length of 0.300 m, as shown in Figure P23.57a. A total charge of Q is slowly placed on the system, causing the spring to stretch to an equilibrium length of 0.400 m, as shown in Figure P23.57b. Determine the value of Q, assuming that all the charge resides on the blocks and that the blocks are like point charges.

58. Two identical metallic blocks resting on a frictionless horizontal surface are connected by a light metallic spring having a spring constant k and an unstretched length L_i, as shown in Figure P23.57a. A total charge of Q is slowly placed on the system, causing the spring to stretch to an equilibrium length L, as shown in Figure P23.57b. Determine the value of Q, assuming that all the charge resides on the blocks and that the blocks are like point charges.

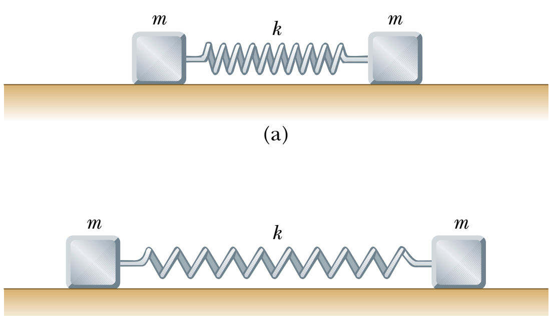


Figure P23.57 Problems 57 and 58.

59. Identical thin rods of length $2a$ carry equal charges, $+Q$, uniformly distributed along their lengths. The rods lie along the x axis with their centers separated by a distance of $b > 2a$ (Fig. P23.59). Show that the magnitude of the force exerted by the left rod on the right one is given by

$$F = \left(\frac{k_e Q^2}{4a^2}\right) \ln\left(\frac{b^2}{b^2 - 4a^2}\right)$$

60. A particle is said to be nonrelativistic as long as its speed is less than one-tenth the speed of light, or less than 3.00×10^7 m/s. (a) How long will an electron remain nonrelativistic if it starts from rest in a region of an electric field of 1.00 N/C? (b) How long will a proton remain nonrelativistic in the same electric field? (c) Electric fields are commonly much larger than

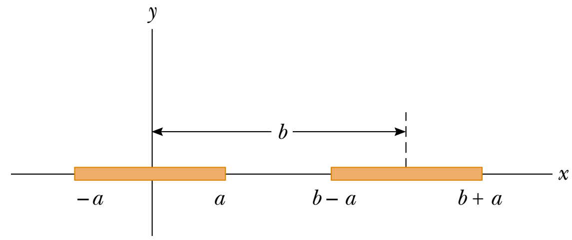


Figure P23.59

1 N/C. Will the charged particle remain nonrelativistic for a shorter or a longer time in a much larger electric field?

61. A line of positive charge is formed into a semicircle of radius $R = 60.0$ cm, as shown in Figure P23.61. The charge per unit length along the semicircle is described by the expression $\lambda = \lambda_0 \cos\theta$. The total charge on the semicircle is 12.0 μC. Calculate the total force on a charge of 3.00 μC placed at the center of curvature.

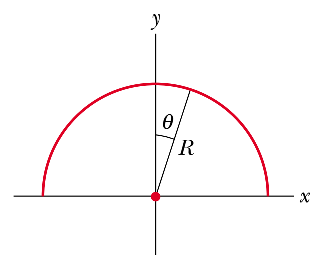


Figure P23.61

62. Two small spheres, each of mass 2.00 g, are suspended by light strings 10.0 cm in length (Fig. P23.62). A uniform electric field is applied in the x direction. The spheres have charges equal to -5.00×10^{-8} C and $+5.00 \times 10^{-8}$ C. Determine the electric field that enables the spheres to be in equilibrium at an angle of $\theta = 10.0°$.

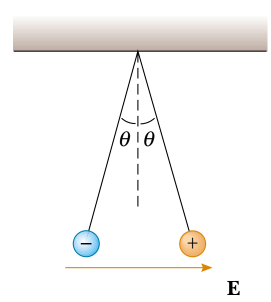


Figure P23.62

63. Two small spheres of mass m are suspended from strings of length ℓ that are connected at a common point. One sphere has charge Q; the other has charge $2Q$. Assume that the angles θ_1 and θ_2 that the strings make with the vertical are small. (a) How are θ_1 and θ_2 related? (b) Show that the distance r between the spheres is

$$r \cong \left(\frac{4k_e Q^2 \ell}{mg}\right)^{1/3}$$

64. Three charges of equal magnitude q are fixed in position at the vertices of an equilateral triangle (Fig. P23.64). A fourth charge Q is free to move along the positive x axis under the influence of the forces exerted by the three fixed charges. Find a value for s for which Q is in equilibrium. You will need to solve a transcendental equation.

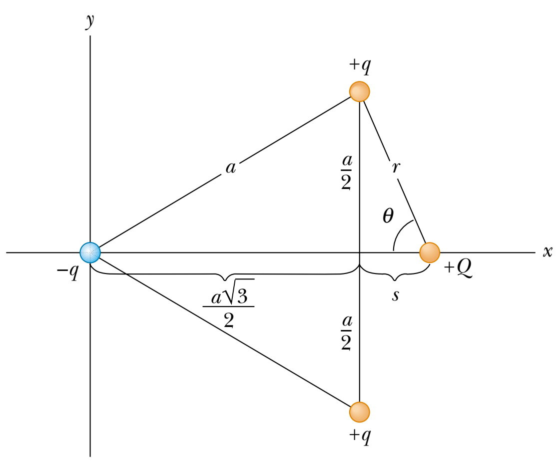


Figure P23.64

65. **Review Problem.** Four identical point charges, each having charge $+q$, are fixed at the corners of a square of side L. A fifth point charge $-Q$ lies a distance z along the line perpendicular to the plane of the square and passing through the center of the square (Fig. P23.65). (a) Show that the force exerted on $-Q$ by the other four charges is

$$\mathbf{F} = -\frac{4k_e qQz}{\left(z^2 + \frac{L^2}{2}\right)^{3/2}}\mathbf{k}$$

Note that this force is directed toward the center of the square whether z is positive ($-Q$ above the square) or negative ($-Q$ below the square). (b) If z is small compared with L, the above expression reduces to $\mathbf{F} \approx -(\text{constant})\, z\mathbf{k}$. Why does this imply that the motion of $-Q$ is simple harmonic, and what would be the period of this motion if the mass of $-Q$ were m?

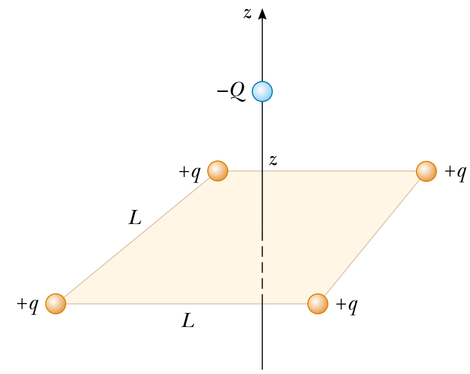


Figure P23.65

66. **Review Problem.** A 1.00-g cork ball with a charge of 2.00 μC is suspended vertically on a 0.500-m-long light string in the presence of a uniform, downward-directed electric field of magnitude $E = 1.00 \times 10^5$ N/C. If the ball is displaced slightly from the vertical, it oscillates like a simple pendulum. (a) Determine the period of this oscillation. (b) Should gravity be included in the calculation for part (a)? Explain.

67. Three charges of equal magnitude q reside at the corners of an equilateral triangle of side length a (Fig. P23.67). (a) Find the magnitude and direction of the electric field at point P, midway between the negative charges, in terms of k_e, q, and a. (b) Where must a $-4q$ charge be placed so that any charge located at P experiences no net electric force? In part (b), let P be the origin and let the distance between the $+q$ charge and P be 1.00 m.

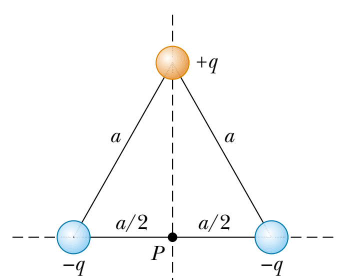


Figure P23.67

68. Two identical beads each have a mass m and charge q. When placed in a hemispherical bowl of radius R with frictionless, nonconducting walls, the beads move, and at equilibrium they are a distance R apart (Fig. P23.68). Determine the charge on each bead.

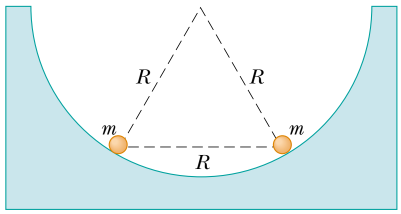


Figure P23.68

69. Eight point charges, each of magnitude q, are located on the corners of a cube of side s, as shown in Figure P23.69. (a) Determine the x, y, and z components of the resultant force exerted on the charge located at point A by the other charges. (b) What are the magnitude and direction of this resultant force?

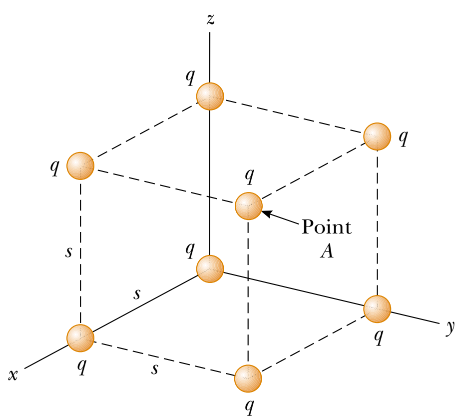


Figure P23.69 Problems 69 and 70.

70. Consider the charge distribution shown in Figure P23.69. (a) Show that the magnitude of the electric field at the center of any face of the cube has a value of $2.18k_e q/s^2$. (b) What is the direction of the electric field at the center of the top face of the cube?

71. A line of charge with a uniform density of 35.0 nC/m lies along the line $y = -15.0$ cm, between the points with coordinates $x = 0$ and $x = 40.0$ cm. Find the electric field it creates at the origin.

72. Three point charges q, $-2q$, and q are located along the x axis, as shown in Figure P23.72. Show that the electric field at P ($y \gg a$) along the y axis is

$$\mathbf{E} = -k_e \frac{3qa^2}{y^4}\,\mathbf{j}$$

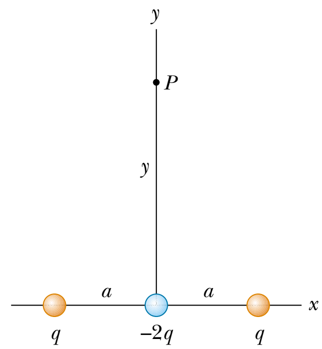


Figure P23.72

This charge distribution, which is essentially that of two electric dipoles, is called an *electric quadrupole.* Note that $\mathbf{E}$ varies as r^{-4} for the quadrupole, compared with variations of r^{-3} for the dipole and r^{-2} for the monopole (a single charge).

73. **Review Problem.** A negatively charged particle $-q$ is placed at the center of a uniformly charged ring, where the ring has a total positive charge Q, as shown in Example 23.8. The particle, confined to move along the x axis, is displaced a *small* distance x along the axis (where $x \ll a$) and released. Show that the particle oscillates with simple harmonic motion with a frequency

$$f = \frac{1}{2\pi}\left(\frac{k_e qQ}{ma^3}\right)^{1/2}$$

74. **Review Problem.** An electric dipole in a uniform electric field is displaced slightly from its equilibrium position, as shown in Figure P23.74, where θ is small and the charges are separated by a distance $2a$. The moment of inertia of the dipole is I. If the dipole is released from this position, show that its angular orientation exhibits simple harmonic motion with a frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{2qaE}{I}}$$

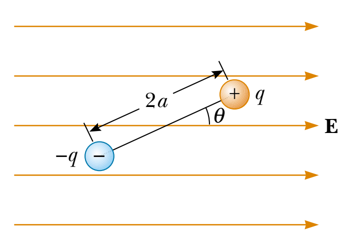
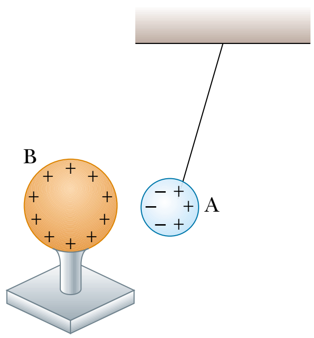


Figure P23.74

Answers to Quick Quizzes

23.1 (b). The amount of charge present after rubbing is the same as that before; it is just distributed differently.

23.2 (d). Object A might be negatively charged, but it also might be electrically neutral with an induced charge separation, as shown in the following figure:



23.3 (b). From Newton's third law, the electric force exerted by object B on object A is equal in magnitude to the force exerted by object A on object B and in the opposite direction—that is, $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$.

23.4 Nothing, if we assume that the source charge producing the field is not disturbed by our actions. Remember that the electric field is created not by the $+3$-μC charge or by the -3-μC charge but by the source charge (unseen in this case).

23.5 *A*, *B*, and *C*. The field is greatest at point *A* because this is where the field lines are closest together. The absence of lines at point *C* indicates that the electric field there is zero.

PUZZLER

Some railway companies are planning to coat the windows of their commuter trains with a very thin layer of metal. (The coating is so thin you can see through it.) They are doing this in response to rider complaints about other passengers' talking loudly on cellular telephones. How can a metallic coating that is only a few hundred nanometers thick overcome this problem? *(Arthur Tilley/FPG International)*

chapter

24

Gauss's Law

Chapter Outline

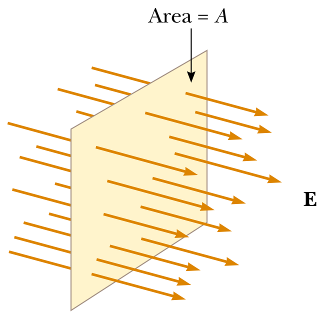


Figure 24.1 Field lines representing a uniform electric field penetrating a plane of area A perpendicular to the field. The electric flux Φ_E through this area is equal to EA.

In the preceding chapter we showed how to use Coulomb's law to calculate the electric field generated by a given charge distribution. In this chapter, we describe *Gauss's law* and an alternative procedure for calculating electric fields. The law is based on the fact that the fundamental electrostatic force between point charges exhibits an inverse-square behavior. Although a consequence of Coulomb's law, Gauss's law is more convenient for calculating the electric fields of highly symmetric charge distributions and makes possible useful qualitative reasoning when we are dealing with complicated problems.

24.1 ELECTRIC FLUX

11.6 The concept of electric field lines is described qualitatively in Chapter 23. We now use the concept of electric flux to treat electric field lines in a more quantitative way.

Consider an electric field that is uniform in both magnitude and direction, as shown in Figure 24.1. The field lines penetrate a rectangular surface of area A, which is perpendicular to the field. Recall from Section 23.6 that the number of lines per unit area (in other words, the *line density*) is proportional to the magnitude of the electric field. Therefore, the total number of lines penetrating the surface is proportional to the product EA. This product of the magnitude of the electric field E and surface area A perpendicular to the field is called the **electric flux** Φ_E (uppercase Greek phi):

$$\Phi_E = EA \tag{24.1}$$

From the SI units of E and A, we see that Φ_E has units of newton–meters squared per coulomb ($\text{N}\cdot\text{m}^2/\text{C}$). **Electric flux is proportional to the number of electric field lines penetrating some surface.**

EXAMPLE 24.1 Flux Through a Sphere

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of $+1.00\ \mu\text{C}$ at its center?

Solution The magnitude of the electric field 1.00 m from this charge is given by Equation 23.4,

$$E = k_e \frac{q}{r^2} = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{1.00 \times 10^{-6}\ \text{C}}{(1.00\ \text{m})^2}$$
$$= 8.99 \times 10^3\ \text{N/C}$$

The field points radially outward and is therefore everywhere perpendicular to the surface of the sphere. The flux through the sphere (whose surface area $A = 4\pi r^2 = 12.6\ \text{m}^2$) is thus

$$\Phi_E = EA = (8.99 \times 10^3\ \text{N/C})(12.6\ \text{m}^2)$$
$$= \boxed{1.13 \times 10^5\ \text{N}\cdot\text{m}^2/\text{C}}$$

Exercise What would be the (a) electric field and (b) flux through the sphere if it had a radius of 0.500 m?

Answer (a) 3.60×10^4 N/C; (b) $1.13 \times 10^5\ \text{N}\cdot\text{m}^2/\text{C}$.

If the surface under consideration is not perpendicular to the field, the flux through it must be less than that given by Equation 24.1. We can understand this by considering Figure 24.2, in which the normal to the surface of area A is at an angle θ to the uniform electric field. Note that the number of lines that cross this area A is equal to the number that cross the area A', which is a projection of area A aligned perpendicular to the field. From Figure 24.2 we see that the two areas are related by $A' = A\cos\theta$. Because the flux through A equals the flux through A', we

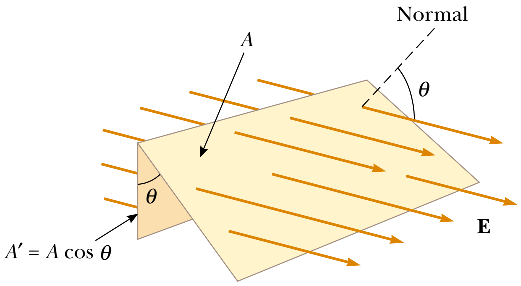


Figure 24.2 Field lines representing a uniform electric field penetrating an area A that is at an angle θ to the field. Because the number of lines that go through the area A' is the same as the number that go through A, the flux through A' is equal to the flux through A and is given by $\Phi_E = EA\cos\theta$.

QuickLab

Shine a desk lamp onto a playing card and notice how the size of the shadow on your desk depends on the orientation of the card with respect to the beam of light. Could a formula like Equation 24.2 be used to describe how much light was being blocked by the card?

conclude that the flux through A is

$$\Phi_E = EA' = EA\cos\theta \qquad \textbf{(24.2)}$$

From this result, we see that the flux through a surface of fixed area A has a maximum value EA when the surface is perpendicular to the field (in other words, when the normal to the surface is parallel to the field, that is, $\theta = 0°$ in Figure 24.2); the flux is zero when the surface is parallel to the field (in other words, when the normal to the surface is perpendicular to the field, that is, $\theta = 90°$).

We assumed a uniform electric field in the preceding discussion. In more general situations, the electric field may vary over a surface. Therefore, our definition of flux given by Equation 24.2 has meaning only over a small element of area. Consider a general surface divided up into a large number of small elements, each of area ΔA. The variation in the electric field over one element can be neglected if the element is sufficiently small. It is convenient to define a vector $\Delta\mathbf{A}_i$ whose magnitude represents the area of the ith element of the surface and whose direction is *defined to be perpendicular* to the surface element, as shown in Figure 24.3. The electric flux $\Delta\Phi_E$ through this element is

$$\Delta\Phi_E = E_i\,\Delta A_i\cos\theta = \mathbf{E}_i\cdot\Delta\mathbf{A}_i$$

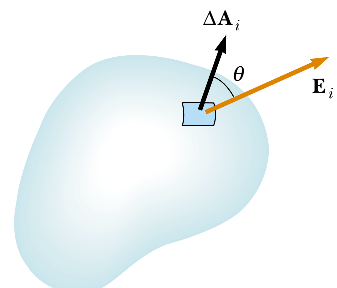


Figure 24.3 A small element of surface area ΔA_i. The electric field makes an angle θ with the vector $\Delta\mathbf{A}_i$, defined as being normal to the surface element, and the flux through the element is equal to $E_i\,\Delta A_i\cos\theta$.

where we have used the definition of the scalar product of two vectors ($\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$). By summing the contributions of all elements, we obtain the total flux through the surface.[1] If we let the area of each element approach zero, then the number of elements approaches infinity and the sum is replaced by an integral. Therefore, the general definition of electric flux is

Definition of electric flux

$$\Phi_E = \lim_{\Delta A_i\to 0}\sum \mathbf{E}_i\cdot\Delta\mathbf{A}_i = \int_{\text{surface}} \mathbf{E}\cdot d\mathbf{A} \qquad \textbf{(24.3)}$$

Equation 24.3 is a *surface integral,* which means it must be evaluated over the surface in question. In general, the value of Φ_E depends both on the field pattern and on the surface.

We are often interested in evaluating the flux through a *closed surface,* which is defined as one that divides space into an inside and an outside region, so that one cannot move from one region to the other without crossing the surface. The surface of a sphere, for example, is a closed surface.

Consider the closed surface in Figure 24.4. The vectors $\Delta\mathbf{A}_i$ point in different directions for the various surface elements, but at each point they are normal to

[1] It is important to note that drawings with field lines have their inaccuracies because a small area element (depending on its location) may happen to have too many or too few field lines penetrating it. We stress that the basic definition of electric flux is $\int \mathbf{E}\cdot d\mathbf{A}$. The use of lines is only an aid for visualizing the concept.

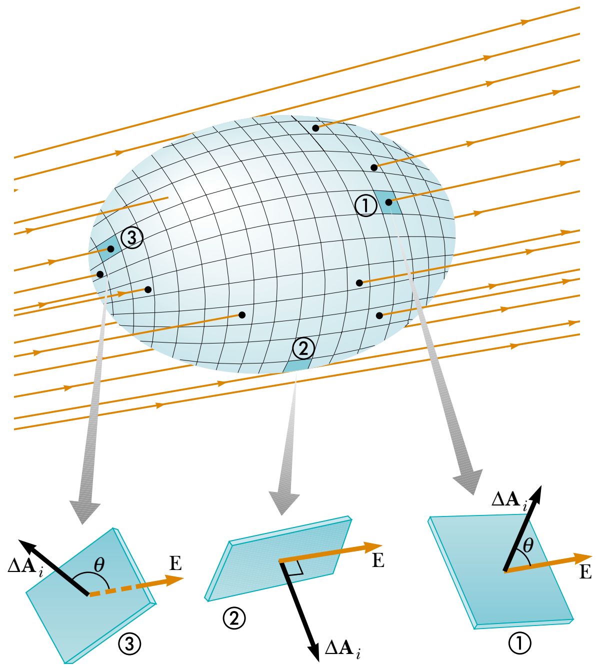
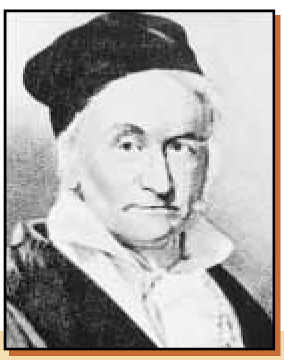


Figure 24.4 A closed surface in an electric field. The area vectors $\Delta\mathbf{A}_i$ are, by convention, normal to the surface and point outward. The flux through an area element can be positive (element ①), zero (element ②), or negative (element ③).



Karl Friedrich Gauss German mathematician and astronomer (1777–1855)

the surface and, by convention, always point outward. At the element labeled ①, the field lines are crossing the surface from the inside to the outside and $\theta < 90°$; hence, the flux $\Delta\Phi_E = \mathbf{E}\cdot\Delta\mathbf{A}_i$ through this element is positive. For element ②, the field lines graze the surface (perpendicular to the vector $\Delta\mathbf{A}_i$); thus, $\theta = 90°$ and the flux is zero. For elements such as ③, where the field lines are crossing the surface from outside to inside, $180° > \theta > 90°$ and the flux is negative because $\cos\theta$ is negative. The *net* flux through the surface is proportional to the net number of lines leaving the surface, where the net number means *the number leaving the surface minus the number entering the surface.* If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative. Using the symbol $\oint$ to represent an integral over a closed surface, we can write the net flux Φ_E through a closed surface as

$$\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = \oint E_n\, dA \qquad \textbf{(24.4)}$$

where E_n represents the component of the electric field normal to the surface. Evaluating the net flux through a closed surface can be very cumbersome. However, if the field is normal to the surface at each point and constant in magnitude, the calculation is straightforward, as it was in Example 24.1. The next example also illustrates this point.

EXAMPLE 24.2 Flux Through a Cube

Consider a uniform electric field $\mathbf{E}$ oriented in the x direction. Find the net electric flux through the surface of a cube of edges ℓ, oriented as shown in Figure 24.5.

Solution The net flux is the sum of the fluxes through all faces of the cube. First, note that the flux through four of the faces (③, ④, and the unnumbered ones) is zero because $\mathbf{E}$ is perpendicular to $d\mathbf{A}$ on these faces.

The net flux through faces ① and ② is

$$\Phi_E = \int_1 \mathbf{E}\cdot d\mathbf{A} + \int_2 \mathbf{E}\cdot d\mathbf{A}$$

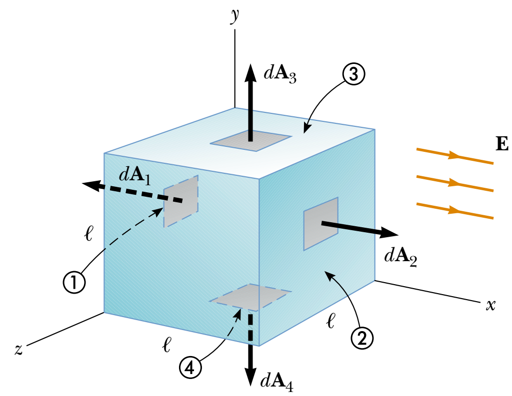


Figure 24.5 A closed surface in the shape of a cube in a uniform electric field oriented parallel to the x axis. The net flux through the closed surface is zero. Side ④ is the bottom of the cube, and side ① is opposite side ②.

For ①, $\mathbf{E}$ is constant and directed inward but $d\mathbf{A}_1$ is directed outward ($\theta = 180°$); thus, the flux through this face is

$$\int_1 \mathbf{E}\cdot d\mathbf{A} = \int_1 E(\cos 180°)\,dA = -E\int_1 dA = -EA = -E\ell^2$$

because the area of each face is $A = \ell^2$.

For ②, $\mathbf{E}$ is constant and outward and in the same direction as $d\mathbf{A}_2 (\theta = 0°)$; hence, the flux through this face is

$$\int_2 \mathbf{E}\cdot d\mathbf{A} = \int_2 E(\cos 0°)\,dA = E\int_2 dA = +EA = E\ell^2$$

Therefore, the net flux over all six faces is

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = \boxed{0}$$

24.2 GAUSS'S LAW

11.6 In this section we describe a general relationship between the net electric flux through a closed surface (often called a *gaussian surface*) and the charge enclosed by the surface. This relationship, known as *Gauss's law,* is of fundamental importance in the study of electric fields.

Let us again consider a positive point charge q located at the center of a sphere of radius r, as shown in Figure 24.6. From Equation 23.4 we know that the magnitude of the electric field everywhere on the surface of the sphere is $E = k_e q/r^2$. As noted in Example 24.1, the field lines are directed radially outward and hence perpendicular to the surface at every point on the surface. That is, at each surface point, $\mathbf{E}$ is parallel to the vector $\Delta\mathbf{A}_i$ representing a local element of area ΔA_i surrounding the surface point. Therefore,

$$\mathbf{E}\cdot\Delta\mathbf{A}_i = E\,\Delta A_i$$

and from Equation 24.4 we find that the net flux through the gaussian surface is

$$\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = \oint E\,dA = E\oint dA$$

where we have moved E outside of the integral because, by symmetry, E is constant over the surface and given by $E = k_e q/r^2$. Furthermore, because the surface is spherical, $\oint dA = A = 4\pi r^2$. Hence, the net flux through the gaussian surface is

$$\Phi_E = \frac{k_e q}{r^2}(4\pi r^2) = 4\pi k_e q$$

Recalling from Section 23.3 that $k_e = 1/(4\pi\epsilon_0)$, we can write this equation in the form

$$\Phi_E = \frac{q}{\epsilon_0} \qquad \textbf{(24.5)}$$

We can verify that this expression for the net flux gives the same result as Example 24.1: $\Phi_E = (1.00\times10^{-6}\text{ C})/(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = 1.13\times10^5\text{ N}\cdot\text{m}^2/\text{C}$.

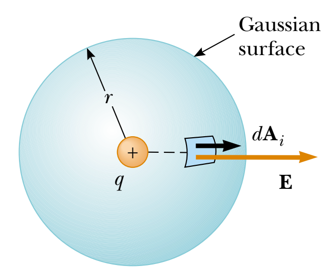


Figure 24.6 A spherical gaussian surface of radius r surrounding a point charge q. When the charge is at the center of the sphere, the electric field is everywhere normal to the surface and constant in magnitude.

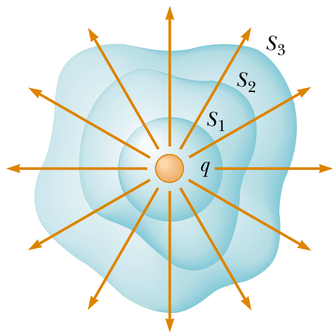


Figure 24.7 Closed surfaces of various shapes surrounding a charge q. The net electric flux is the same through all surfaces.

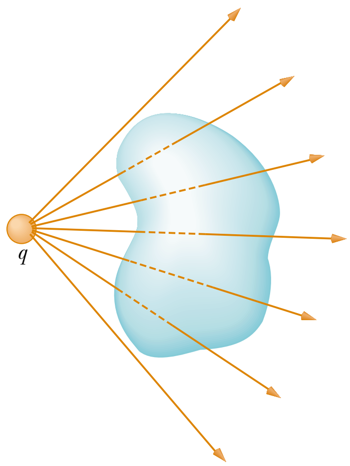


Figure 24.8 A point charge located *outside* a closed surface. The number of lines entering the surface equals the number leaving the surface.

The net electric flux through a closed surface is zero if there is no charge inside

Note from Equation 24.5 that the net flux through the spherical surface is proportional to the charge inside. The flux is independent of the radius r because the area of the spherical surface is proportional to r^2, whereas the electric field is proportional to $1/r^2$. Thus, in the product of area and electric field, the dependence on r cancels.

Now consider several closed surfaces surrounding a charge q, as shown in Figure 24.7. Surface S_1 is spherical, but surfaces S_2 and S_3 are not. From Equation 24.5, the flux that passes through S_1 has the value q/ϵ_0. As we discussed in the previous section, flux is proportional to the number of electric field lines passing through a surface. The construction shown in Figure 24.7 shows that the number of lines through S_1 is equal to the number of lines through the nonspherical surfaces S_2 and S_3. Therefore, we conclude that the net flux through *any* closed surface is independent of the shape of that surface. **The net flux through any closed surface surrounding a point charge q is given by q/ϵ_0.**

Now consider a point charge located *outside* a closed surface of arbitrary shape, as shown in Figure 24.8. As you can see from this construction, any electric field line that enters the surface leaves the surface at another point. The number of electric field lines entering the surface equals the number leaving the surface. Therefore, we conclude that **the net electric flux through a closed surface that surrounds no charge is zero.** If we apply this result to Example 24.2, we can easily see that the net flux through the cube is zero because there is no charge inside the cube.

Quick Quiz 24.1

Suppose that the charge in Example 24.1 is just outside the sphere, 1.01 m from its center. What is the total flux through the sphere?

Let us extend these arguments to two generalized cases: (1) that of many point charges and (2) that of a continuous distribution of charge. We once again use the superposition principle, which states that **the electric field due to many charges is the vector sum of the electric fields produced by the individual charges.** Therefore, we can express the flux through any closed surface as

$$\oint \mathbf{E} \cdot d\mathbf{A} = \oint (\mathbf{E}_1 + \mathbf{E}_2 + \cdots) \cdot d\mathbf{A}$$

where $\mathbf{E}$ is the total electric field at any point on the surface produced by the vector addition of the electric fields at that point due to the individual charges.

Consider the system of charges shown in Figure 24.9. The surface S surrounds only one charge, q_1; hence, the net flux through S is q_1/ϵ_0. The flux through S due to charges q_2 and q_3 outside it is zero because each electric field line that enters S at one point leaves it at another. The surface S' surrounds charges q_2 and q_3; hence, the net flux through it is $(q_2 + q_3)/\epsilon_0$. Finally, the net flux through surface S'' is zero because there is no charge inside this surface. That is, *all* the electric field lines that enter S'' at one point leave at another.

Gauss's law, which is a generalization of what we have just described, states that the net flux through *any* closed surface is

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} \tag{24.6}$$

Gauss's law

where q_{in} represents the net charge inside the surface and $\mathbf{E}$ represents the electric field at any point on the surface.

A formal proof of Gauss's law is presented in Section 24.6. When using Equation 24.6, you should note that although the charge q_{in} is the net charge inside the gaussian surface, $\mathbf{E}$ represents the *total electric field,* which includes contributions from charges both inside and outside the surface.

Gauss's law is useful for evaluating E when the charge distribution has high symmetry

In principle, Gauss's law can be solved for $\mathbf{E}$ to determine the electric field due to a system of charges or a continuous distribution of charge. In practice, however, this type of solution is applicable only in a limited number of highly symmetric situations. As we shall see in the next section, Gauss's law can be used to evaluate the electric field for charge distributions that have spherical, cylindrical, or planar symmetry. If one chooses the gaussian surface surrounding the charge distribution carefully, the integral in Equation 24.6 can be simplified. You should also note that a gaussian surface is a mathematical construction and need not coincide with any real physical surface.

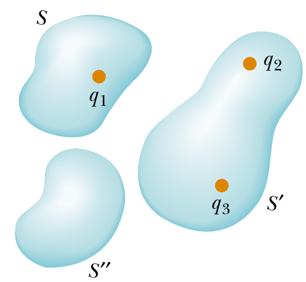


Figure 24.9 The net electric flux through any closed surface depends only on the charge *inside* that surface. The net flux through surface S is q_1/ϵ_0, the net flux through surface S' is $(q_2 + q_3)/\epsilon_0$, and the net flux through surface S'' is zero.

Quick Quiz 24.2

For a gaussian surface through which the net flux is zero, the following four statements *could be true.* Which of the statements *must be true?* (a) There are no charges inside the surface. (b) The net charge inside the surface is zero. (c) The electric field is zero everywhere on the surface. (d) The number of electric field lines entering the surface equals the number leaving the surface.

Conceptual Example 24.3

A spherical gaussian surface surrounds a point charge q. Describe what happens to the total flux through the surface if (a) the charge is tripled, (b) the radius of the sphere is doubled, (c) the surface is changed to a cube, and (d) the charge is moved to another location inside the surface.

Solution (a) The flux through the surface is tripled because flux is proportional to the amount of charge inside the surface.

(b) The flux does not change because all electric field lines from the charge pass through the sphere, regardless of its radius.

(c) The flux does not change when the shape of the gaussian surface changes because all electric field lines from the charge pass through the surface, regardless of its shape.

(d) The flux does not change when the charge is moved to another location inside that surface because Gauss's law refers to the total charge enclosed, regardless of where the charge is located inside the surface.

24.3 APPLICATION OF GAUSS'S LAW TO CHARGED INSULATORS

As mentioned earlier, Gauss's law is useful in determining electric fields when the charge distribution is characterized by a high degree of symmetry. The following examples demonstrate ways of choosing the gaussian surface over which the surface integral given by Equation 24.6 can be simplified and the electric field determined. In choosing the surface, we should always take advantage of the symmetry of the charge distribution so that we can remove E from the integral and solve for it. The goal in this type of calculation is to determine a surface that satisfies one or more of the following conditions:

1. The value of the electric field can be argued by symmetry to be constant over the surface.
2. The dot product in Equation 24.6 can be expressed as a simple algebraic product $E\,dA$ because $\mathbf{E}$ and $d\mathbf{A}$ are parallel.
3. The dot product in Equation 24.6 is zero because $\mathbf{E}$ and $d\mathbf{A}$ are perpendicular.
4. The field can be argued to be zero over the surface.

All four of these conditions are used in examples throughout the remainder of this chapter.

EXAMPLE 24.4 The Electric Field Due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge q.

Solution A single charge represents the simplest possible charge distribution, and we use this familiar case to show how to solve for the electric field with Gauss's law. We choose a spherical gaussian surface of radius r centered on the point charge, as shown in Figure 24.10. The electric field due to a positive point charge is directed radially outward by symmetry and is therefore normal to the surface at every point. Thus, as in condition (2), $\mathbf{E}$ is parallel to $d\mathbf{A}$ at each point. Therefore, $\mathbf{E}\cdot d\mathbf{A} = E\,dA$ and Gauss's law gives

$$\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = \oint E\,dA = \frac{q}{\epsilon_0}$$

By symmetry, E is constant everywhere on the surface, which satisfies condition (1), so it can be removed from the integral. Therefore,

$$\oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

where we have used the fact that the surface area of a sphere is $4\pi r^2$. Now, we solve for the electric field:

$$E = \frac{q}{4\pi\epsilon_0 r^2} = \boxed{k_e\frac{q}{r^2}}$$

This is the familiar electric field due to a point charge that we developed from Coulomb's law in Chapter 23.

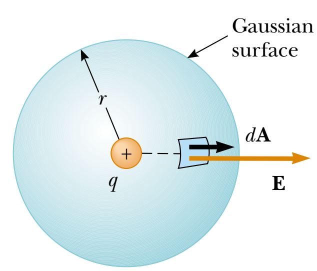


Figure 24.10 The point charge q is at the center of the spherical gaussian surface, and $\mathbf{E}$ is parallel to $d\mathbf{A}$ at every point on the surface.

EXAMPLE 24.5 A Spherically Symmetric Charge Distribution

11.6

An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q (Fig. 24.11). (a) Calculate the magnitude of the electric field at a point outside the sphere.

Solution Because the charge distribution is spherically symmetric, we again select a spherical gaussian surface of radius r, concentric with the sphere, as shown in Figure 24.11a. For this choice, conditions (1) and (2) are satisfied, as they were for the point charge in Example 24.4. Following the line of reasoning given in Example 24.4, we find that

$$E = \boxed{k_e \frac{Q}{r^2}} \qquad \text{(for } r > a\text{)}$$

Note that this result is identical to the one we obtained for a point charge. Therefore, we conclude that, for a uniformly charged sphere, the field in the region external to the sphere is *equivalent* to that of a point charge located at the center of the sphere.

(b) Find the magnitude of the electric field at a point inside the sphere.

Solution In this case we select a spherical gaussian surface having radius $r < a$, concentric with the insulated sphere (Fig. 24.11b). Let us denote the volume of this smaller sphere by V'. To apply Gauss's law in this situation, it is important to recognize that the charge q_{in} within the gaussian surface of volume V' is less than Q. To calculate q_{in}, we use the fact that $q_{\text{in}} = \rho V'$:

$$q_{\text{in}} = \rho V' = \rho(\tfrac{4}{3}\pi r^3)$$

By symmetry, the magnitude of the electric field is constant everywhere on the spherical gaussian surface and is normal to the surface at each point—both conditions (1) and (2) are satisfied. Therefore, Gauss's law in the region $r < a$ gives

$$\oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0}$$

Solving for E gives

$$E = \frac{q_{\text{in}}}{4\pi\epsilon_0 r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0}r$$

Because $\rho = Q/\frac{4}{3}\pi a^3$ by definition and since $k_e = 1/(4\pi\epsilon_0)$, this expression for E can be written as

$$E = \frac{Qr}{4\pi\epsilon_0 a^3} = \boxed{\frac{k_e Q}{a^3}r} \qquad \text{(for } r < a\text{)}$$

Note that this result for E differs from the one we obtained in part (a). It shows that $E \rightarrow 0$ as $r \rightarrow 0$. Therefore, the result eliminates the problem that would exist at $r = 0$ if E varied as $1/r^2$ inside the sphere as it does outside the sphere. That is, if $E \propto 1/r^2$ for $r < a$, the field would be infinite at $r = 0$, which is physically impossible. Note also that the expressions for parts (a) and (b) match when $r = a$.

A plot of E versus r is shown in Figure 24.12.

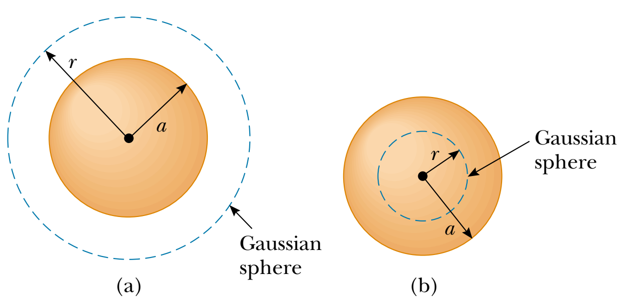


Figure 24.11 A uniformly charged insulating sphere of radius a and total charge Q. (a) The magnitude of the electric field at a point exterior to the sphere is k_eQ/r^2. (b) The magnitude of the electric field inside the insulating sphere is due only to the charge *within* the gaussian sphere defined by the dashed circle and is k_eQr/a^3.

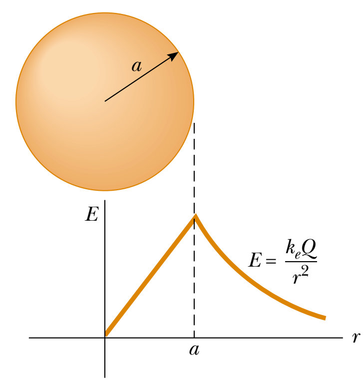


Figure 24.12 A plot of E versus r for a uniformly charged insulating sphere. The electric field inside the sphere ($r < a$) varies linearly with r. The field outside the sphere ($r > a$) is the same as that of a point charge Q located at $r = 0$.

EXAMPLE 24.6 The Electric Field Due to a Thin Spherical Shell

A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface (Fig. 24.13a). Find the electric field at points (a) outside and (b) inside the shell.

Solution (a) The calculation for the field outside the shell is identical to that for the solid sphere shown in Example 24.5a. If we construct a spherical gaussian surface of radius $r > a$ concentric with the shell (Fig. 24.13b), the charge inside this surface is Q. Therefore, the field at a point outside the shell is equivalent to that due to a point charge Q located at the center:

$$E = \boxed{k_e \frac{Q}{r^2}} \qquad \text{(for } r > a\text{)}$$

(b) The electric field inside the spherical shell is zero. This follows from Gauss's law applied to a spherical surface of radius $r < a$ concentric with the shell (Fig. 24.13c). Because of the spherical symmetry of the charge distribution and because the net charge inside the surface is zero—satisfaction of conditions (1) and (2) again—application of Gauss's law shows that $E = 0$ in the region $r < a$.

We obtain the same results using Equation 23.6 and integrating over the charge distribution. This calculation is rather complicated. Gauss's law allows us to determine these results in a much simpler way.

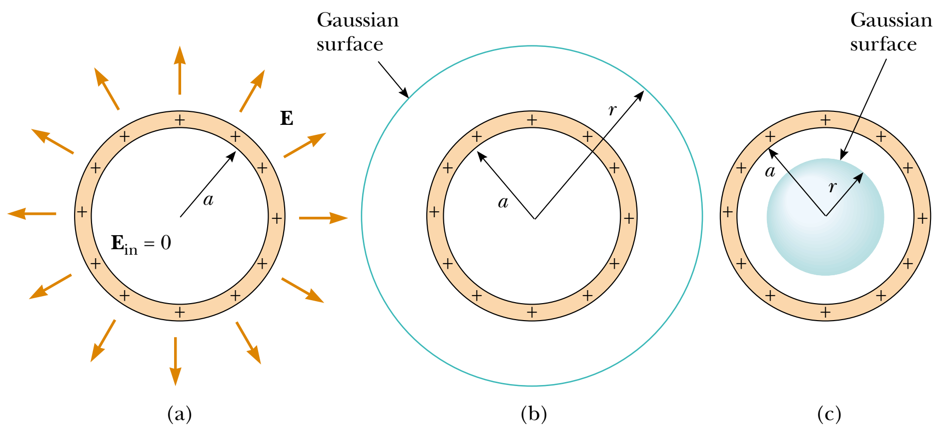


Figure 24.13 (a) The electric field inside a uniformly charged spherical shell is zero. The field outside is the same as that due to a point charge Q located at the center of the shell. (b) Gaussian surface for $r > a$. (c) Gaussian surface for $r < a$.

EXAMPLE 24.7 A Cylindrically Symmetric Charge Distribution

11.7

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ (Fig. 24.14a).

Solution The symmetry of the charge distribution requires that **E** be perpendicular to the line charge and directed outward, as shown in Figure 24.14a and b. To reflect the symmetry of the charge distribution, we select a cylindrical gaussian surface of radius r and length ℓ that is coaxial with the line charge. For the curved part of this surface, **E** is constant in magnitude and perpendicular to the surface at each point—satisfaction of conditions (1) and (2). Furthermore, the flux through the ends of the gaussian cylinder is zero because **E** is parallel to these surfaces—the first application we have seen of condition (3).

We take the surface integral in Gauss's law over the entire gaussian surface. Because of the zero value of $\mathbf{E} \cdot d\mathbf{A}$ for the ends of the cylinder, however, we can restrict our attention to only the curved surface of the cylinder.

The total charge inside our gaussian surface is $\lambda\ell$. Applying Gauss's law and conditions (1) and (2), we find that for the curved surface

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = E\oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda\ell}{\epsilon_0}$$

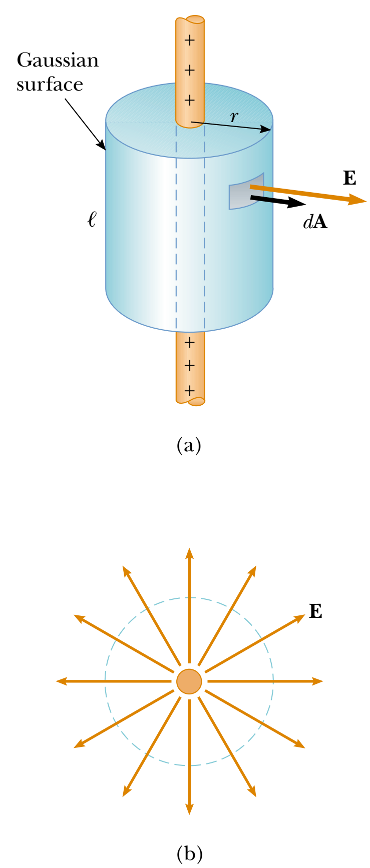


Figure 24.14 (a) An infinite line of charge surrounded by a cylindrical gaussian surface concentric with the line. (b) An end view shows that the electric field at the cylindrical surface is constant in magnitude and perpendicular to the surface.

The area of the curved surface is $A = 2\pi r\ell$; therefore,

$$E(2\pi r\ell) = \frac{\lambda\ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \boxed{2k_e\frac{\lambda}{r}} \quad \textbf{(24.7)}$$

Thus, we see that the electric field due to a cylindrically symmetric charge distribution varies as $1/r$, whereas the field external to a spherically symmetric charge distribution varies as $1/r^2$. Equation 24.7 was also derived in Chapter 23 (see Problem 35[b]), by integration of the field of a point charge.

If the line charge in this example were of finite length, the result for E would not be that given by Equation 24.7. A finite line charge does not possess sufficient symmetry for us to make use of Gauss's law. This is because the magnitude of the electric field is no longer constant over the surface of the gaussian cylinder—the field near the ends of the line would be different from that far from the ends. Thus, condition (1) would not be satisfied in this situation. Furthermore, **E** is not perpendicular to the cylindrical surface at all points—the field vectors near the ends would have a component parallel to the line. Thus, condition (2) would not be satisfied. When there is insufficient symmetry in the charge distribution, as in this situation, it is necessary to use Equation 23.6 to calculate **E**.

For points close to a finite line charge and far from the ends, Equation 24.7 gives a good approximation of the value of the field.

It is left for you to show (see Problem 29) that the electric field inside a uniformly charged rod of finite radius and infinite length is proportional to r.

EXAMPLE 24.8 A Nonconducting Plane of Charge

Find the electric field due to a nonconducting, infinite plane of positive charge with uniform surface charge density σ.

Solution By symmetry, **E** must be perpendicular to the plane and must have the same magnitude at all points equidistant from the plane. The fact that the direction of **E** is away from positive charges indicates that the direction of **E** on one side of the plane must be opposite its direction on the other side, as shown in Figure 24.15. A gaussian surface that reflects the symmetry is a small cylinder whose axis is perpendicular to the plane and whose ends each have an area A and are equidistant from the plane. Because **E** is parallel to the curved surface—and, therefore, perpendicular to $d\mathbf{A}$ everywhere on the surface—condition (3) is satisfied and there is no contribution to the surface integral from this surface. For the flat ends of the cylinder, conditions (1) and (2) are satisfied. The flux through each end of the cylinder is EA; hence, the total flux through the entire gaussian surface is just that through the ends, $\Phi_E = 2EA$.

Noting that the total charge inside the surface is $q_{\text{in}} = \sigma A$, we use Gauss's law and find that

$$\Phi_E = 2EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \boxed{\frac{\sigma}{2\epsilon_0}} \quad \textbf{(24.8)}$$

Because the distance from each flat end of the cylinder to the plane does not appear in Equation 24.8, we conclude that $E = \sigma/2\epsilon_0$ at any distance from the plane. That is, the field is uniform everywhere.

An important charge configuration related to this example consists of two parallel planes, one positively charged and the other negatively charged, and each with a surface charge density σ (see Problem 58). In this situation, the electric fields due to the two planes add in the region between the planes, resulting in a field of magnitude σ/ϵ_0, and cancel elsewhere to give a field of zero.

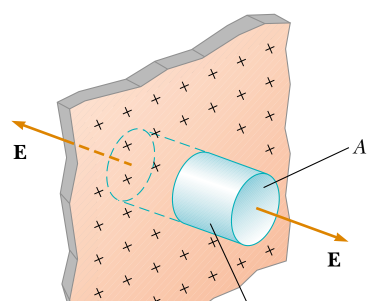


Figure 24.15 A cylindrical gaussian surface penetrating an infinite plane of charge. The flux is EA through each end of the gaussian surface and zero through its curved surface.

CONCEPTUAL EXAMPLE 24.9

Explain why Gauss's law cannot be used to calculate the electric field near an electric dipole, a charged disk, or a triangle with a point charge at each corner.

Solution The charge distributions of all these configurations do not have sufficient symmetry to make the use of Gauss's law practical. We cannot find a closed surface surrounding any of these distributions that satisfies one or more of conditions (1) through (4) listed at the beginning of this section.

24.4 CONDUCTORS IN ELECTROSTATIC EQUILIBRIUM

As we learned in Section 23.2, a good electrical conductor contains charges (electrons) that are not bound to any atom and therefore are free to move about within the material. When there is no net motion of charge within a conductor, the conductor is in **electrostatic equilibrium.** As we shall see, a conductor in electrostatic equilibrium has the following properties:

Properties of a conductor in electrostatic equilibrium

1. The electric field is zero everywhere inside the conductor.
2. If an isolated conductor carries a charge, the charge resides on its surface.
3. The electric field just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude σ/ϵ_0, where σ is the surface charge density at that point.
4. On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest.

We verify the first three properties in the discussion that follows. The fourth property is presented here without further discussion so that we have a complete list of properties for conductors in electrostatic equilibrium.

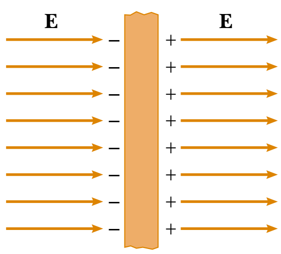


Figure 24.16 A conducting slab in an external electric field **E**. The charges induced on the two surfaces of the slab produce an electric field that opposes the external field, giving a resultant field of zero inside the slab.

We can understand the first property by considering a conducting slab placed in an external field **E** (Fig. 24.16). We can argue that the electric field inside the conductor *must* be zero under the assumption that we have electrostatic equilibrium. If the field were not zero, free charges in the conductor would accelerate under the action of the field. This motion of electrons, however, would mean that the conductor is not in electrostatic equilibrium. Thus, the existence of electrostatic equilibrium is consistent only with a zero field in the conductor.

Let us investigate how this zero field is accomplished. Before the external field is applied, free electrons are uniformly distributed throughout the conductor. When the external field is applied, the free electrons accelerate to the left in Figure 24.16, causing a plane of negative charge to be present on the left surface. The movement of electrons to the left results in a plane of positive charge on the right surface. These planes of charge create an additional electric field inside the conductor that opposes the external field. As the electrons move, the surface charge density increases until the magnitude of the internal field equals that of the external field, and the net result is a net field of zero inside the conductor. The time it takes a good conductor to reach equilibrium is of the order of 10^{-16} s, which for most purposes can be considered instantaneous.

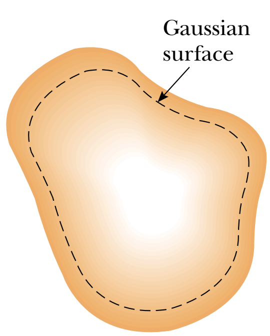
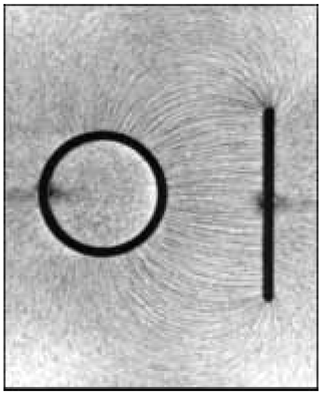


Figure 24.17 A conductor of arbitrary shape. The broken line represents a gaussian surface just inside the conductor.

We can use Gauss's law to verify the second property of a conductor in electrostatic equilibrium. Figure 24.17 shows an arbitrarily shaped conductor. A gaussian surface is drawn inside the conductor and can be as close to the conductor's surface as we wish. As we have just shown, the electric field everywhere inside the conductor is zero when it is in electrostatic equilibrium. Therefore, the electric field must be zero at every point on the gaussian surface, in accordance with condition (4) in Section 24.3. Thus, the net flux through this gaussian surface is zero. From this result and Gauss's law, we conclude that the net charge inside the gaussian sur-



Electric field pattern surrounding a charged conducting plate placed near an oppositely charged conducting cylinder. Small pieces of thread suspended in oil align with the electric field lines. Note that (1) the field lines are perpendicular to both conductors and (2) there are no lines inside the cylinder ($E = 0$).

face is zero. Because there can be no net charge inside the gaussian surface (which is arbitrarily close to the conductor's surface), **any net charge on the conductor must reside on its surface.** Gauss's law does not indicate how this excess charge is distributed on the conductor's surface.

We can also use Gauss's law to verify the third property. We draw a gaussian surface in the shape of a small cylinder whose end faces are parallel to the surface of the conductor (Fig. 24.18). Part of the cylinder is just outside the conductor, and part is inside. The field is normal to the conductor's surface from the condition of electrostatic equilibrium. (If **E** had a component parallel to the conductor's surface, the free charges would move along the surface; in such a case, the conductor would not be in equilibrium.) Thus, we satisfy condition (3) in Section 24.3 for the curved part of the cylindrical gaussian surface—there is no flux through this part of the gaussian surface because **E** is parallel to the surface. There is no flux through the flat face of the cylinder inside the conductor because here $\mathbf{E} = 0$—satisfaction of condition (4). Hence, the net flux through the gaussian surface is that through only the flat face outside the conductor, where the field is perpendicular to the gaussian surface. Using conditions (1) and (2) for this face, the flux is EA, where E is the electric field just outside the conductor and A is the area of the cylinder's face. Applying Gauss's law to this surface, we obtain

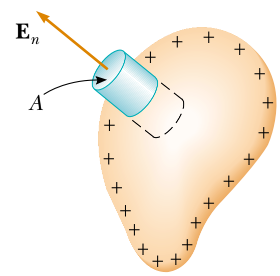


Figure 24.18 A gaussian surface in the shape of a small cylinder is used to calculate the electric field just outside a charged conductor. The flux through the gaussian surface is E_nA. Remember that **E** is zero inside the conductor.

$$\Phi_E = \oint E\, dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

where we have used the fact that $q_{\text{in}} = \sigma A$. Solving for E gives

$$E = \frac{\sigma}{\epsilon_0} \qquad \textbf{(24.9)}$$

Electric field just outside a charged conductor

EXAMPLE 24.10 A Sphere Inside a Spherical Shell

A solid conducting sphere of radius a carries a net positive charge $2Q$. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge $-Q$. Using Gauss's law, find the electric field in the regions labeled ①, ②, ③, and ④ in Figure 24.19 and the charge distribution on the shell when the entire system is in electrostatic equilibrium.

Solution First note that the charge distributions on both the sphere and the shell are characterized by spherical symmetry around their common center. To determine the electric field at various distances r from this center, we construct a spherical gaussian surface for each of the four regions of interest. Such a surface for region ② is shown in Figure 24.19.

To find E inside the solid sphere (region ①), consider a

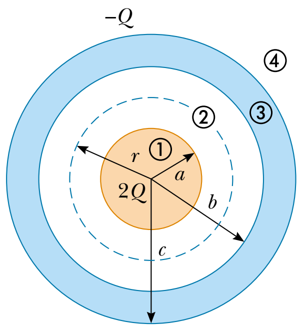


Figure 24.19 A solid conducting sphere of radius a and carrying a charge $2Q$ surrounded by a conducting spherical shell carrying a charge $-Q$.

gaussian surface of radius $r < a$. Because there can be no charge inside a conductor in electrostatic equilibrium, we see that $q_{\text{in}} = 0$; thus, on the basis of Gauss's law and symmetry, $E_1 = 0$ for $r < a$.

In region ②—between the surface of the solid sphere and the inner surface of the shell—we construct a spherical gaussian surface of radius r where $a < r < b$ and note that the charge inside this surface is $+2Q$ (the charge on the solid sphere). Because of the spherical symmetry, the electric field lines must be directed radially outward and be constant in magnitude on the gaussian surface. Following Example 24.4 and using Gauss's law, we find that

$$E_2 A = E_2(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0} = \frac{2Q}{\epsilon_0}$$

$$E_2 = \frac{2Q}{4\pi\epsilon_0 r^2} = \boxed{\frac{2k_e Q}{r^2}} \qquad (\text{for } a < r < b)$$

In region ④, where $r > c$, the spherical gaussian surface we construct surrounds a total charge of $q_{\text{in}} = 2Q + (-Q) = Q$. Therefore, application of Gauss's law to this surface gives

$$E_4 = \boxed{\frac{k_e Q}{r^2}} \qquad (\text{for } r > c)$$

In region ③, the electric field must be zero because the spherical shell is also a conductor in equilibrium. If we construct a gaussian surface of radius r where $b < r < c$, we see that q_{in} must be zero because $E_3 = 0$. From this argument, we conclude that the charge on the inner surface of the spherical shell must be $-2Q$ to cancel the charge $+2Q$ on the solid sphere. Because the net charge on the shell is $-Q$, we conclude that its outer surface must carry a charge $+Q$.

Quick Quiz 24.3

How would the electric flux through a gaussian surface surrounding the shell in Example 24.10 change if the solid sphere were off-center but still inside the shell?

Optional Section

24.5 EXPERIMENTAL VERIFICATION OF GAUSS'S LAW AND COULOMB'S LAW

When a net charge is placed on a conductor, the charge distributes itself on the surface in such a way that the electric field inside the conductor is zero. Gauss's law shows that there can be no net charge inside the conductor in this situation. In this section, we investigate an experimental verification of the absence of this charge.

We have seen that Gauss's law is equivalent to Equation 23.6, the expression for the electric field of a distribution of charge. Because this equation arises from Coulomb's law, we can claim theoretically that Gauss's law and Coulomb's law are equivalent. Hence, it is possible to test the validity of both laws by attempting to detect a net charge inside a conductor or, equivalently, a nonzero electric field inside the conductor. If a nonzero field is detected within the conductor, Gauss's law and Coulomb's law are invalid. Many experiments, including early work by Faraday, Cavendish, and Maxwell, have been performed to detect the field inside a conductor. In all reported cases, no electric field could be detected inside a conductor.

Here is one of the experiments that can be performed.[2] A positively charged metal ball at the end of a silk thread is lowered through a small opening into an uncharged hollow conductor that is insulated from ground (Fig. 24.20a). The positively charged ball induces a negative charge on the inner wall of the hollow conductor, leaving an equal positive charge on the outer wall (Fig. 24.20b). The presence of positive charge on the outer wall is indicated by the deflection of the needle of an electrometer (a device used to measure charge and that measures charge only on the outer surface of the conductor). The ball is then lowered and allowed to touch the inner surface of the hollow conductor (Fig. 24.20c). Charge is transferred between the ball and the inner surface so that neither is charged after contact is made. The needle deflection remains unchanged while this happens, indicating that the charge on the outer surface is unaffected. When the ball is removed, the electrometer reading remains the same (Fig. 24.20d). Furthermore, the ball is found to be uncharged; this verifies that charge was transferred between the ball and the inner surface of the hollow conductor. The overall effect is that the charge that was originally on the ball now appears on the hollow conductor. The fact that the deflection of the needle on the electrometer measuring the charge on the outer surface remained unchanged regardless of what was happening inside the hollow conductor indicates that the net charge on the system always resided on the outer surface of the conductor.

If we now apply another positive charge to the metal ball and place it near the outside of the conductor, it is repelled by the conductor. This demonstrates that $\mathbf{E} \neq 0$ outside the conductor, a finding consistent with the fact that the conductor carries a net charge. If the charged metal ball is now lowered into the interior of the charged hollow conductor, it exhibits no evidence of an electric force. This shows that $\mathbf{E} = 0$ inside the hollow conductor.

This experiment verifies the predictions of Gauss's law and therefore verifies Coulomb's law. The equivalence of Gauss's law and Coulomb's law is due to the inverse-square behavior of the electric force. Thus, we can interpret this experiment as verifying the exponent of 2 in the $1/r^2$ behavior of the electric force. Experiments by Williams, Faller, and Hill in 1971 showed that the exponent of r in Coulomb's law is $(2 + \delta)$, where $\delta = (2.7 \pm 3.1) \times 10^{-16}$!

In the experiment we have described, the charged ball hanging in the hollow conductor would show no deflection even in the case in which an external electric field is applied to the entire system. The field inside the conductor is still zero. This ability of conductors to "block" external electric fields is utilized in many places, from electromagnetic shielding for computer components to thin metal coatings on the glass in airport control towers to keep radar originating outside the tower from disrupting the electronics inside. Cellular telephone users riding trains like the one pictured at the beginning of the chapter have to speak loudly to be heard above the noise of the train. In response to complaints from other passengers, the train companies are considering coating the windows with a thin metallic conductor. This coating, combined with the metal frame of the train car, blocks cellular telephone transmissions into and out of the train.

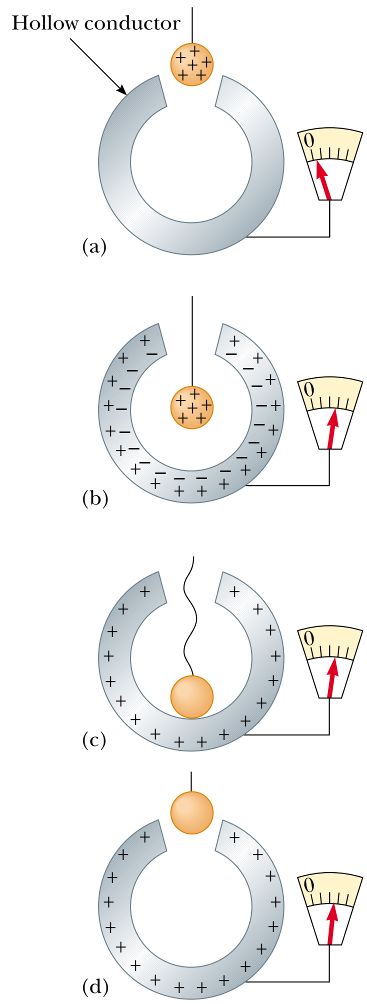


Figure 24.20 An experiment showing that any charge transferred to a conductor resides on its surface in electrostatic equilibrium. The hollow conductor is insulated from ground, and the small metal ball is supported by an insulating thread.

QuickLab

Wrap a radio or cordless telephone in aluminum foil and see if it still works. Does it matter if the foil touches the antenna?

[2] The experiment is often referred to as *Faraday's ice-pail experiment* because Faraday, the first to perform it, used an ice pail for the hollow conductor.

Optional Section

24.6 FORMAL DERIVATION OF GAUSS'S LAW

One way of deriving Gauss's law involves *solid angles*. Consider a spherical surface of radius r containing an area element ΔA. The solid angle $\Delta\Omega$ (uppercase Greek omega) subtended at the center of the sphere by this element is defined to be

$$\Delta\Omega \equiv \frac{\Delta A}{r^2}$$

From this equation, we see that $\Delta\Omega$ has no dimensions because ΔA and r^2 both have dimensions L^2. The dimensionless unit of a solid angle is the **steradian.** (You may want to compare this equation to Equation 10.1b, the definition of the radian.) Because the surface area of a sphere is $4\pi r^2$, the total solid angle subtended by the sphere is

$$\Omega = \frac{4\pi r^2}{r^2} = 4\pi \text{ steradians}$$

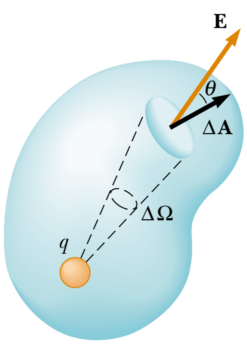


Figure 24.21 A closed surface of arbitrary shape surrounds a point charge q. The net electric flux through the surface is independent of the shape of the surface.

Now consider a point charge q surrounded by a closed surface of arbitrary shape (Fig. 24.21). The total electric flux through this surface can be obtained by evaluating $\mathbf{E} \cdot \Delta\mathbf{A}$ for each small area element ΔA and summing over all elements. The flux through each element is

$$\Delta\Phi_E = \mathbf{E} \cdot \Delta\mathbf{A} = E\,\Delta A \cos\theta = k_e q \frac{\Delta A \cos\theta}{r^2}$$

where r is the distance from the charge to the area element, θ is the angle between the electric field $\mathbf{E}$ and $\Delta\mathbf{A}$ for the element, and $E = k_e q/r^2$ for a point charge. In Figure 24.22, we see that the projection of the area element perpendicular to the radius vector is $\Delta A \cos\theta$. Thus, the quantity $\Delta A \cos\theta/r^2$ is equal to the solid angle $\Delta\Omega$ that the surface element ΔA subtends at the charge q. We also see that $\Delta\Omega$ is equal to the solid angle subtended by the area element of a spherical surface of radius r. Because the total solid angle at a point is 4π steradians, the total flux

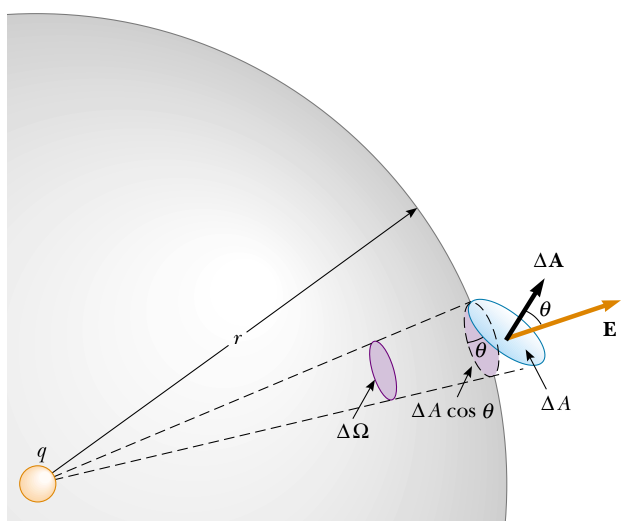


Figure 24.22 The area element ΔA subtends a solid angle $\Delta\Omega = (\Delta A \cos\theta)/r^2$ at the charge q.

through the closed surface is

$$\Phi_E = k_e q \oint \frac{dA \cos \theta}{r^2} = k_e q \oint d\Omega = 4\pi k_e q = \frac{q}{\epsilon_0}$$

Thus we have derived Gauss's law, Equation 24.6. Note that this result is independent of the shape of the closed surface and independent of the position of the charge within the surface.

SUMMARY

Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle θ with the normal to a surface of area A, the electric flux through the surface is

$$\Phi_E = EA \cos \theta \tag{24.2}$$

In general, the electric flux through a surface is

$$\Phi_E = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A} \tag{24.3}$$

You need to be able to apply Equations 24.2 and 24.3 in a variety of situations, particularly those in which symmetry simplifies the calculation.

Gauss's law says that the net electric flux Φ_E through any closed gaussian surface is equal to the *net* charge inside the surface divided by ϵ_0:

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} \tag{24.6}$$

Using Gauss's law, you can calculate the electric field due to various symmetric charge distributions. Table 24.1 lists some typical results.

TABLE 24.1 Typical Electric Field Calculations Using Gauss's Law

Charge Distribution	Electric Field	Location
Insulating sphere of radius R, uniform charge density, and total charge Q	$k_e \dfrac{Q}{r^2}$	$r > R$
	$k_e \dfrac{Q}{R^3} r$	$r < R$
Thin spherical shell of radius R and total charge Q	$k_e \dfrac{Q}{r^2}$	$r > R$
	0	$r < R$
Line charge of infinite length and charge per unit length λ	$2k_e \dfrac{\lambda}{r}$	Outside the line
Nonconducting, infinite charged plane having surface charge density σ	$\dfrac{\sigma}{2\epsilon_0}$	Everywhere outside the plane
Conductor having surface charge density σ	$\dfrac{\sigma}{\epsilon_0}$	Just outside the conductor
	0	Inside the conductor

A conductor in **electrostatic equilibrium** has the following properties:

1. The electric field is zero everywhere inside the conductor.
2. Any net charge on the conductor resides entirely on its surface.
3. The electric field just outside the conductor is perpendicular to its surface and has a magnitude σ/ϵ_0, where σ is the surface charge density at that point.
4. On an irregularly shaped conductor, the surface charge density is greatest where the radius of curvature of the surface is the smallest.

Problem-Solving Hints

Gauss's law, as we have seen, is very powerful in solving problems involving highly symmetric charge distributions. In this chapter, you encountered three kinds of symmetry: planar, cylindrical, and spherical. It is important to review Examples 24.4 through 24.10 and to adhere to the following procedure when using Gauss's law:

- Select a gaussian surface that has a symmetry to match that of the charge distribution and satisfies one or more of the conditions listed in Section 24.3. For point charges or spherically symmetric charge distributions, the gaussian surface should be a sphere centered on the charge as in Examples 24.4, 24.5, 24.6, and 24.10. For uniform line charges or uniformly charged cylinders, your gaussian surface should be a cylindrical surface that is coaxial with the line charge or cylinder as in Example 24.7. For planes of charge, a useful choice is a cylindrical gaussian surface that straddles the plane, as shown in Example 24.8. These choices enable you to simplify the surface integral that appears in Gauss's law and represents the total electric flux through that surface.
- Evaluate the q_{in}/ϵ_0 term in Gauss's law, which amounts to calculating the total electric charge q_{in} inside the gaussian surface. If the charge density is uniform (that is, if λ, σ, or ρ is constant), simply multiply that charge density by the length, area, or volume enclosed by the gaussian surface. If the charge distribution is *nonuniform,* integrate the charge density over the region enclosed by the gaussian surface. For example, if the charge is distributed along a line, integrate the expression $dq = \lambda\, dx$, where dq is the charge on an infinitesimal length element dx. For a plane of charge, integrate $dq = \sigma\, dA$, where dA is an infinitesimal element of area. For a volume of charge, integrate $dq = \rho\, dV$, where dV is an infinitesimal element of volume.
- Once the terms in Gauss's law have been evaluated, solve for the electric field on the gaussian surface if the charge distribution is given in the problem. Conversely, if the electric field is known, calculate the charge distribution that produces the field.

Questions

1. The Sun is lower in the sky during the winter than it is in the summer. How does this change the flux of sunlight hitting a given area on the surface of the Earth? How does this affect the weather?

2. If the electric field in a region of space is zero, can you conclude no electric charges are in that region? Explain.

3. If more electric field lines are leaving a gaussian surface than entering, what can you conclude about the net charge enclosed by that surface?

4. A uniform electric field exists in a region of space in which there are no charges. What can you conclude about the net electric flux through a gaussian surface placed in this region of space?

5. If the total charge inside a closed surface is known but the distribution of the charge is unspecified, can you use Gauss's law to find the electric field? Explain.
6. Explain why the electric flux through a closed surface with a given enclosed charge is independent of the size or shape of the surface.
7. Consider the electric field due to a nonconducting infinite plane having a uniform charge density. Explain why the electric field does not depend on the distance from the plane in terms of the spacing of the electric field lines.
8. Use Gauss's law to explain why electric field lines must begin or end on electric charges. (*Hint:* Change the size of the gaussian surface.)
9. On the basis of the repulsive nature of the force between like charges and the freedom of motion of charge within the conductor, explain why excess charge on an isolated conductor must reside on its surface.
10. A person is placed in a large, hollow metallic sphere that is insulated from ground. If a large charge is placed on the sphere, will the person be harmed upon touching the inside of the sphere? Explain what will happen if the person also has an initial charge whose sign is opposite that of the charge on the sphere.
11. How would the observations described in Figure 24.20 differ if the hollow conductor were grounded? How would they differ if the small charged ball were an insulator rather than a conductor?
12. What other experiment might be performed on the ball in Figure 24.20 to show that its charge was transferred to the hollow conductor?
13. What would happen to the electrometer reading if the charged ball in Figure 24.20 touched the inner wall of the conductor? the outer wall?
14. You may have heard that one of the safer places to be during a lightning storm is inside a car. Why would this be the case?
15. Two solid spheres, both of radius R, carry identical total charges Q. One sphere is a good conductor, while the other is an insulator. If the charge on the insulating sphere is uniformly distributed throughout its interior volume, how do the electric fields outside these two spheres compare? Are the fields identical inside the two spheres?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging   ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/**   = Computer useful in solving problem   = Interactive Physics
   = paired numerical/symbolic problems

Section 24.1 Electric Flux

1. An electric field with a magnitude of 3.50 kN/C is applied along the x axis. Calculate the electric flux through a rectangular plane 0.350 m wide and 0.700 m long if (a) the plane is parallel to the yz plane; (b) the plane is parallel to the xy plane; and (c) the plane contains the y axis, and its normal makes an angle of 40.0° with the x axis.
2. A vertical electric field of magnitude 2.00×10^4 N/C exists above the Earth's surface on a day when a thunderstorm is brewing. A car with a rectangular size of approximately 6.00 m by 3.00 m is traveling along a roadway sloping downward at 10.0°. Determine the electric flux through the bottom of the car.
3. A 40.0-cm-diameter loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is measured to be 5.20×10^5 N·m²/C. What is the magnitude of the electric field?
4. A spherical shell is placed in a uniform electric field. Find the total electric flux through the shell.
5. Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 7.80 \times 10^4$ N/C, as shown in Figure P24.5. Calculate the electric flux through (a) the vertical rectangular surface, (b) the slanted surface, and (c) the entire surface of the box.

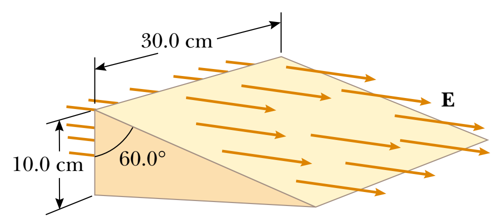


Figure P24.5

6. A uniform electric field $a\mathbf{i} + b\mathbf{j}$ intersects a surface of area A. What is the flux through this area if the surface lies (a) in the yz plane? (b) in the xz plane? (c) in the xy plane?
7. A point charge q is located at the center of a uniform ring having linear charge density λ and radius a, as shown in Figure P24.7. Determine the total electric flux

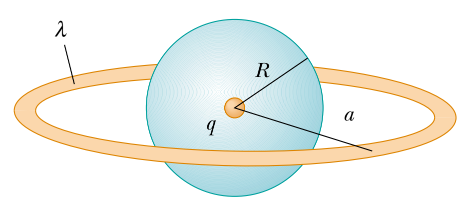


Figure P24.7

through a sphere centered at the point charge and having radius R, where $R < a$.

8. A pyramid with a 6.00-m-square base and height of 4.00 m is placed in a vertical electric field of 52.0 N/C. Calculate the total electric flux through the pyramid's four slanted surfaces.

9. A cone with base radius R and height h is located on a horizontal table. A horizontal uniform field E penetrates the cone, as shown in Figure P24.9. Determine the electric flux that enters the left-hand side of the cone.

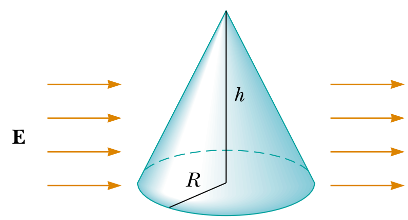


Figure P24.9

Section 24.2 Gauss's Law

10. The electric field everywhere on the surface of a thin spherical shell of radius 0.750 m is measured to be equal to 890 N/C and points radially toward the center of the sphere. (a) What is the net charge within the sphere's surface? (b) What can you conclude about the nature and distribution of the charge inside the spherical shell?

11. The following charges are located inside a submarine: 5.00 μC, -9.00 μC, 27.0 μC, and -84.0 μC. (a) Calculate the net electric flux through the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

12. Four closed surfaces, S_1 through S_4, together with the charges $-2Q$, Q, and $-Q$ are sketched in Figure P24.12. Find the electric flux through each surface.

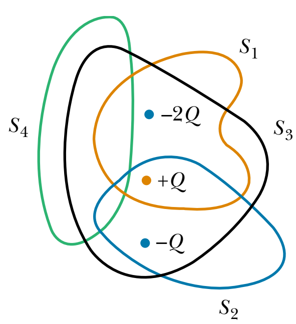


Figure P24.12

13. (a) A point charge q is located a distance d from an infinite plane. Determine the electric flux through the plane due to the point charge. (b) A point charge q is located a *very small* distance from the center of a *very large* square on the line perpendicular to the square and going through its center. Determine the approximate electric flux through the square due to the point charge. (c) Explain why the answers to parts (a) and (b) are identical.

14. Calculate the total electric flux through the paraboloidal surface due to a constant electric field of magnitude E_0 in the direction shown in Figure P24.14.

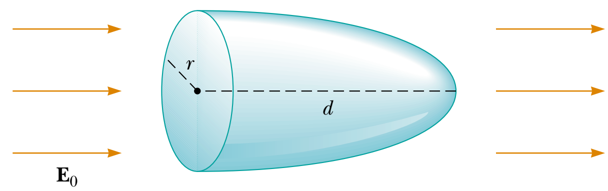


Figure P24.14

WEB **15.** A point charge Q is located just above the center of the flat face of a hemisphere of radius R, as shown in Figure P24.15. What is the electric flux (a) through the curved surface and (b) through the flat face?

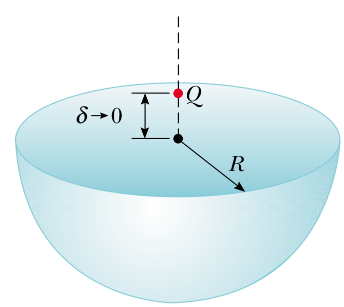


Figure P24.15

16. A point charge of 12.0 μC is placed at the center of a spherical shell of radius 22.0 cm. What is the total electric flux through (a) the surface of the shell and (b) any hemispherical surface of the shell? (c) Do the results depend on the radius? Explain.

17. A point charge of 0.046 2 μC is inside a pyramid. Determine the total electric flux through the surface of the pyramid.

18. An infinitely long line charge having a uniform charge per unit length λ lies a distance d from point O, as shown in Figure P24.18. Determine the total electric flux through the surface of a sphere of radius R centered at O resulting from this line charge. (*Hint:* Consider both cases: when $R < d$, and when $R > d$.)

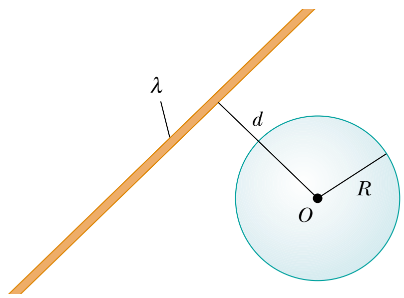


Figure P24.18

19. A point charge $Q = 5.00\ \mu C$ is located at the center of a cube of side $L = 0.100$ m. In addition, six other identical point charges having $q = -1.00\ \mu C$ are positioned symmetrically around Q, as shown in Figure P24.19. Determine the electric flux through one face of the cube.

20. A point charge Q is located at the center of a cube of side L. In addition, six other identical negative point charges are positioned symmetrically around Q, as shown in Figure P24.19. Determine the electric flux through one face of the cube.

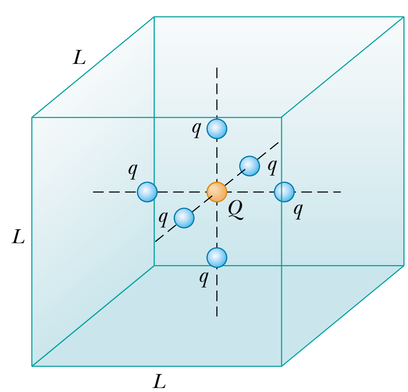


Figure P24.19 Problems 19 and 20.

21. Consider an infinitely long line charge having uniform charge per unit length λ. Determine the total electric flux through a closed right circular cylinder of length L and radius R that is parallel to the line charge, if the distance between the axis of the cylinder and the line charge is d. (*Hint:* Consider both cases: when $R < d$, and when $R > d$.)

22. A $10.0\text{-}\mu C$ charge located at the origin of a cartesian coordinate system is surrounded by a nonconducting hollow sphere of radius 10.0 cm. A drill with a radius of 1.00 mm is aligned along the z axis, and a hole is drilled in the sphere. Calculate the electric flux through the hole.

23. A charge of 170 μC is at the center of a cube of side 80.0 cm. (a) Find the total flux through each face of the cube. (b) Find the flux through the whole surface of the cube. (c) Would your answers to parts (a) or (b) change if the charge were not at the center? Explain.

24. The total electric flux through a closed surface in the shape of a cylinder is $8.60 \times 10^4\ N \cdot m^2/C$. (a) What is the net charge within the cylinder? (b) From the information given, what can you say about the charge within the cylinder? (c) How would your answers to parts (a) and (b) change if the net flux were $-8.60 \times 10^4\ N \cdot m^2/C$?

25. The line ag is a diagonal of a cube (Fig. P24.25). A point charge q is located on the extension of line ag, very close to vertex a of the cube. Determine the electric flux through each of the sides of the cube that meet at the point a.

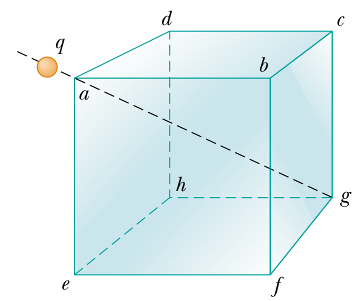


Figure P24.25

Section 24.3 Application of Gauss's Law to Charged Insulators

26. Determine the magnitude of the electric field at the surface of a lead-208 nucleus, which contains 82 protons and 126 neutrons. Assume that the lead nucleus has a volume 208 times that of one proton, and consider a proton to be a sphere of radius 1.20×10^{-15} m.

27. A solid sphere of radius 40.0 cm has a total positive charge of 26.0 μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere.

28. A cylindrical shell of radius 7.00 cm and length 240 cm has its charge uniformly distributed on its curved surface. The magnitude of the electric field at a point 19.0 cm radially outward from its axis (measured from the midpoint of the shell) is 36.0 kN/C. Use approximate relationships to find (a) the net charge on the shell and (b) the electric field at a point 4.00 cm from the axis, measured radially outward from the midpoint of the shell.

WEB 29. Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the axis where $r < R$.

30. A nonconducting wall carries a uniform charge density of 8.60 μC/cm^2. What is the electric field 7.00 cm in front of the wall? Does your result change as the distance from the wall is varied?

31. Consider a thin spherical shell of radius 14.0 cm with a total charge of 32.0 μC distributed uniformly on its surface. Find the electric field (a) 10.0 cm and (b) 20.0 cm from the center of the charge distribution.

32. In nuclear fission, a nucleus of uranium-238, which contains 92 protons, divides into two smaller spheres, each having 46 protons and a radius of 5.90×10^{-15} m. What is the magnitude of the repulsive electric force pushing the two spheres apart?

33. Fill two rubber balloons with air. Suspend both of them from the same point on strings of equal length. Rub each with wool or your hair, so that they hang apart with a noticeable separation between them. Make order-of-magnitude estimates of (a) the force on each, (b) the charge on each, (c) the field each creates at the center of the other, and (d) the total flux of electric field created by each balloon. In your solution, state the quantities you take as data and the values you measure or estimate for them.

34. An insulating sphere is 8.00 cm in diameter and carries a 5.70-μC charge uniformly distributed throughout its interior volume. Calculate the charge enclosed by a concentric spherical surface with radius (a) $r = 2.00$ cm and (b) $r = 6.00$ cm.

35. A uniformly charged, straight filament 7.00 m in length has a total positive charge of 2.00 μC. An uncharged cardboard cylinder 2.00 cm in length and 10.0 cm in radius surrounds the filament at its center, with the filament as the axis of the cylinder. Using reasonable approximations, find (a) the electric field at the surface of the cylinder and (b) the total electric flux through the cylinder.

36. The charge per unit length on a long, straight filament is -90.0 μC/m. Find the electric field (a) 10.0 cm, (b) 20.0 cm, and (c) 100 cm from the filament, where distances are measured perpendicular to the length of the filament.

37. A large flat sheet of charge has a charge per unit area of 9.00 μC/m^2. Find the electric field just above the surface of the sheet, measured from its midpoint.

Section 24.4 Conductors in Electrostatic Equilibrium

38. On a clear, sunny day, a vertical electrical field of about 130 N/C points down over flat ground. What is the surface charge density on the ground for these conditions?

39. A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of 30.0 nC/m. Find the electric field (a) 3.00 cm, (b) 10.0 cm, and (c) 100 cm from the axis of the rod, where distances are measured perpendicular to the rod.

40. A very large, thin, flat plate of aluminum of area A has a total charge Q uniformly distributed over its surfaces. If the same charge is spread uniformly over the *upper* surface of an otherwise identical glass plate, compare the electric fields just above the center of the upper surface of each plate.

41. A square plate of copper with 50.0-cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed perpendicularly to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

42. A hollow conducting sphere is surrounded by a larger concentric, spherical, conducting shell. The inner sphere has a charge $-Q$, and the outer sphere has a charge $3Q$. The charges are in electrostatic equilibrium. Using Gauss's law, find the charges and the electric fields everywhere.

43. Two identical conducting spheres each having a radius of 0.500 cm are connected by a light 2.00-m-long conducting wire. Determine the tension in the wire if 60.0 μC is placed on one of the conductors. (*Hint:* Assume that the surface distribution of charge on each sphere is uniform.)

44. The electric field on the surface of an irregularly shaped conductor varies from 56.0 kN/C to 28.0 kN/C. Calculate the local surface charge density at the point on the surface where the radius of curvature of the surface is (a) greatest and (b) smallest.

45. A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 2λ. From this information, use Gauss's law to find (a) the charge per unit length on the inner and outer surfaces of the cylinder and (b) the electric field outside the cylinder, a distance r from the axis.

46. A conducting spherical shell of radius 15.0 cm carries a net charge of -6.40 μC uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

WEB **47.** A thin conducting plate 50.0 cm on a side lies in the xy plane. If a total charge of 4.00×10^{-8} C is placed on the plate, find (a) the charge density on the plate, (b) the electric field just above the plate, and (c) the electric field just below the plate.

48. A conducting spherical shell having an inner radius of a and an outer radius of b carries a net charge Q. If a point charge q is placed at the center of this shell, determine the surface charge density on (a) the inner surface of the shell and (b) the outer surface of the shell.

49. A solid conducting sphere of radius 2.00 cm has a charge 8.00 μC. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge -4.00 μC. Find the electric field at (a) $r = 1.00$ cm, (b) $r = 3.00$ cm, (c) $r = 4.50$ cm, and (d) $r = 7.00$ cm from the center of this charge configuration.

50. A positive point charge is at a distance of $R/2$ from the center of an uncharged thin conducting spherical shell of radius R. Sketch the electric field lines set up by this arrangement both inside and outside the shell.

(Optional)

Section 24.5 Experimental Verification of Gauss's Law and Coulomb's Law

Section 24.6 Formal Derivation of Gauss's Law

51. A sphere of radius R surrounds a point charge Q, located at its center. (a) Show that the electric flux through a circular cap of half-angle θ (Fig. P24.51) is

$$\Phi_E = \frac{Q}{2\epsilon_0}(1 - \cos\theta)$$

What is the flux for (b) $\theta = 90°$ and (c) $\theta = 180°$?

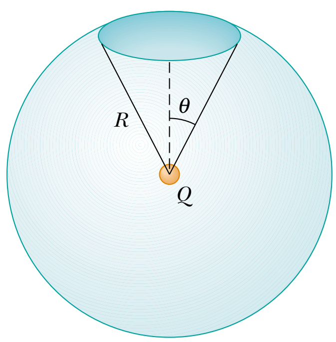


Figure P24.51

ADDITIONAL PROBLEMS

52. A nonuniform electric field is given by the expression $\mathbf{E} = ay\mathbf{i} + bz\mathbf{j} + cx\mathbf{k}$, where a, b, and c are constants. Determine the electric flux through a rectangular surface in the xy plane, extending from $x = 0$ to $x = w$ and from $y = 0$ to $y = h$.

53. A solid insulating sphere of radius a carries a net positive charge $3Q$, uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius b and outer radius c, and having a net charge $-Q$, as shown in Figure P24.53. (a) Construct a spherical gaussian surface of radius $r > c$ and find the net charge enclosed by this surface. (b) What is the direction of the electric field at $r > c$? (c) Find the electric field at $r > c$. (d) Find the electric field in the region with radius r where $c > r > b$. (e) Construct a spherical gaussian surface of radius r, where $c > r > b$, and find the net charge enclosed by this surface. (f) Construct a spherical gaussian surface of radius r, where $b > r > a$, and find the net charge enclosed by this surface. (g) Find the electric field in the region $b > r > a$. (h) Construct a spherical gaussian surface of radius $r < a$, and find an expression for the net charge enclosed by this surface, as a function of r. Note that the charge inside this surface is less than $3Q$. (i) Find the electric field in the region $r < a$. (j) Determine the charge on the inner surface of the conducting shell. (k) Determine the charge on the outer surface of the conducting shell. (l) Make a plot of the magnitude of the electric field versus r.

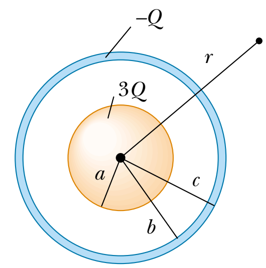


Figure P24.53

54. Consider two identical conducting spheres whose surfaces are separated by a small distance. One sphere is given a large net positive charge, while the other is given a small net positive charge. It is found that the force between them is attractive even though both spheres have net charges of the same sign. Explain how this is possible.

WEB **55.** A solid, insulating sphere of radius a has a uniform charge density ρ and a total charge Q. Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are b and c, as shown in Figure P24.55. (a) Find the magnitude of the electric field in the regions $r < a$, $a < r < b$, $b < r < c$, and $r > c$. (b) Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.

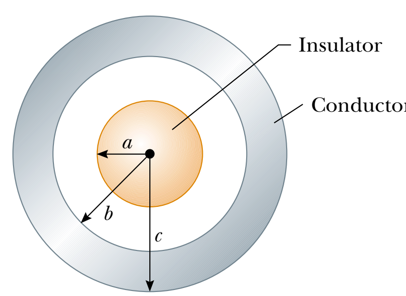


Figure P24.55 Problems 55 and 56.

56. For the configuration shown in Figure P24.55, suppose that $a = 5.00$ cm, $b = 20.0$ cm, and $c = 25.0$ cm. Furthermore, suppose that the electric field at a point 10.0 cm from the center is 3.60×10^3 N/C radially inward, while the electric field at a point 50.0 cm from the center is 2.00×10^2 N/C radially outward. From this information, find (a) the charge on the insulating sphere,

(b) the net charge on the hollow conducting sphere, and (c) the total charge on the inner and outer surfaces of the hollow conducting sphere.

57. An infinitely long cylindrical insulating shell of inner radius a and outer radius b has a uniform volume charge density ρ (C/m^3). A line of charge density λ (C/m) is placed along the axis of the shell. Determine the electric field intensity everywhere.

58. Two infinite, nonconducting sheets of charge are parallel to each other, as shown in Figure P24.58. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density $-\sigma$. Calculate the value of the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets. (*Hint:* See Example 24.8.)

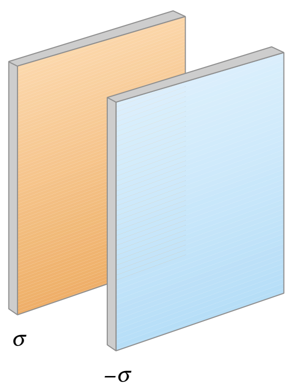


Figure P24.58

WEB **59.** Repeat the calculations for Problem 58 when both sheets have *positive* uniform surface charge densities of value σ.

60. A sphere of radius $2a$ is made of a nonconducting material that has a uniform volume charge density ρ. (Assume that the material does not affect the electric field.) A spherical cavity of radius a is now removed from the sphere, as shown in Figure P24.60. Show that the electric field within the cavity is uniform and is given by $E_x = 0$ and $E_y = \rho a/3\epsilon_0$. (*Hint:* The field within the cavity is the superposition of the field due to the original uncut sphere, plus the field due to a sphere the size of the cavity with a uniform negative charge density $-\rho$.)

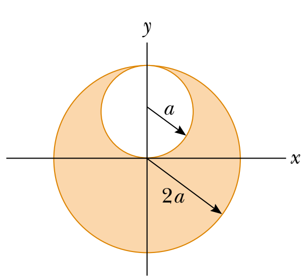


Figure P24.60

61. Review Problem. An early (incorrect) model of the hydrogen atom, suggested by J. J. Thomson, proposed that a positive cloud of charge $+e$ was uniformly distributed throughout the volume of a sphere of radius R, with the electron an equal-magnitude negative point charge $-e$ at the center. (a) Using Gauss's law, show that the electron would be in equilibrium at the center and, if displaced from the center a distance $r < R$, would experience a restoring force of the form $F = -Kr$, where K is a constant. (b) Show that $K = k_e e^2/R^3$. (c) Find an expression for the frequency f of simple harmonic oscillations that an electron of mass m_e would undergo if displaced a short distance ($< R$) from the center and released. (d) Calculate a numerical value for R that would result in a frequency of electron vibration of 2.47×10^{15} Hz, the frequency of the light in the most intense line in the hydrogen spectrum.

62. A closed surface with dimensions $a = b = 0.400$ m and $c = 0.600$ m is located as shown in Figure P24.62. The electric field throughout the region is nonuniform and given by $\mathbf{E} = (3.0 + 2.0x^2)\,\mathbf{i}$ N/C, where x is in meters. Calculate the net electric flux leaving the closed surface. What net charge is enclosed by the surface?

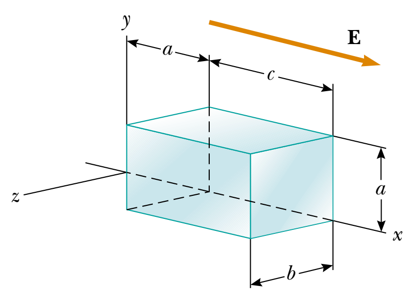


Figure P24.62

63. A solid insulating sphere of radius R has a nonuniform charge density that varies with r according to the expression $\rho = Ar^2$, where A is a constant and $r < R$ is measured from the center of the sphere. (a) Show that the electric field outside ($r > R$) the sphere is $E = AR^5/5\epsilon_0 r^2$. (b) Show that the electric field inside ($r < R$) the sphere is $E = Ar^3/5\epsilon_0$. (*Hint:* Note that the total charge Q on the sphere is equal to the integral of $\rho\, dV$, where r extends from 0 to R; also note that the charge q within a radius $r < R$ is less than Q. To evaluate the integrals, note that the volume element dV for a spherical shell of radius r and thickness dr is equal to $4\pi r^2\, dr$.)

64. A point charge Q is located on the axis of a disk of radius R at a distance b from the plane of the disk (Fig. P24.64). Show that if one fourth of the electric flux from the charge passes through the disk, then $R = \sqrt{3}b$.

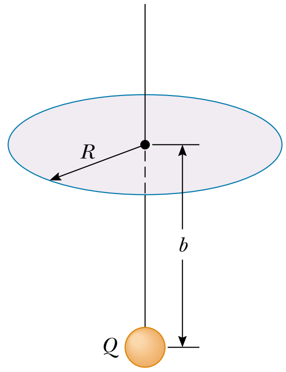


Figure P24.64

65. A spherically symmetric charge distribution has a charge density given by $\rho = a/r$, where a is constant. Find the electric field as a function of r. (*Hint:* Note that the charge within a sphere of radius R is equal to the integral of $\rho\, dV$, where r extends from 0 to R. To evaluate the integral, note that the volume element dV for a spherical shell of radius r and thickness dr is equal to $4\pi r^2\, dr$.)

66. An infinitely long insulating cylinder of radius R has a volume charge density that varies with the radius as

$$\rho = \rho_0\left(a - \frac{r}{b}\right)$$

where ρ_0, a, and b are positive constants and r is the distance from the axis of the cylinder. Use Gauss's law to determine the magnitude of the electric field at radial distances (a) $r < R$ and (b) $r > R$.

67. **Review Problem.** A slab of insulating material (infinite in two of its three dimensions) has a uniform positive charge density ρ. An edge view of the slab is shown in Figure P24.67. (a) Show that the magnitude of the electric field a distance x from its center and inside the slab is $E = \rho x/\epsilon_0$. (b) Suppose that an electron of charge $-e$ and mass m_e is placed inside the slab. If it is released from rest at a distance x from the center, show that the electron exhibits simple harmonic motion with a frequency described by the expression

$$f = \frac{1}{2\pi}\sqrt{\frac{\rho e}{m_e \epsilon_0}}$$

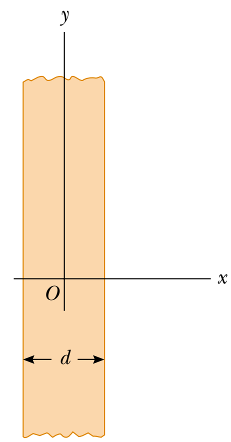


Figure P24.67 Problems 67 and 68.

68. A slab of insulating material has a nonuniform positive charge density $\rho = Cx^2$, where x is measured from the center of the slab, as shown in Figure P24.67, and C is a constant. The slab is infinite in the y and z directions. Derive expressions for the electric field in (a) the exterior regions and (b) the interior region of the slab $(-d/2 < x < d/2)$.

69. (a) Using the mathematical similarity between Coulomb's law and Newton's law of universal gravitation, show that Gauss's law for gravitation can be written as

$$\oint \mathbf{g} \cdot d\mathbf{A} = -4\pi G m_{\text{in}}$$

where m_{in} is the mass inside the gaussian surface and $\mathbf{g} = \mathbf{F}_g/m$ represents the gravitational field at any point on the gaussian surface. (b) Determine the gravitational field at a distance r from the center of the Earth where $r < R_E$, assuming that the Earth's mass density is uniform.

Answers to Quick Quizzes

24.1 Zero, because there is no net charge within the surface.

24.2 (b) and (d). Statement (a) is not necessarily true because an equal number of positive and negative charges could be present inside the surface. Statement (c) is not necessarily true, as can be seen from Figure 24.8: A nonzero electric field exists everywhere on the surface, but the charge is not enclosed within the surface; thus, the net flux is zero.

24.3 Any gaussian surface surrounding the system encloses the same amount of charge, regardless of how the components of the system are moved. Thus, the flux through the gaussian surface would be the same as it is when the sphere and shell are concentric.

PUZZLER

Jennifer is holding on to an electrically charged sphere that reaches an electric potential of about 100 000 V. The device that generates this high electric potential is called a *Van de Graaff generator.* What causes Jennifer's hair to stand on end like the needles of a porcupine? Why is she safe in this situation in view of the fact that 110 V from a wall outlet can kill you? *(Henry Leap and Jim Lehman)*

chapter

25 Electric Potential

Chapter Outline

The concept of potential energy was introduced in Chapter 8 in connection with such conservative forces as the force of gravity and the elastic force exerted by a spring. By using the law of conservation of energy, we were able to avoid working directly with forces when solving various problems in mechanics. In this chapter we see that the concept of potential energy is also of great value in the study of electricity. Because the electrostatic force given by Coulomb's law is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy. This idea enables us to define a scalar quantity known as *electric potential.* Because the electric potential at any point in an electric field is a scalar function, we can use it to describe electrostatic phenomena more simply than if we were to rely only on the concepts of the electric field and electric forces. In later chapters we shall see that the concept of electric potential is of great practical value.

25.1 POTENTIAL DIFFERENCE AND ELECTRIC POTENTIAL

11.8 When a test charge q_0 is placed in an electric field $\mathbf{E}$ created by some other charged object, the electric force acting on the test charge is $q_0\mathbf{E}$. (If the field is produced by more than one charged object, this force acting on the test charge is the vector sum of the individual forces exerted on it by the various other charged objects.) The force $q_0\mathbf{E}$ is conservative because the individual forces described by Coulomb's law are conservative. When the test charge is moved in the field by some external agent, the work done by the field on the charge is equal to the negative of the work done by the external agent causing the displacement. For an infinitesimal displacement $d\mathbf{s}$, the work done by the electric field on the charge is $\mathbf{F}\cdot d\mathbf{s} = q_0\mathbf{E}\cdot d\mathbf{s}$. As this amount of work is done by the field, the potential energy of the charge–field system is decreased by an amount $dU = -q_0\mathbf{E}\cdot d\mathbf{s}$. For a finite displacement of the charge from a point A to a point B, the change in potential energy of the system $\Delta U = U_B - U_A$ is

$$\Delta U = -q_0 \int_A^B \mathbf{E}\cdot d\mathbf{s} \qquad \textbf{(25.1)}$$

Change in potential energy

The integration is performed along the path that q_0 follows as it moves from A to B, and the integral is called either a *path integral* or a *line integral* (the two terms are synonymous). Because the force $q_0\mathbf{E}$ is conservative, **this line integral does not depend on the path taken from *A* to *B*.**

Quick Quiz 25.1

If the path between A and B does not make any difference in Equation 25.1, why don't we just use the expression $\Delta U = -q_0 Ed$, where d is the straight-line distance between A and B?

The potential energy per unit charge U/q_0 is independent of the value of q_0 and has a unique value at every point in an electric field. This quantity U/q_0 is called the **electric potential** (or simply the **potential**) V. Thus, the electric potential at any point in an electric field is

$$V = \frac{U}{q_0} \qquad \textbf{(25.2)}$$

The fact that potential energy is a scalar quantity means that electric potential also is a scalar quantity.

The **potential difference** $\Delta V = V_B - V_A$ between any two points A and B in an electric field is defined as the change in potential energy of the system divided by the test charge q_0:

Potential difference

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{25.3}$$

Potential difference should not be confused with difference in potential energy. The potential difference is proportional to the change in potential energy, and we see from Equation 25.3 that the two are related by $\Delta U = q_0 \Delta V$.

Electric potential is a scalar characteristic of an electric field, independent of the charges that may be placed in the field. However, when we speak of potential energy, we are referring to the charge–field system. Because we are usually interested in knowing the electric potential at the location of a charge and the potential energy resulting from the interaction of the charge with the field, we follow the common convention of speaking of the potential energy as if it belonged to the charge.

Because the change in potential energy of a charge is the negative of the work done by the electric field on the charge (as noted in Equation 25.1), the potential difference ΔV between points A and B equals the work per unit charge that an external agent must perform to move a test charge from A to B without changing the kinetic energy of the test charge.

Just as with potential energy, only *differences* in electric potential are meaningful. To avoid having to work with potential differences, however, we often take the value of the electric potential to be zero at some convenient point in an electric field. This is what we do here: arbitrarily establish the electric potential to be zero at a point that is infinitely remote from the charges producing the field. Having made this choice, we can state that the **electric potential at an arbitrary point in an electric field equals the work required per unit charge to bring a positive test charge from infinity to that point.** Thus, if we take point A in Equation 25.3 to be at infinity, the electric potential at any point P is

$$V_P = -\int_\infty^P \mathbf{E} \cdot d\mathbf{s} \tag{25.4}$$

In reality, V_P represents the potential difference ΔV between the point P and a point at infinity. (Eq. 25.4 is a special case of Eq. 25.3.)

Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a **volt** (V):

Definition of volt

$$1\ \mathrm{V} \equiv 1\ \frac{\mathrm{J}}{\mathrm{C}}$$

That is, 1 J of work must be done to move a 1-C charge through a potential difference of 1 V.

Equation 25.3 shows that potential difference also has units of electric field times distance. From this, it follows that the SI unit of electric field (N/C) can also be expressed in volts per meter:

$$1\ \frac{\mathrm{N}}{\mathrm{C}} = 1\ \frac{\mathrm{V}}{\mathrm{m}}$$

A unit of energy commonly used in atomic and nuclear physics is the **electron volt** (eV), which is defined as **the energy an electron (or proton) gains or loses by moving through a potential difference of 1 V.** Because 1 V = 1 J/C and because the fundamental charge is approximately 1.60×10^{-19} C, the electron volt is related to the joule as follows:

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ C}\cdot\text{V} = 1.60 \times 10^{-19} \text{ J} \quad \textbf{(25.5)}$$

The electron volt

For instance, an electron in the beam of a typical television picture tube may have a speed of 3.5×10^7 m/s. This corresponds to a kinetic energy of 5.6×10^{-16} J, which is equivalent to 3.5×10^3 eV. Such an electron has to be accelerated from rest through a potential difference of 3.5 kV to reach this speed.

25.2 POTENTIAL DIFFERENCES IN A UNIFORM ELECTRIC FIELD

Equations 25.1 and 25.3 hold in all electric fields, whether uniform or varying, but they can be simplified for a uniform field. First, consider a uniform electric field directed along the negative y axis, as shown in Figure 25.1a. Let us calculate the potential difference between two points A and B separated by a distance d, where d is measured parallel to the field lines. Equation 25.3 gives

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\int_A^B E\cos 0^\circ\, ds = -\int_A^B E\, ds$$

Because E is constant, we can remove it from the integral sign; this gives

$$\Delta V = -E\int_A^B ds = -Ed \quad \textbf{(25.6)}$$

Potential difference in a uniform electric field

The minus sign indicates that point B is at a lower electric potential than point A; that is, $V_B < V_A$. **Electric field lines always point in the direction of decreasing electric potential,** as shown in Figure 25.1a.

Now suppose that a test charge q_0 moves from A to B. We can calculate the change in its potential energy from Equations 25.3 and 25.6:

$$\Delta U = q_0\,\Delta V = -q_0 Ed \quad \textbf{(25.7)}$$

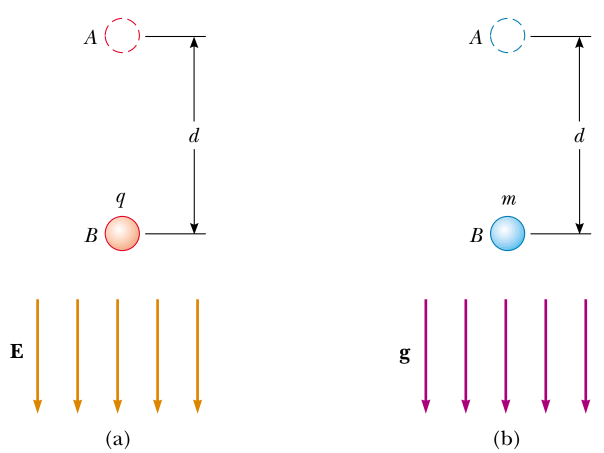


Figure 25.1 (a) When the electric field **E** is directed downward, point B is at a lower electric potential than point A. A positive test charge that moves from point A to point B loses electric potential energy. (b) A mass m moving downward in the direction of the gravitational field **g** loses gravitational potential energy.

QuickLab

It takes an electric field of about 30 000 V/cm to cause a spark in dry air. Shuffle across a rug and reach toward a doorknob. By estimating the length of the spark, determine the electric potential difference between your finger and the doorknob after shuffling your feet but before touching the knob. (If it is very humid on the day you attempt this, it may not work. Why?)

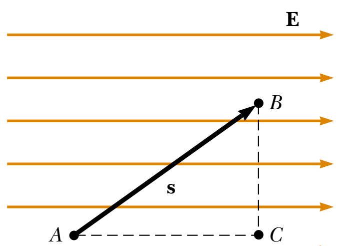


Figure 25.2 A uniform electric field directed along the positive x axis. Point B is at a lower electric potential than point A. Points B and C are at the *same* electric potential.

From this result, we see that if q_0 is positive, then ΔU is negative. We conclude that **a positive charge loses electric potential energy when it moves in the direction of the electric field.** This means that an electric field does work on a positive charge when the charge moves in the direction of the electric field. (This is analogous to the work done by the gravitational field on a falling mass, as shown in Figure 25.1b.) If a positive test charge is released from rest in this electric field, it experiences an electric force $q_0\mathbf{E}$ in the direction of $\mathbf{E}$ (downward in Fig. 25.1a). Therefore, it accelerates downward, gaining kinetic energy. **As the charged particle gains kinetic energy, it loses an equal amount of potential energy.**

If q_0 is negative, then ΔU is positive and the situation is reversed: **A negative charge gains electric potential energy when it moves in the direction of the electric field.** If a negative charge is released from rest in the field $\mathbf{E}$, it accelerates in a direction opposite the direction of the field.

Now consider the more general case of a charged particle that is free to move between any two points in a uniform electric field directed along the x axis, as shown in Figure 25.2. (In this situation, the charge is not being moved by an external agent as before.) If $\mathbf{s}$ represents the displacement vector between points A and B, Equation 25.3 gives

$$\Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\mathbf{E}\cdot\int_A^B d\mathbf{s} = -\mathbf{E}\cdot\mathbf{s} \qquad \textbf{(25.8)}$$

where again we are able to remove $\mathbf{E}$ from the integral because it is constant. The change in potential energy of the charge is

$$\Delta U = q_0\,\Delta V = -q_0\,\mathbf{E}\cdot\mathbf{s} \qquad \textbf{(25.9)}$$

11.9 Finally, we conclude from Equation 25.8 that all points in a plane perpendicular to a uniform electric field are at the same electric potential. We can see this in Figure 25.2, where the potential difference $V_B - V_A$ is equal to the potential difference $V_C - V_A$. (Prove this to yourself by working out the dot product $\mathbf{E}\cdot\mathbf{s}$ for $\mathbf{s}_{A\to B}$, where the angle θ between $\mathbf{E}$ and $\mathbf{s}$ is arbitrary as shown in Figure 25.2, and the dot product for $\mathbf{s}_{A\to C}$, where $\theta = 0$.) Therefore, $V_B = V_C$. **The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.**

An equipotential surface

Note that because $\Delta U = q_0\Delta V$, no work is done in moving a test charge between any two points on an equipotential surface. The equipotential surfaces of a uniform electric field consist of a family of planes that are all perpendicular to the field. Equipotential surfaces for fields with other symmetries are described in later sections.

Quick Quiz 25.2

The labeled points in Figure 25.3 are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves from A to B; from B to C; from C to D; from D to E.

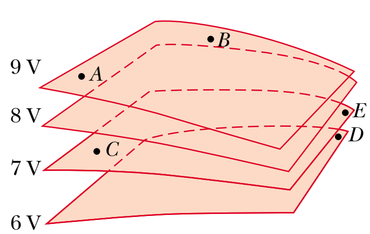


Figure 25.3 Four equipotential surfaces.

EXAMPLE 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge

A battery produces a specified potential difference between conductors attached to the battery terminals. A 12-V battery is connected between two parallel plates, as shown in Figure 25.4. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and if we do not consider points near the plate edges.) Find the magnitude of the electric field between the plates.

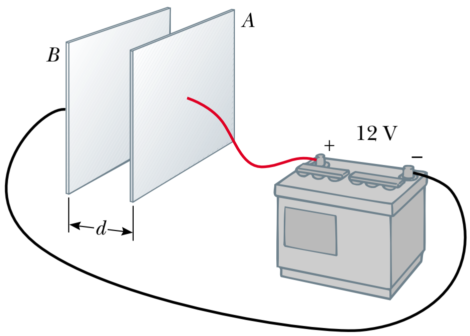


Figure 25.4 A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference ΔV divided by the plate separation d.

Solution The electric field is directed from the positive plate (A) to the negative one (B), and the positive plate is at a higher electric potential than the negative plate is. The potential difference between the plates must equal the potential difference between the battery terminals. We can understand this by noting that all points on a conductor in equilibrium are at the same electric potential[1]; no potential difference exists between a terminal and any portion of the plate to which it is connected. Therefore, the magnitude of the electric field between the plates is, from Equation 25.6,

$$E = \frac{|V_B - V_A|}{d} = \frac{12\text{ V}}{0.30 \times 10^{-2}\text{ m}} = \boxed{4.0 \times 10^3\text{ V/m}}$$

This configuration, which is shown in Figure 25.4 and called a *parallel-plate capacitor*, is examined in greater detail in Chapter 26.

EXAMPLE 25.2 Motion of a Proton in a Uniform Electric Field

A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m and is directed along the positive x axis (Fig. 25.5). The proton undergoes a displacement of 0.50 m in the direction of $\mathbf{E}$. (a) Find the change in electric potential between points A and B.

Solution Because the proton (which, as you remember, carries a positive charge) moves in the direction of the field, we expect it to move to a position of lower electric potential.

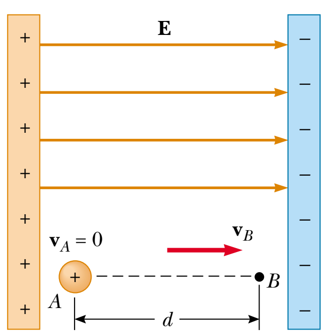


Figure 25.5 A proton accelerates from A to B in the direction of the electric field.

From Equation 25.6, we have

$$\Delta V = -Ed = -(8.0 \times 10^4\text{ V/m})(0.50\text{ m})$$
$$= \boxed{-4.0 \times 10^4\text{ V}}$$

(b) Find the change in potential energy of the proton for this displacement.

Solution

$$\Delta U = q_0\,\Delta V = e\,\Delta V$$
$$= (1.6 \times 10^{-19}\text{ C})(-4.0 \times 10^4\text{ V})$$
$$= \boxed{-6.4 \times 10^{-15}\text{ J}}$$

The negative sign means the potential energy of the proton decreases as it moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time loses electric potential energy (because energy is conserved).

Exercise Use the concept of conservation of energy to find the speed of the proton at point B.

Answer 2.77×10^6 m/s.

[1] The electric field vanishes within a conductor in electrostatic equilibrium; thus, the path integral $\int \mathbf{E} \cdot d\mathbf{s}$ between any two points in the conductor must be zero. A more complete discussion of this point is given in Section 25.6.

25.3 ELECTRIC POTENTIAL AND POTENTIAL ENERGY DUE TO POINT CHARGES

Consider an isolated positive point charge q. Recall that such a charge produces an electric field that is directed radially outward from the charge. To find the electric potential at a point located a distance r from the charge, we begin with the general expression for potential difference:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

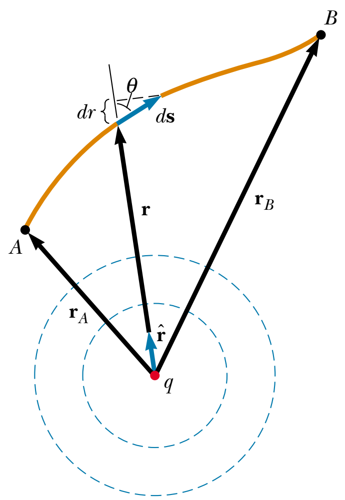


Figure 25.6 The potential difference between points A and B due to a point charge q depends *only* on the initial and final radial coordinates r_A and r_B. The two dashed circles represent cross-sections of spherical equipotential surfaces.

where A and B are the two arbitrary points shown in Figure 25.6. At any field point, the electric field due to the point charge is $\mathbf{E} = k_e q\hat{\mathbf{r}}/r^2$ (Eq. 23.4), where $\hat{\mathbf{r}}$ is a unit vector directed from the charge toward the field point. The quantity $\mathbf{E} \cdot d\mathbf{s}$ can be expressed as

$$\mathbf{E} \cdot d\mathbf{s} = k_e \frac{q}{r^2}\, \hat{\mathbf{r}} \cdot d\mathbf{s}$$

Because the magnitude of $\hat{\mathbf{r}}$ is 1, the dot product $\hat{\mathbf{r}} \cdot d\mathbf{s} = ds \cos\theta$, where θ is the angle between $\hat{\mathbf{r}}$ and $d\mathbf{s}$. Furthermore, $ds \cos\theta$ is the projection of $d\mathbf{s}$ onto $\mathbf{r}$; thus, $ds \cos\theta = dr$. That is, any displacement $d\mathbf{s}$ along the path from point A to point B produces a change dr in the magnitude of $\mathbf{r}$, the radial distance to the charge creating the field. Making these substitutions, we find that $\mathbf{E} \cdot d\mathbf{s} = (k_e q/r^2)\,dr$; hence, the expression for the potential difference becomes

$$V_B - V_A = -\int E_r\, dr = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r}\bigg]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A}\right] \tag{25.10}$$

The integral of $\mathbf{E} \cdot d\mathbf{s}$ is *independent* of the path between points A and B—as it must be because the electric field of a point charge is conservative. Furthermore, Equation 25.10 expresses the important result that the potential difference between any two points A and B in a field created by a point charge depends only on the radial coordinates r_A and r_B. It is customary to choose the reference of electric potential to be zero at $r_A = \infty$. With this reference, the electric potential created by a point charge at any distance r from the charge is

Electric potential created by a point charge

$$V = k_e \frac{q}{r} \tag{25.11}$$

Electric potential is graphed in Figure 25.7 as a function of r, the radial distance from a positive charge in the xy plane. Consider the following analogy to gravitational potential: Imagine trying to roll a marble toward the top of a hill shaped like Figure 25.7a. The gravitational force experienced by the marble is analogous to the repulsive force experienced by a positively charged object as it approaches another positively charged object. Similarly, the electric potential graph of the region surrounding a negative charge is analogous to a "hole" with respect to any approaching positively charged objects. A charged object must be infinitely distant from another charge before the surface is "flat" and has an electric potential of zero.

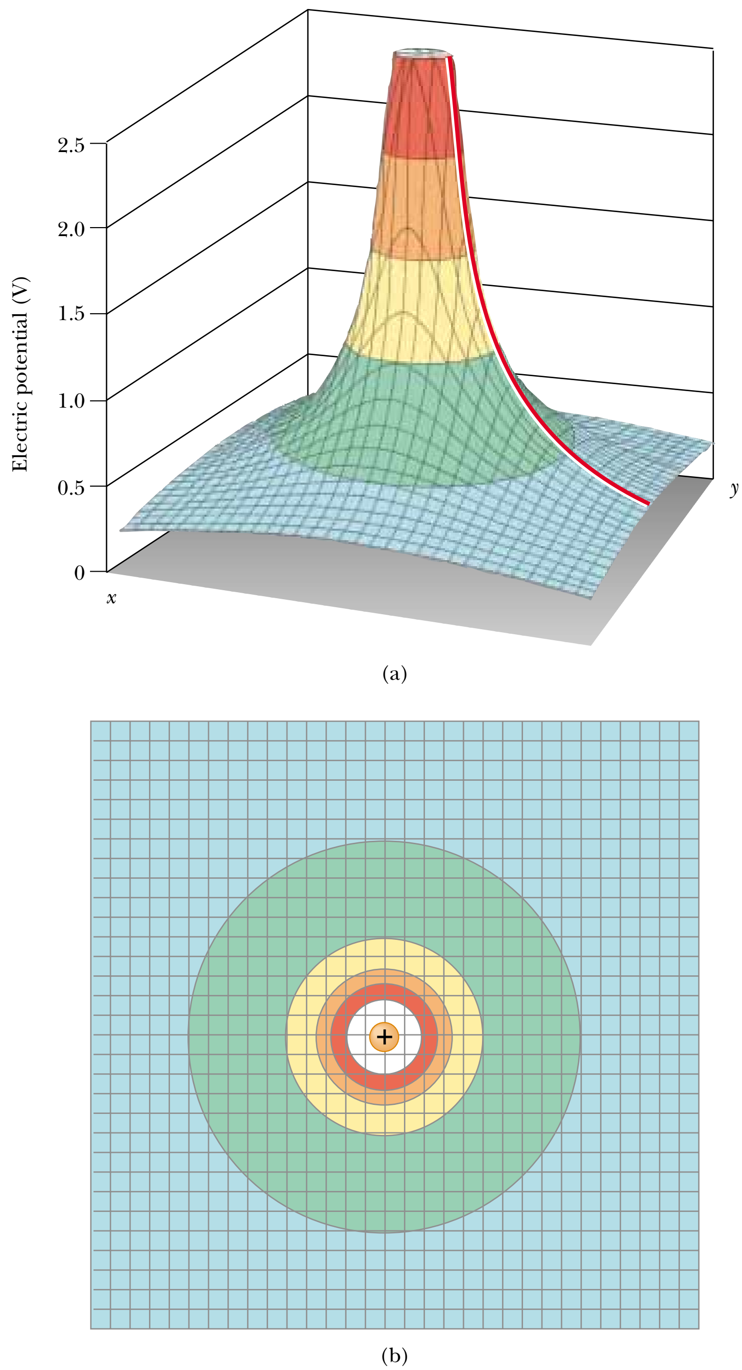


Figure 25.7 (a) The electric potential in the plane around a single positive charge is plotted on the vertical axis. (The electric potential function for a negative charge would look like a hole instead of a hill.) The red line shows the $1/r$ nature of the electric potential, as given by Equation 25.11. (b) View looking straight down the vertical axis of the graph in part (a), showing concentric circles where the electric potential is constant. These circles are cross sections of equipotential spheres having the charge at the center.

Quick Quiz 25.3

A spherical balloon contains a positively charged object at its center. As the balloon is inflated to a greater volume while the charged object remains at the center, does the electric potential at the surface of the balloon increase, decrease, or remain the same? How about the magnitude of the electric field? The electric flux?

We obtain the electric potential resulting from two or more point charges by applying the superposition principle. That is, the total electric potential at some point P due to several point charges is the sum of the potentials due to the individual charges. For a group of point charges, we can write the total electric potential at P in the form

Electric potential due to several point charges

$$V = k_e \sum_i \frac{q_i}{r_i} \qquad \textbf{(25.12)}$$

where the potential is again taken to be zero at infinity and r_i is the distance from the point P to the charge q_i. Note that the sum in Equation 25.12 is an algebraic sum of scalars rather than a vector sum (which we use to calculate the electric field of a group of charges). Thus, it is often much easier to evaluate V than to evaluate $\mathbf{E}$. The electric potential around a dipole is illustrated in Figure 25.8.

We now consider the potential energy of a system of two charged particles. If V_1 is the electric potential at a point P due to charge q_1, then the work an external agent must do to bring a second charge q_2 from infinity to P without acceleration is q_2V_1. By definition, this work equals the potential energy U of the two-particle system when the particles are separated by a distance r_{12} (Fig. 25.9). Therefore, we can express the potential energy as[2]

Electric potential energy due to two charges

$$U = k_e \frac{q_1 q_2}{r_{12}} \qquad \textbf{(25.13)}$$

Note that if the charges are of the same sign, U is positive. This is consistent with the fact that positive work must be done by an external agent on the system to bring the two charges near one another (because like charges repel). If the charges are of opposite sign, U is negative; this means that negative work must be done against the attractive force between the unlike charges for them to be brought near each other.

If more than two charged particles are in the system, we can obtain the total potential energy by calculating U for every pair of charges and summing the terms algebraically. As an example, the total potential energy of the system of three charges shown in Figure 25.10 is

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \qquad \textbf{(25.14)}$$

Physically, we can interpret this as follows: Imagine that q_1 is fixed at the position shown in Figure 25.10 but that q_2 and q_3 are at infinity. The work an external agent must do to bring q_2 from infinity to its position near q_1 is $k_e q_1 q_2 / r_{12}$, which is the first term in Equation 25.14. The last two terms represent the work required to bring q_3 from infinity to its position near q_1 and q_2. (The result is independent of the order in which the charges are transported.)

[2] The expression for the electric potential energy of a system made up of two point charges, Equation 25.13, is of the *same* form as the equation for the gravitational potential energy of a system made up of two point masses, Gm_1m_2/r (see Chapter 14). The similarity is not surprising in view of the fact that both expressions are derived from an inverse-square force law.

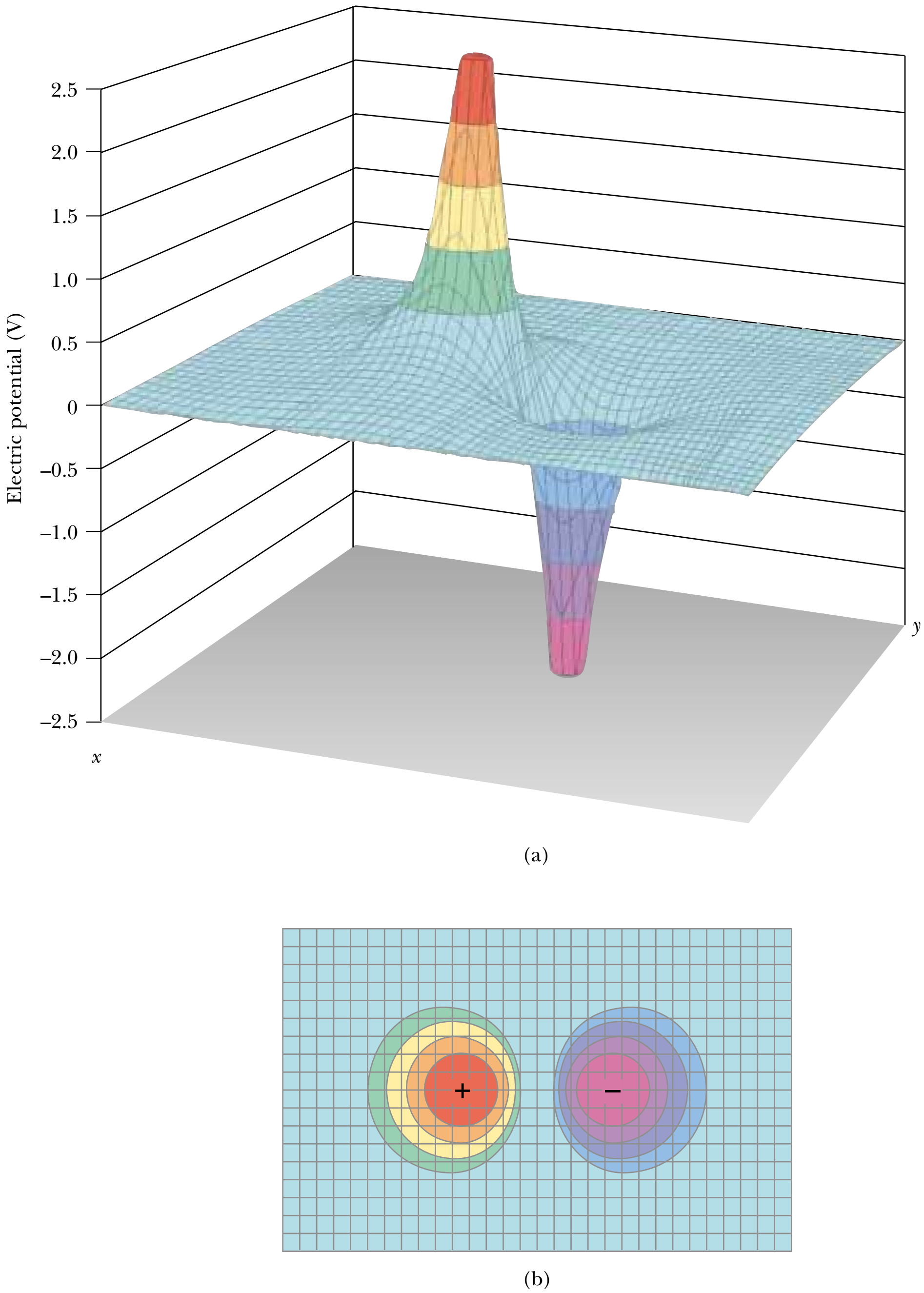


Figure 25.8 (a) The electric potential in the plane containing a dipole. (b) Top view of the function graphed in part (a).

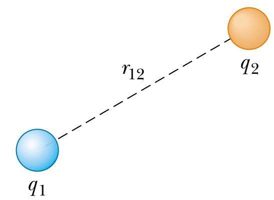


Figure 25.9 If two point charges are separated by a distance r_{12}, the potential energy of the pair of charges is given by $k_e q_1 q_2 / r_{12}$.

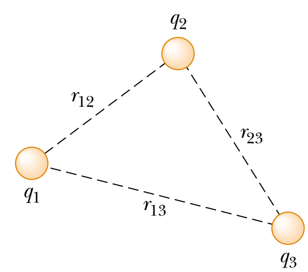


Figure 25.10 Three point charges are fixed at the positions shown. The potential energy of this system of charges is given by Equation 25.14.

EXAMPLE 25.3 The Electric Potential Due to Two Point Charges

A charge $q_1 = 2.00\ \mu C$ is located at the origin, and a charge $q_2 = -6.00\ \mu C$ is located at (0, 3.00) m, as shown in Figure 25.11a. (a) Find the total electric potential due to these charges at the point P, whose coordinates are (4.00, 0) m.

Solution For two charges, the sum in Equation 25.12 gives

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= 8.99 \times 10^9\ \frac{N \cdot m^2}{C^2}\left(\frac{2.00 \times 10^{-6}\ C}{4.00\ m} + \frac{-6.00 \times 10^{-6}\ C}{5.00\ m}\right)$$

$$= \boxed{-6.29 \times 10^3\ V}$$

(b) Find the change in potential energy of a 3.00-μC charge as it moves from infinity to point P (Fig. 25.11b).

Solution When the charge is at infinity, $U_i = 0$, and when the charge is at P, $U_f = q_3 V_P$; therefore,

$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6}\ C)(-6.29 \times 10^3\ V)$$

$$= \boxed{-18.9 \times 10^{-3}\ J}$$

Therefore, because $W = -\Delta U$, positive work would have to be done by an external agent to remove the charge from point P back to infinity.

Exercise Find the total potential energy of the system illustrated in Figure 25.11b.

Answer -5.48×10^{-2} J.

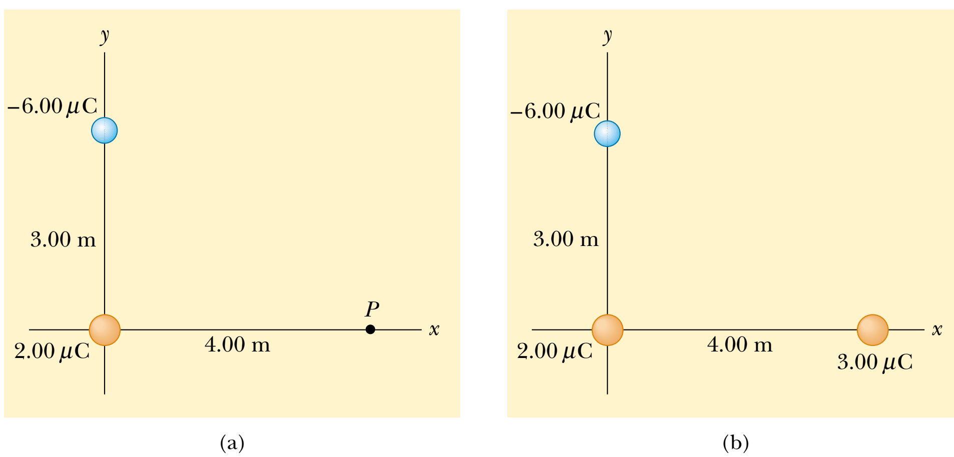


Figure 25.11 (a) The electric potential at P due to the two charges is the algebraic sum of the potentials due to the individual charges. (b) What is the potential energy of the three-charge system?

25.4 OBTAINING THE VALUE OF THE ELECTRIC FIELD FROM THE ELECTRIC POTENTIAL

The electric field **E** and the electric potential V are related as shown in Equation 25.3. We now show how to calculate the value of the electric field if the electric potential is known in a certain region.

From Equation 25.3 we can express the potential difference dV between two points a distance ds apart as

$$dV = -\mathbf{E} \cdot d\mathbf{s} \tag{25.15}$$

If the electric field has only one component E_x, then $\mathbf{E} \cdot d\mathbf{s} = E_x\,dx$. Therefore, Equation 25.15 becomes $dV = -E_x\,dx$, or

$$E_x = -\frac{dV}{dx} \tag{25.16}$$

That is, the magnitude of the electric field in the direction of some coordinate is equal to the negative of the derivative of the electric potential with respect to that coordinate. Recall from the discussion following Equation 25.8 that the electric potential does not change for any displacement perpendicular to an electric field. This is consistent with the notion, developed in Section 25.2, that equipotential surfaces are perpendicular to the field, as shown in Figure 25.12. A small positive charge placed at rest on an electric field line begins to move along the direction of $\mathbf{E}$ because that is the direction of the force exerted on the charge by the charge distribution creating the electric field (and hence is the direction of $\mathbf{a}$). Because the charge starts with zero velocity, it moves in the direction of the change in velocity—that is, in the direction of $\mathbf{a}$. In Figures 25.12a and 25.12b, a charge placed at rest in the field will move in a straight line because its acceleration vector is always parallel to its velocity vector. The magnitude of $\mathbf{v}$ increases, but its direction does not change. The situation is different in Figure 25.12c. A positive charge placed at some point near the dipole first moves in a direction parallel to $\mathbf{E}$ at that point. Because the direction of the electric field is different at different locations, however, the force acting on the charge changes direction, and $\mathbf{a}$ is no longer parallel to $\mathbf{v}$. This causes the moving charge to change direction and speed, but it does not necessarily follow the electric field lines. Recall that it is not the velocity vector but rather the acceleration vector that is proportional to force.

If the charge distribution creating an electric field has spherical symmetry such that the volume charge density depends only on the radial distance r, then the electric field is radial. In this case, $\mathbf{E} \cdot d\mathbf{s} = E_r\, dr$, and thus we can express dV in the form $dV = -E_r\, dr$. Therefore,

$$E_r = -\frac{dV}{dr} \tag{25.17}$$

For example, the electric potential of a point charge is $V = k_e q/r$. Because V is a function of r only, the potential function has spherical symmetry. Applying Equation 25.17, we find that the electric field due to the point charge is $E_r = k_e q/r^2$, a familiar result. Note that the potential changes only in the radial direction, not in

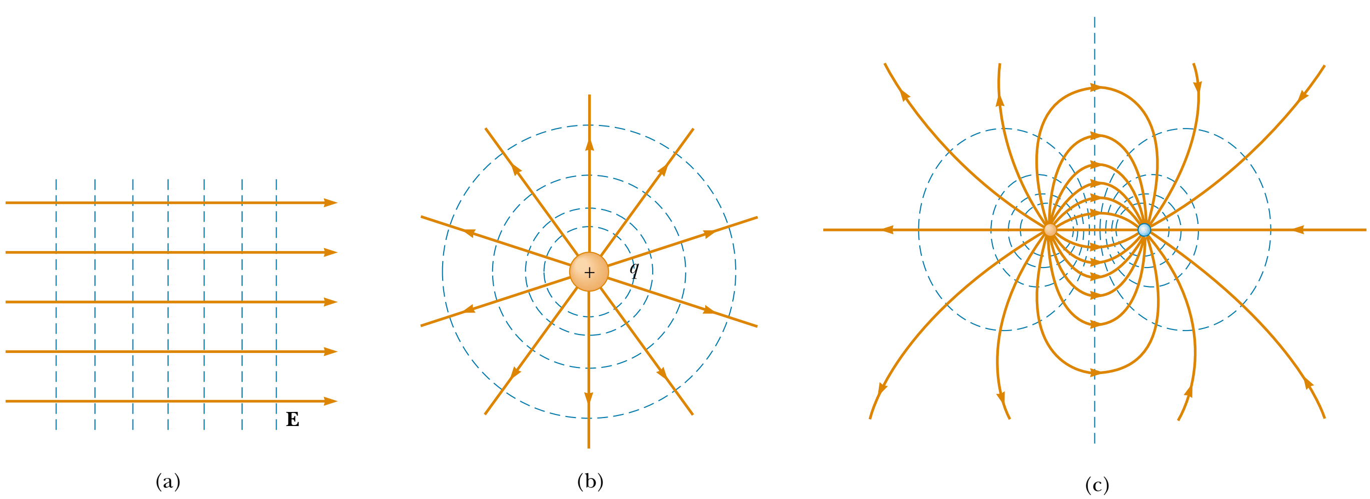


Figure 25.12 Equipotential surfaces (dashed blue lines) and electric field lines (red lines) for (a) a uniform electric field produced by an infinite sheet of charge, (b) a point charge, and (c) an electric dipole. In all cases, the equipotential surfaces are *perpendicular* to the electric field lines at every point. Compare these drawings with Figures 25.2, 25.7b, and 25.8b.

Equipotential surfaces are perpendicular to the electric field lines

any direction perpendicular to r. Thus, V (like E_r) is a function only of r. Again, this is consistent with the idea that **equipotential surfaces are perpendicular to field lines.** In this case the equipotential surfaces are a family of spheres concentric with the spherically symmetric charge distribution (Fig. 25.12b).

The equipotential surfaces for an electric dipole are sketched in Figure 25.12c. When a test charge undergoes a displacement $d\mathbf{s}$ along an equipotential surface, then $dV = 0$ because the potential is constant along an equipotential surface. From Equation 25.15, then, $dV = -\mathbf{E} \cdot d\mathbf{s} = 0$; thus, $\mathbf{E}$ must be perpendicular to the displacement along the equipotential surface. This shows that the equipotential surfaces must *always* be *perpendicular* to the electric field lines.

In general, the electric potential is a function of all three spatial coordinates. If $V(r)$ is given in terms of the cartesian coordinates, the electric field components E_x, E_y, and E_z can readily be found from $V(x, y, z)$ as the partial derivatives[3]

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

For example, if $V = 3x^2y + y^2 + yz$, then

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(3x^2y + y^2 + yz) = \frac{\partial}{\partial x}(3x^2y) = 3y\frac{d}{dx}(x^2) = 6xy$$

EXAMPLE 25.4 The Electric Potential Due to a Dipole

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$, as shown in Figure 25.13. The dipole is along the x axis and is centered at the origin. (a) Calculate the electric potential at point P.

Solution For point P in Figure 25.13,

$$V = k_e \sum \frac{q_i}{r_i} = k_e\left(\frac{q}{x-a} - \frac{q}{x+a}\right) = \boxed{\frac{2k_eqa}{x^2 - a^2}}$$

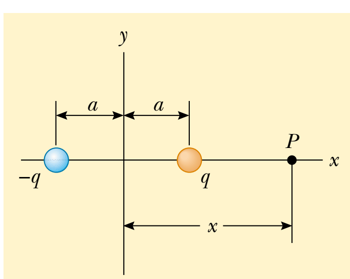


Figure 25.13 An electric dipole located on the x axis.

(How would this result change if point P happened to be located to the left of the negative charge?)

(b) Calculate V and E_x at a point far from the dipole.

Solution If point P is far from the dipole, such that $x \gg a$, then a^2 can be neglected in the term $x^2 - a^2$, and V becomes

$$V \approx \boxed{\frac{2k_eqa}{x^2}} \qquad (x \gg a)$$

Using Equation 25.16 and this result, we can calculate the electric field at a point far from the dipole:

$$E_x = -\frac{dV}{dx} = \boxed{\frac{4k_eqa}{x^3}} \qquad (x \gg a)$$

(c) Calculate V and E_x if point P is located anywhere between the two charges.

Solution

$$V = k_e \sum \frac{q_i}{r_i} = k_e\left(\frac{q}{a-x} - \frac{q}{x+a}\right) = -\frac{2k_eqx}{x^2 - a^2}$$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_eqx}{x^2 - a^2}\right) = 2k_eq\left(\frac{-x^2 - a^2}{(x^2 - a^2)^2}\right)$$

[3] In vector notation, $\mathbf{E}$ is often written

$$\mathbf{E} = -\nabla V = -\left(\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}\right)V$$

where ∇ is called the *gradient operator.*

We can check these results by considering the situation at the center of the dipole, where $x = 0$, $V = 0$, and $E_x = -2k_eq/a^2$.

Exercise Verify the electric field result in part (c) by calculating the sum of the individual electric field vectors at the origin due to the two charges.

25.5 ELECTRIC POTENTIAL DUE TO CONTINUOUS CHARGE DISTRIBUTIONS

We can calculate the electric potential due to a continuous charge distribution in two ways. If the charge distribution is known, we can start with Equation 25.11 for the electric potential of a point charge. We then consider the potential due to a small charge element dq, treating this element as a point charge (Fig. 25.14). The electric potential dV at some point P due to the charge element dq is

$$dV = k_e \frac{dq}{r} \tag{25.18}$$

where r is the distance from the charge element to point P. To obtain the total potential at point P, we integrate Equation 25.18 to include contributions from all elements of the charge distribution. Because each element is, in general, a different distance from point P and because k_e is constant, we can express V as

$$V = k_e \int \frac{dq}{r} \tag{25.19}$$

In effect, we have replaced the sum in Equation 25.12 with an integral. Note that this expression for V uses a particular reference: The electric potential is taken to be zero when point P is infinitely far from the charge distribution.

If the electric field is already known from other considerations, such as Gauss's law, we can calculate the electric potential due to a continuous charge distribution using Equation 25.3. If the charge distribution is highly symmetric, we first evaluate $\mathbf{E}$ at any point using Gauss's law and then substitute the value obtained into Equation 25.3 to determine the potential difference ΔV between any two points. We then choose the electric potential V to be zero at some convenient point.

We illustrate both methods with several examples.

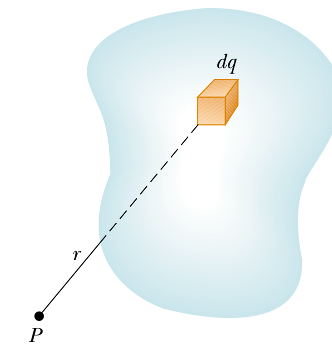


Figure 25.14 The electric potential at the point P due to a continuous charge distribution can be calculated by dividing the charged body into segments of charge dq and summing the electric potential contributions over all segments.

EXAMPLE 25.5 Electric Potential Due to a Uniformly Charged Ring

(a) Find an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q.

Solution Let us orient the ring so that its plane is perpendicular to an x axis and its center is at the origin. We can then take point P to be at a distance x from the center of the ring, as shown in Figure 25.15. The charge element dq is at a distance $\sqrt{x^2 + a^2}$ from point P. Hence, we can express V as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

Because each element dq is at the same distance from point P, we can remove $\sqrt{x^2 + a^2}$ from the integral, and V reduces to

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{x^2 + a^2}}} \tag{25.20}$$

The only variable in this expression for V is x. This is not surprising because our calculation is valid only for points along the x axis, where y and z are both zero.

(b) Find an expression for the magnitude of the electric field at point P.

Solution From symmetry, we see that along the x axis $\mathbf{E}$ can have only an x component. Therefore, we can use Equation 25.16:

$$E_x = -\frac{dV}{dx} = -k_eQ\frac{d}{dx}(x^2 + a^2)^{-1/2}$$
$$= -k_eQ(-\tfrac{1}{2})(x^2 + a^2)^{-3/2}(2x)$$
$$= \boxed{\frac{k_eQx}{(x^2 + a^2)^{3/2}}} \qquad \textbf{(25.21)}$$

This result agrees with that obtained by direct integration (see Example 23.8). Note that $E_x = 0$ at $x = 0$ (the center of the ring). Could you have guessed this from Coulomb's law?

Exercise What is the electric potential at the center of the ring? What does the value of the field at the center tell you about the value of V at the center?

Answer $V = k_eQ/a$. Because $E_x = -dV/dx = 0$ at the center, V has either a maximum or minimum value; it is, in fact, a maximum.

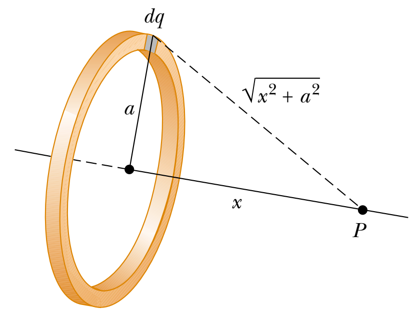


Figure 25.15 A uniformly charged ring of radius a lies in a plane perpendicular to the x axis. All segments dq of the ring are the same distance from any point P lying on the x axis.

EXAMPLE 25.6 Electric Potential Due to a Uniformly Charged Disk

Find (a) the electric potential and (b) the magnitude of the electric field along the perpendicular central axis of a uniformly charged disk of radius a and surface charge density σ.

Solution (a) Again, we choose the point P to be at a distance x from the center of the disk and take the plane of the disk to be perpendicular to the x axis. We can simplify the problem by dividing the disk into a series of charged rings. The electric potential of each ring is given by Equation 25.20. Consider one such ring of radius r and width dr, as indicated in Figure 25.16. The surface area of the ring is $dA = 2\pi r\,dr$; from the definition of surface charge density (see Section 23.5), we know that the charge on the ring is $dq = \sigma\,dA = \sigma 2\pi r\,dr$. Hence, the potential at the point P due to this ring is

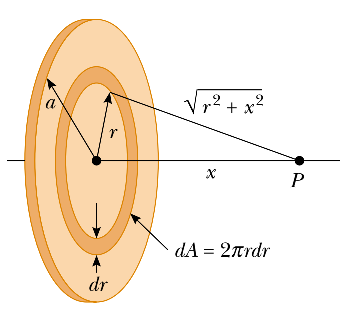


Figure 25.16 A uniformly charged disk of radius a lies in a plane perpendicular to the x axis. The calculation of the electric potential at any point P on the x axis is simplified by dividing the disk into many rings each of area $2\pi r\,dr$.

$$dV = \frac{k_e\,dq}{\sqrt{r^2 + x^2}} = \frac{k_e\sigma 2\pi r\,dr}{\sqrt{r^2 + x^2}}$$

To find the *total* electric potential at P, we sum over all rings making up the disk. That is, we integrate dV from $r = 0$ to $r = a$:

$$V = \pi k_e\sigma\int_0^a \frac{2r\,dr}{\sqrt{r^2 + x^2}} = \pi k_e\sigma\int_0^a (r^2 + x^2)^{-1/2}\,2r\,dr$$

This integral is of the form $u^n\,du$ and has the value $u^{n+1}/(n+1)$, where $n = -\frac{1}{2}$ and $u = r^2 + x^2$. This gives

$$V = \boxed{2\pi k_e\sigma[(x^2 + a^2)^{1/2} - x]} \qquad \textbf{(25.22)}$$

(b) As in Example 25.5, we can find the electric field at any axial point from

$$E_x = -\frac{dV}{dx} = \boxed{2\pi k_e\sigma\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)} \qquad \textbf{(25.23)}$$

The calculation of V and $\mathbf{E}$ for an arbitrary point off the axis is more difficult to perform, and we do not treat this situation in this text.

EXAMPLE 25.7 Electric Potential Due to a Finite Line of Charge

A rod of length ℓ located along the x axis has a total charge Q and a uniform linear charge density $\lambda = Q/\ell$. Find the electric potential at a point P located on the y axis a distance a from the origin (Fig. 25.17).

Solution The length element dx has a charge $dq = \lambda\, dx$. Because this element is a distance $r = \sqrt{x^2 + a^2}$ from point P, we can express the potential at point P due to this element as

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{x^2 + a^2}}$$

To obtain the total potential at P, we integrate this expression over the limits $x = 0$ to $x = \ell$. Noting that k_e and λ are constants, we find that

$$V = k_e \lambda \int_0^\ell \frac{dx}{\sqrt{x^2 + a^2}} = k_e \frac{Q}{\ell} \int_0^\ell \frac{dx}{\sqrt{x^2 + a^2}}$$

This integral has the following value (see Appendix B):

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

Evaluating V, we find that

$$V = \frac{k_e Q}{\ell} \ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right) \qquad \textbf{(25.24)}$$

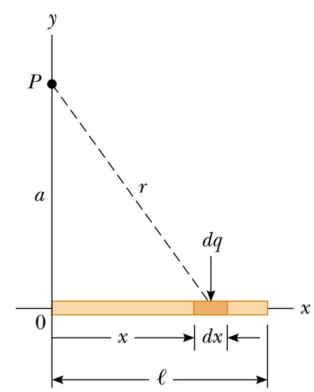


Figure 25.17 A uniform line charge of length ℓ located along the x axis. To calculate the electric potential at P, the line charge is divided into segments each of length dx and each carrying a charge $dq = \lambda\, dx$.

EXAMPLE 25.8 Electric Potential Due to a Uniformly Charged Sphere

An insulating solid sphere of radius R has a uniform positive volume charge density and total charge Q. (a) Find the electric potential at a point outside the sphere, that is, for $r > R$. Take the potential to be zero at $r = \infty$.

Solution In Example 24.5, we found that the magnitude of the electric field outside a uniformly charged sphere of radius R is

$$E_r = k_e \frac{Q}{r^2} \qquad (\text{for } r > R)$$

where the field is directed radially outward when Q is positive. In this case, to obtain the electric potential at an exterior point, such as B in Figure 25.18, we use Equation 25.4 and the expression for E_r given above:

$$V_B = -\int_\infty^r E_r\, dr = -k_e Q \int_\infty^r \frac{dr}{r^2}$$

$$\boxed{V_B = k_e \frac{Q}{r}} \qquad (\text{for } r > R)$$

Note that the result is identical to the expression for the electric potential due to a point charge (Eq. 25.11).

Because the potential must be continuous at $r = R$, we can use this expression to obtain the potential at the surface of the sphere. That is, the potential at a point such as C shown in Figure 25.18 is

$$V_C = k_e \frac{Q}{R} \qquad (\text{for } r = R)$$

(b) Find the potential at a point inside the sphere, that is, for $r < R$.

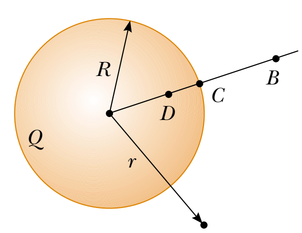


Figure 25.18 A uniformly charged insulating sphere of radius R and total charge Q. The electric potentials at points B and C are equivalent to those produced by a point charge Q located at the center of the sphere, but this is not true for point D.

Solution In Example 24.5 we found that the electric field inside an insulating uniformly charged sphere is

$$E_r = \frac{k_e Q}{R^3} r \qquad (\text{for } r < R)$$

We can use this result and Equation 25.3 to evaluate the potential difference $V_D - V_C$ at some interior point D:

$$V_D - V_C = -\int_R^r E_r \, dr = -\frac{k_e Q}{R^3} \int_R^r r \, dr = \frac{k_e Q}{2R^3} (R^2 - r^2)$$

Substituting $V_C = k_e Q/R$ into this expression and solving for V_D, we obtain

$$V_D = \frac{k_e Q}{2R} \left(3 - \frac{r^2}{R^2}\right) \qquad (\text{for } r < R) \qquad \textbf{(25.25)}$$

At $r = R$, this expression gives a result that agrees with that for the potential at the surface, that is, V_C. A plot of V versus r for this charge distribution is given in Figure 25.19.

Exercise What are the magnitude of the electric field and the electric potential at the center of the sphere?

Answer $E = 0$; $V_0 = 3k_e Q/2R$.

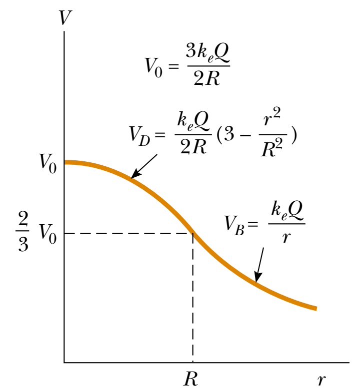


Figure 25.19 A plot of electric potential V versus distance r from the center of a uniformly charged insulating sphere of radius R. The curve for V_D inside the sphere is parabolic and joins smoothly with the curve for V_B outside the sphere, which is a hyperbola. The potential has a maximum value V_0 at the center of the sphere. We could make this graph three dimensional (similar to Figures 25.7a and 25.8a) by spinning it around the vertical axis.

25.6 ELECTRIC POTENTIAL DUE TO A CHARGED CONDUCTOR

In Section 24.4 we found that when a solid conductor in equilibrium carries a net charge, the charge resides on the outer surface of the conductor. Furthermore, we showed that the electric field just outside the conductor is perpendicular to the surface and that the field inside is zero.

We now show that **every point on the surface of a charged conductor in equilibrium is at the same electric potential.** Consider two points A and B on the surface of a charged conductor, as shown in Figure 25.20. Along a surface path connecting these points, $\mathbf{E}$ is always perpendicular to the displacement $d\mathbf{s}$; there-

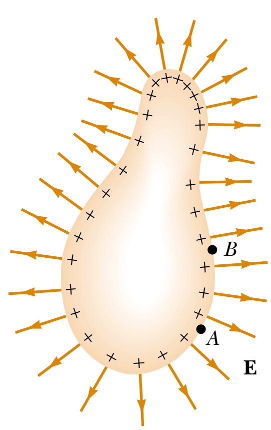


Figure 25.20 An arbitrarily shaped conductor carrying a positive charge. When the conductor is in electrostatic equilibrium, all of the charge resides at the surface, $\mathbf{E} = 0$ inside the conductor, and the direction of $\mathbf{E}$ just outside the conductor is perpendicular to the surface. The electric potential is constant inside the conductor and is equal to the potential at the surface. Note from the spacing of the plus signs that the surface charge density is nonuniform.

fore $\mathbf{E} \cdot d\mathbf{s} = 0$. Using this result and Equation 25.3, we conclude that the potential difference between A and B is necessarily zero:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

This result applies to any two points on the surface. Therefore, V is constant everywhere on the surface of a charged conductor in equilibrium. That is,

the surface of any charged conductor in electrostatic equilibrium is an equipotential surface. Furthermore, because the electric field is zero inside the conductor, we conclude from the relationship $E_r = -dV/dr$ that the electric potential is constant everywhere inside the conductor and equal to its value at the surface.

The surface of a charged conductor is an equipotential surface

Because this is true about the electric potential, no work is required to move a test charge from the interior of a charged conductor to its surface.

Consider a solid metal conducting sphere of radius R and total positive charge Q, as shown in Figure 25.21a. The electric field outside the sphere is k_eQ/r^2 and points radially outward. From Example 25.8, we know that the electric potential at the interior and surface of the sphere must be k_eQ/R relative to infinity. The potential outside the sphere is k_eQ/r. Figure 25.21b is a plot of the electric potential as a function of r, and Figure 25.21c shows how the electric field varies with r.

When a net charge is placed on a spherical conductor, the surface charge density is uniform, as indicated in Figure 25.21a. However, if the conductor is nonspherical, as in Figure 25.20, the surface charge density is high where the radius of curvature is small and the surface is convex (as noted in Section 24.4), and it is low where the radius of curvature is small and the surface is concave. Because the electric field just outside the conductor is proportional to the surface charge density, we see that **the electric field is large near convex points having small radii of curvature and reaches very high values at sharp points.**

Figure 25.22 shows the electric field lines around two spherical conductors: one carrying a net charge Q, and a larger one carrying zero net charge. In this case, the surface charge density is not uniform on either conductor. The sphere having zero net charge has negative charges induced on its side that faces the

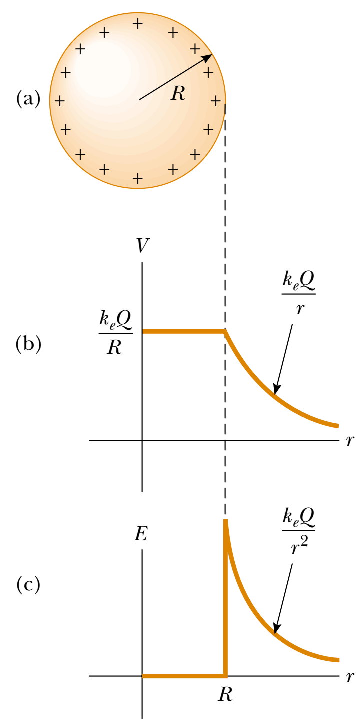
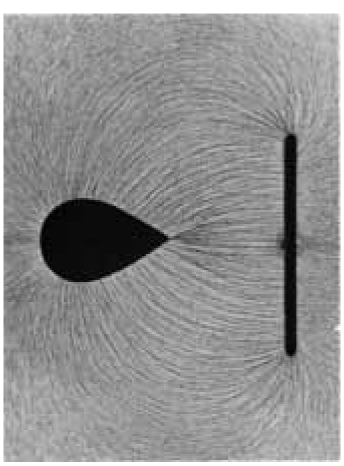


Figure 25.21 (a) The excess charge on a conducting sphere of radius R is uniformly distributed on its surface. (b) Electric potential versus distance r from the center of the charged conducting sphere. (c) Electric field magnitude versus distance r from the center of the charged conducting sphere.



Electric field pattern of a charged conducting plate placed near an oppositely charged pointed conductor. Small pieces of thread suspended in oil align with the electric field lines. The field surrounding the pointed conductor is most intense near the pointed end and at other places where the radius of curvature is small.

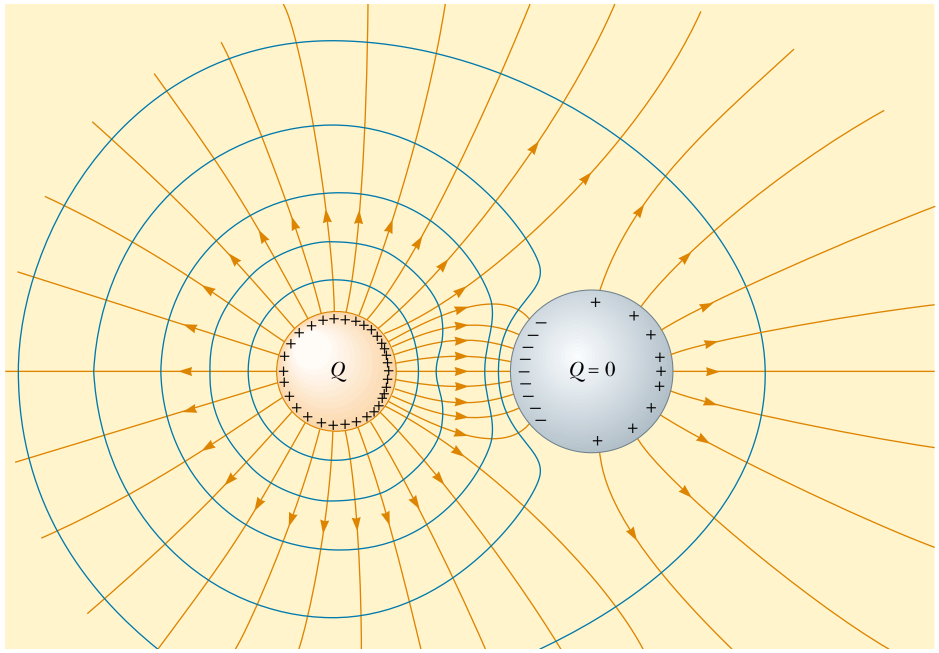


Figure 25.22 The electric field lines (in red) around two spherical conductors. The smaller sphere has a net charge Q, and the larger one has zero net charge. The blue curves are cross-sections of equipotential surfaces.

charged sphere and positive charges induced on its side opposite the charged sphere. The blue curves in the figure represent the cross-sections of the equipotential surfaces for this charge configuration. As usual, the field lines are perpendicular to the conducting surfaces at all points, and the equipotential surfaces are perpendicular to the field lines everywhere. Trying to move a positive charge in the region of these conductors would be like moving a marble on a hill that is flat on top (representing the conductor on the left) and has another flat area partway down the side of the hill (representing the conductor on the right).

EXAMPLE 25.9 Two Connected Charged Spheres

Two spherical conductors of radii r_1 and r_2 are separated by a distance much greater than the radius of either sphere. The spheres are connected by a conducting wire, as shown in Figure 25.23. The charges on the spheres in equilibrium are q_1 and q_2, respectively, and they are uniformly charged. Find the ratio of the magnitudes of the electric fields at the surfaces of the spheres.

Solution Because the spheres are connected by a conducting wire, they must both be at the same electric potential:

$$V = k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2}$$

Therefore, the ratio of charges is

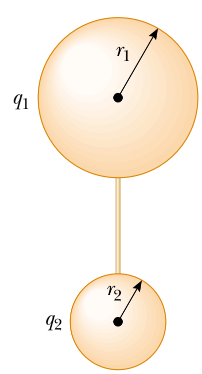


Figure 25.23 Two charged spherical conductors connected by a conducting wire. The spheres are at the *same* electric potential V.

$$(1) \qquad \frac{q_1}{q_2} = \frac{r_1}{r_2}$$

Because the spheres are very far apart and their surfaces uniformly charged, we can express the magnitude of the electric fields at their surfaces as

$$E_1 = k_e \frac{q_1}{{r_1}^2} \quad \text{and} \quad E_2 = k_e \frac{q_2}{{r_2}^2}$$

Taking the ratio of these two fields and making use of Equation (1), we find that

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

Hence, the field is more intense in the vicinity of the smaller sphere even though the electric potentials of both spheres are the same.

A Cavity Within a Conductor

Now consider a conductor of arbitrary shape containing a cavity as shown in Figure 25.24. Let us assume that no charges are inside the cavity. **In this case, the electric field inside the cavity must be zero** regardless of the charge distribution on the outside surface of the conductor. Furthermore, the field in the cavity is zero even if an electric field exists outside the conductor.

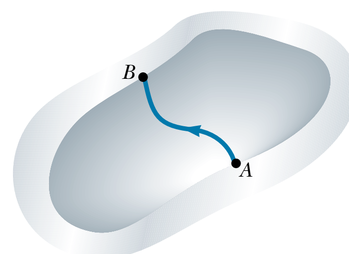


Figure 25.24 A conductor in electrostatic equilibrium containing a cavity. The electric field in the cavity is zero, regardless of the charge on the conductor.

To prove this point, we use the fact that every point on the conductor is at the same electric potential, and therefore any two points A and B on the surface of the cavity must be at the same potential. Now imagine that a field $\mathbf{E}$ exists in the cavity and evaluate the potential difference $V_B - V_A$ defined by Equation 25.3:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

If $\mathbf{E}$ is nonzero, we can always find a path between A and B for which $\mathbf{E} \cdot d\mathbf{s}$ is a positive number; thus, the integral must be positive. However, because $V_B - V_A = 0$, the integral of $\mathbf{E} \cdot d\mathbf{s}$ must be zero for all paths between any two points on the conductor, which implies that $\mathbf{E}$ is zero everywhere. This contradiction can be reconciled only if $\mathbf{E}$ is zero inside the cavity. Thus, we conclude that a cavity surrounded by conducting walls is a field-free region as long as no charges are inside the cavity.

Corona Discharge

A phenomenon known as **corona discharge** is often observed near a conductor such as a high-voltage power line. When the electric field in the vicinity of the conductor is sufficiently strong, electrons are stripped from air molecules. This causes the molecules to be ionized, thereby increasing the air's ability to conduct. The observed glow (or corona discharge) results from the recombination of free electrons with the ionized air molecules. If a conductor has an irregular shape, the electric field can be very high near sharp points or edges of the conductor; consequently, the ionization process and corona discharge are most likely to occur around such points.

Quick Quiz 25.4

(a) Is it possible for the magnitude of the electric field to be zero at a location where the electric potential is not zero? (b) Can the electric potential be zero where the electric field is nonzero?

Optional Section

25.7 THE MILLIKAN OIL-DROP EXPERIMENT

During the period from 1909 to 1913, Robert Millikan performed a brilliant set of experiments in which he measured e, the elementary charge on an electron, and demonstrated the quantized nature of this charge. His apparatus, diagrammed in Figure 25.25, contains two parallel metallic plates. Charged oil droplets from an atomizer are allowed to pass through a small hole in the upper plate. A horizontally directed light beam (not shown in the diagram) is used to illuminate the oil droplets, which are viewed through a telescope whose long axis is at right angles to the light beam. When the droplets are viewed in this manner, they appear as shining stars against a dark background, and the rate at which individual drops fall can be determined.[4]

Let us assume that a single drop having a mass m and carrying a charge q is being viewed and that its charge is negative. If no electric field is present between the plates, the two forces acting on the charge are the force of gravity $m\mathbf{g}$ acting downward and a viscous drag force $\mathbf{F}_D$ acting upward as indicated in Figure 25.26a. The drag force is proportional to the drop's speed. When the drop reaches its terminal speed v, the two forces balance each other ($mg = F_D$).

Now suppose that a battery connected to the plates sets up an electric field between the plates such that the upper plate is at the higher electric potential. In this case, a third force $q\mathbf{E}$ acts on the charged drop. Because q is negative and $\mathbf{E}$ is directed downward, this electric force is directed upward, as shown in Figure 25.26b. If this force is sufficiently great, the drop moves upward and the drag force $\mathbf{F}'_D$ acts downward. When the upward electric force $q\mathbf{E}$ balances the sum of the gravitational force and the downward drag force $\mathbf{F}'_D$, the drop reaches a new terminal speed v' in the upward direction.

With the field turned on, a drop moves slowly upward, typically at rates of hundredths of a centimeter per second. The rate of fall in the absence of a field is comparable. Hence, one can follow a single droplet for hours, alternately rising and falling, by simply turning the electric field on and off.

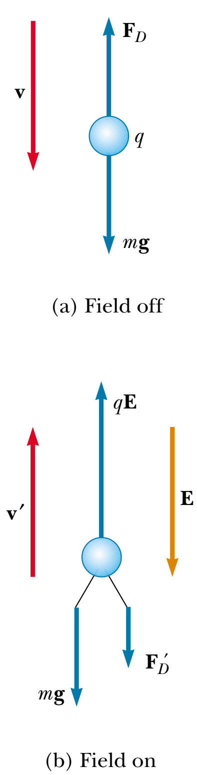


Figure 25.26 The forces acting on a negatively charged oil droplet in the Millikan experiment.

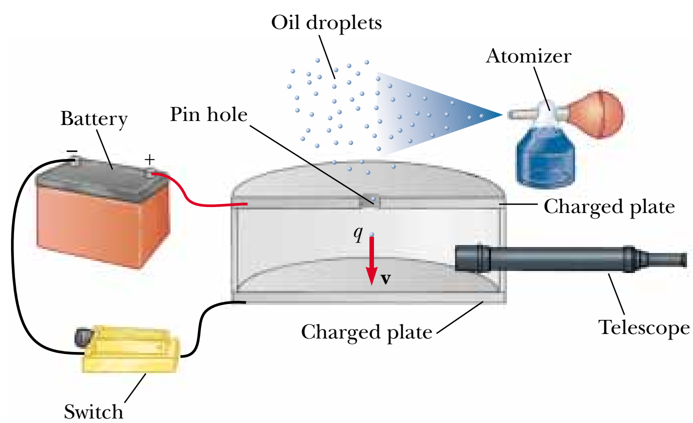


Figure 25.25 Schematic drawing of the Millikan oil-drop apparatus.

[4] At one time, the oil droplets were termed "Millikan's Shining Stars." Perhaps this description has lost its popularity because of the generations of physics students who have experienced hallucinations, near blindness, migraine headaches, and so forth, while repeating Millikan's experiment!

After recording measurements on thousands of droplets, Millikan and his co-workers found that all droplets, to within about 1% precision, had a charge equal to some integer multiple of the elementary charge e:

$$q = ne \qquad n = 0, -1, -2, -3, \ldots$$

where $e = 1.60 \times 10^{-19}$ C. Millikan's experiment yields conclusive evidence that charge is quantized. For this work, he was awarded the Nobel Prize in Physics in 1923.

Optional Section

25.8 APPLICATIONS OF ELECTROSTATICS

The practical application of electrostatics is represented by such devices as lightning rods and electrostatic precipitators and by such processes as xerography and the painting of automobiles. Scientific devices based on the principles of electrostatics include electrostatic generators, the field-ion microscope, and ion-drive rocket engines.

The Van de Graaff Generator

In Section 24.5 we described an experiment that demonstrates a method for trans-
11.10 ferring charge to a hollow conductor (the Faraday ice-pail experiment). When a charged conductor is placed in contact with the inside of a hollow conductor, all of the charge of the charged conductor is transferred to the hollow conductor. In principle, the charge on the hollow conductor and its electric potential can be increased without limit by repetition of the process.

In 1929 Robert J. Van de Graaff (1901–1967) used this principle to design and build an electrostatic generator. This type of generator is used extensively in nuclear physics research. A schematic representation of the generator is given in Figure 25.27. Charge is delivered continuously to a high-potential electrode by means of a moving belt of insulating material. The high-voltage electrode is a hollow conductor mounted on an insulating column. The belt is charged at point A by means of a corona discharge between comb-like metallic needles and a grounded grid. The needles are maintained at a positive electric potential of typically 10^4 V. The positive charge on the moving belt is transferred to the hollow conductor by a second comb of needles at point B. Because the electric field inside the hollow conductor is negligible, the positive charge on the belt is easily transferred to the conductor regardless of its potential. In practice, it is possible to increase the electric potential of the hollow conductor until electrical discharge occurs through the air. Because the "breakdown" electric field in air is about 3×10^6 V/m, a sphere 1 m in radius can be raised to a maximum potential of 3×10^6 V. The potential can be increased further by increasing the radius of the hollow conductor and by placing the entire system in a container filled with high-pressure gas.

Van de Graaff generators can produce potential differences as large as 20 million volts. Protons accelerated through such large potential differences receive enough energy to initiate nuclear reactions between themselves and various target nuclei. Smaller generators are often seen in science classrooms and museums. If a person insulated from the ground touches the sphere of a Van de Graaff generator, his or her body can be brought to a high electric potential. The hair acquires a net positive charge, and each strand is repelled by all the others. The result is a

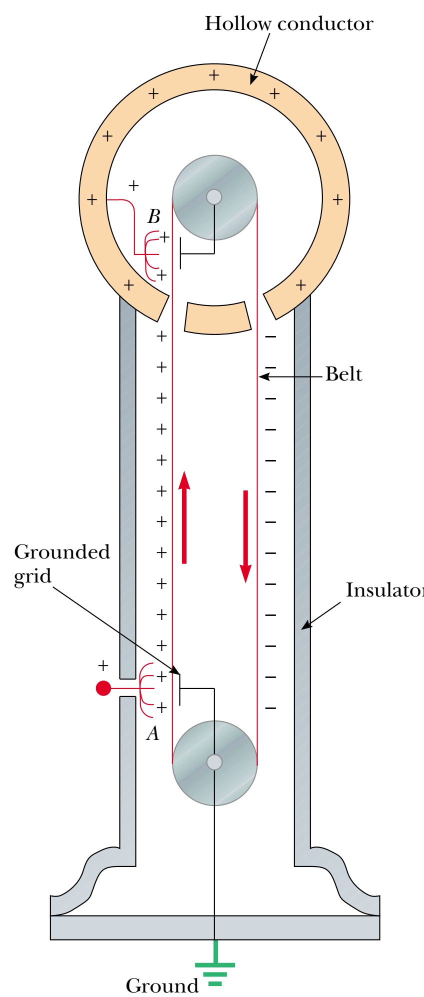


Figure 25.27 Schematic diagram of a Van de Graaff generator. Charge is transferred to the hollow conductor at the top by means of a moving belt. The charge is deposited on the belt at point A and transferred to the hollow conductor at point B.

scene such as that depicted in the photograph at the beginning of this chapter. In addition to being insulated from ground, the person holding the sphere is safe in this demonstration because the total charge on the sphere is very small (on the order of 1 μC). If this amount of charge accidentally passed from the sphere through the person to ground, the corresponding current would do no harm.

The Electrostatic Precipitator

QuickLab

Sprinkle some salt and pepper on an open dish and mix the two together. Now pull a comb through your hair several times and bring the comb to within 1 cm of the salt and pepper. What happens? How is what happens here related to the operation of an electrostatic precipitator?

One important application of electrical discharge in gases is the *electrostatic precipitator.* This device removes particulate matter from combustion gases, thereby reducing air pollution. Precipitators are especially useful in coal-burning power plants and in industrial operations that generate large quantities of smoke. Current systems are able to eliminate more than 99% of the ash from smoke.

Figure 25.28a shows a schematic diagram of an electrostatic precipitator. A high potential difference (typically 40 to 100 kV) is maintained between a wire running down the center of a duct and the walls of the duct, which are grounded. The wire is maintained at a negative electric potential with respect to the walls, so the electric field is directed toward the wire. The values of the field near the wire become high enough to cause a corona discharge around the wire; the discharge ionizes some air molecules to form positive ions, electrons, and such negative ions as O_2^-. The air to be cleaned enters the duct and moves near the wire. As the electrons and negative ions created by the discharge are accelerated toward the outer wall by the electric field, the dirt particles in the air become charged by collisions and ion capture. Because most of the charged dirt particles are negative, they too are drawn to the duct walls by the electric field. When the duct is periodically shaken, the particles break loose and are collected at the bottom.

(a) (b) (c)

Figure 25.28 (a) Schematic diagram of an electrostatic precipitator. The high negative electric potential maintained on the central coiled wire creates an electrical discharge in the vicinity of the wire. Compare the air pollution when the electrostatic precipitator is (b) operating and (c) turned off.

In addition to reducing the level of particulate matter in the atmosphere (compare Figs. 25.28b and c), the electrostatic precipitator recovers valuable materials in the form of metal oxides.

Xerography and Laser Printers

The basic idea of xerography[5] was developed by Chester Carlson, who was granted a patent for the xerographic process in 1940. The one feature of this process that makes it unique is the use of a photoconductive material to form an image. (A *photoconductor* is a material that is a poor electrical conductor in the dark but that becomes a good electrical conductor when exposed to light.)

The xerographic process is illustrated in Figure 25.29a to d. First, the surface of a plate or drum that has been coated with a thin film of photoconductive material (usually selenium or some compound of selenium) is given a positive electrostatic charge in the dark. An image of the page to be copied is then focused by a lens onto the charged surface. The photoconducting surface becomes conducting only in areas where light strikes it. In these areas, the light produces charge carriers in the photoconductor that move the positive charge off the drum. However, positive

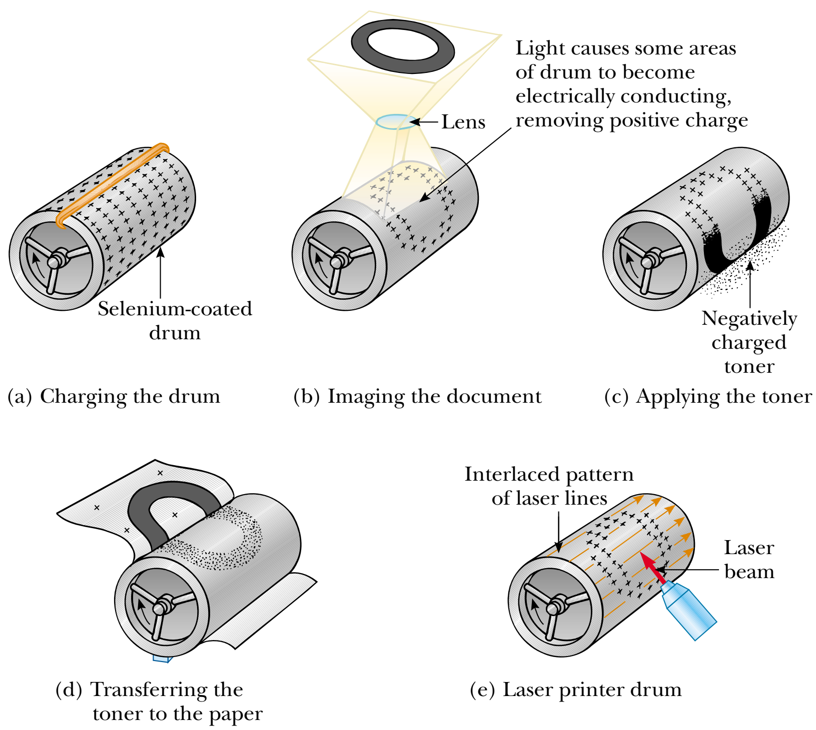


Figure 25.29 The xerographic process: (a) The photoconductive surface of the drum is positively charged. (b) Through the use of a light source and lens, an image is formed on the surface in the form of positive charges. (c) The surface containing the image is covered with a negatively charged powder, which adheres only to the image area. (d) A piece of paper is placed over the surface and given a positive charge. This transfers the image to the paper as the negatively charged powder particles migrate to the paper. The paper is then heat-treated to "fix" the powder. (e) A laser printer operates similarly except the image is produced by turning a laser beam on and off as it sweeps across the selenium-coated drum.

[5] The prefix *xero-* is from the Greek word meaning "dry." Note that no liquid ink is used anywhere in xerography.

charges remain on those areas of the photoconductor not exposed to light, leaving a latent image of the object in the form of a positive surface charge distribution.

Next, a negatively charged powder called a *toner* is dusted onto the photoconducting surface. The charged powder adheres only to those areas of the surface that contain the positively charged image. At this point, the image becomes visible. The toner (and hence the image) are then transferred to the surface of a sheet of positively charged paper.

Finally, the toner is "fixed" to the surface of the paper as the toner melts while passing through high-temperature rollers. This results in a permanent copy of the original.

A laser printer (Fig. 25.29e) operates by the same principle, with the exception that a computer-directed laser beam is used to illuminate the photoconductor instead of a lens.

SUMMARY

When a positive test charge q_0 is moved between points A and B in an electric field $\mathbf{E}$, the **change in the potential energy** is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad \textbf{(25.1)}$$

The **electric potential** $V = U/q_0$ is a scalar quantity and has units of joules per coulomb (J/C), where $1\ \text{J/C} \equiv 1\ \text{V}$.

The **potential difference** ΔV between points A and B in an electric field $\mathbf{E}$ is defined as

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad \textbf{(25.3)}$$

The potential difference between two points A and B in a uniform electric field $\mathbf{E}$ is

$$\Delta V = -Ed \qquad \textbf{(25.6)}$$

where d is the magnitude of the displacement in the direction parallel to $\mathbf{E}$.

An **equipotential surface** is one on which all points are at the same electric potential. Equipotential surfaces are perpendicular to electric field lines.

If we define $V = 0$ at $r_A = \infty$, the electric potential due to a point charge at any distance r from the charge is

$$V = k_e \frac{q}{r} \qquad \textbf{(25.11)}$$

We can obtain the electric potential associated with a group of point charges by summing the potentials due to the individual charges.

The **potential energy associated with a pair of point charges** separated by a distance r_{12} is

$$U = k_e \frac{q_1 q_2}{r_{12}} \qquad \textbf{(25.13)}$$

This energy represents the work required to bring the charges from an infinite separation to the separation r_{12}. We obtain the potential energy of a distribution of point charges by summing terms like Equation 25.13 over all pairs of particles.

TABLE 25.1 Electric Potential Due to Various Charge Distributions

Charge Distribution	Electric Potential	Location
Uniformly charged ring of radius a	$V = k_e \dfrac{Q}{\sqrt{x^2 + a^2}}$	Along perpendicular central axis of ring, distance x from ring center
Uniformly charged disk of radius a	$V = 2\pi k_e \sigma[(x^2 + a^2)^{1/2} - x]$	Along perpendicular central axis of disk, distance x from disk center
Uniformly charged, *insulating* solid sphere of radius R and total charge Q	$V = k_e \dfrac{Q}{r}$	$r \geq R$
	$V = \dfrac{k_e Q}{2R}\left(3 - \dfrac{r^2}{R^2}\right)$	$r < R$
Isolated *conducting* sphere of radius R and total charge Q	$V = k_e \dfrac{Q}{r}$	$r > R$
	$V = k_e \dfrac{Q}{R}$	$r \leq R$

If we know the electric potential as a function of coordinates x, y, z, we can obtain the components of the electric field by taking the negative derivative of the electric potential with respect to the coordinates. For example, the x component of the electric field is

$$E_x = -\frac{dV}{dx} \tag{25.16}$$

The **electric potential due to a continuous charge distribution** is

$$V = k_e \int \frac{dq}{r} \tag{25.19}$$

Every point on the surface of a charged conductor in electrostatic equilibrium is at the same electric potential. The potential is constant everywhere inside the conductor and equal to its value at the surface.

Table 25.1 lists electric potentials due to several charge distributions.

Problem-Solving Hints

Calculating Electric Potential

- Remember that electric potential is a scalar quantity, so components need not be considered. Therefore, when using the superposition principle to evaluate the electric potential at a point due to a system of point charges, simply take the algebraic sum of the potentials due to the various charges. However, you must keep track of signs. The potential is positive for positive charges, and it is negative for negative charges.
- Just as with gravitational potential energy in mechanics, only *changes* in electric potential are significant; hence, the point where you choose the poten-

tial to be zero is arbitrary. When dealing with point charges or a charge distribution of finite size, we usually define $V = 0$ to be at a point infinitely far from the charges.

- You can evaluate the electric potential at some point P due to a continuous distribution of charge by dividing the charge distribution into infinitesimal elements of charge dq located at a distance r from P. Then, treat one charge element as a point charge, such that the potential at P due to the element is $dV = k_e dq/r$. Obtain the total potential at P by integrating dV over the entire charge distribution. In performing the integration for most problems, you must express dq and r in terms of a single variable. To simplify the integration, consider the geometry involved in the problem carefully. Review Examples 25.5 through 25.7 for guidance.
- Another method that you can use to obtain the electric potential due to a finite continuous charge distribution is to start with the definition of potential difference given by Equation 25.3. If you know or can easily obtain $\mathbf{E}$ (from Gauss's law), then you can evaluate the line integral of $\mathbf{E} \cdot d\mathbf{s}$. An example of this method is given in Example 25.8.
- Once you know the electric potential at a point, you can obtain the electric field at that point by remembering that the electric field component in a specified direction is equal to the negative of the derivative of the electric potential in that direction. Example 25.4 illustrates this procedure.

QUESTIONS

1. Distinguish between electric potential and electric potential energy.
2. A negative charge moves in the direction of a uniform electric field. Does the potential energy of the charge increase or decrease? Does it move to a position of higher or lower potential?
3. Give a physical explanation of the fact that the potential energy of a pair of like charges is positive whereas the potential energy of a pair of unlike charges is negative.
4. A uniform electric field is parallel to the x axis. In what direction can a charge be displaced in this field without any external work being done on the charge?
5. Explain why equipotential surfaces are always perpendicular to electric field lines.
6. Describe the equipotential surfaces for (a) an infinite line of charge and (b) a uniformly charged sphere.
7. Explain why, under static conditions, all points in a conductor must be at the same electric potential.
8. The electric field inside a hollow, uniformly charged sphere is zero. Does this imply that the potential is zero inside the sphere? Explain.
9. The potential of a point charge is defined to be zero at an infinite distance. Why can we not define the potential of an infinite line of charge to be zero at $r = \infty$?
10. Two charged conducting spheres of different radii are connected by a conducting wire, as shown in Figure 25.23. Which sphere has the greater charge density?
11. What determines the maximum potential to which the dome of a Van de Graaff generator can be raised?
12. Explain the origin of the glow sometimes observed around the cables of a high-voltage power line.
13. Why is it important to avoid sharp edges or points on conductors used in high-voltage equipment?
14. How would you shield an electronic circuit or laboratory from stray electric fields? Why does this work?
15. Why is it relatively safe to stay in an automobile with a metal body during a severe thunderstorm?
16. Walking across a carpet and then touching someone can result in a shock. Explain why this occurs.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
= paired numerical/symbolic problems

Section 25.1 Potential Difference and Electric Potential

1. How much work is done (by a battery, generator, or some other source of electrical energy) in moving Avogadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the potential is -5.00 V? (The potential in each case is measured relative to a common reference point.)

2. An ion accelerated through a potential difference of 115 V experiences an increase in kinetic energy of 7.37×10^{-17} J. Calculate the charge on the ion.

3. (a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V. (b) Calculate the speed of an electron that is accelerated through the same potential difference.

4. Review Problem. Through what potential difference would an electron need to be accelerated for it to achieve a speed of 40.0% of the speed of light, starting from rest? The speed of light is $c = 3.00 \times 10^8$ m/s; review Section 7.7.

5. What potential difference is needed to stop an electron having an initial speed of 4.20×10^5 m/s?

Section 25.2 Potential Differences in a Uniform Electric Field

6. A uniform electric field of magnitude 250 V/m is directed in the positive x direction. A $+12.0$-μC charge moves from the origin to the point $(x, y) =$ (20.0 cm, 50.0 cm). (a) What was the change in the potential energy of this charge? (b) Through what potential difference did the charge move?

7. The difference in potential between the accelerating plates of a TV set is about 25 000 V. If the distance between these plates is 1.50 cm, find the magnitude of the uniform electric field in this region.

8. Suppose an electron is released from rest in a uniform electric field whose magnitude is 5.90×10^3 V/m. (a) Through what potential difference will it have passed after moving 1.00 cm? (b) How fast will the electron be moving after it has traveled 1.00 cm?

WEB **9.** An electron moving parallel to the x axis has an initial speed of 3.70×10^6 m/s at the origin. Its speed is reduced to 1.40×10^5 m/s at the point $x = 2.00$ cm. Calculate the potential difference between the origin and that point. Which point is at the higher potential?

10. A uniform electric field of magnitude 325 V/m is directed in the *negative* y direction as shown in Figure P25.10. The coordinates of point A are $(-0.200, -0.300)$ m, and those of point B are (0.400, 0.500) m. Calculate the potential difference $V_B - V_A$, using the blue path.

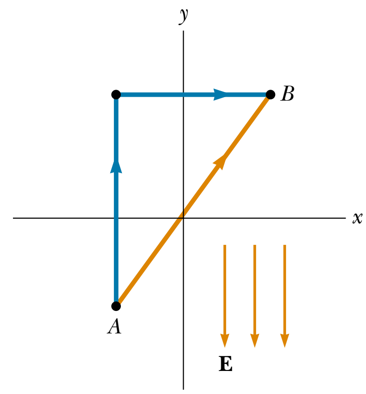


Figure P25.10

11. A 4.00-kg block carrying a charge $Q = 50.0$ μC is connected to a spring for which $k = 100$ N/m. The block lies on a frictionless horizontal track, and the system is immersed in a uniform electric field of magnitude $E = 5.00 \times 10^5$ V/m, directed as shown in Figure P25.11. If the block is released from rest when the spring is unstretched (at $x = 0$), (a) by what maximum amount does the spring expand? (b) What is the equilibrium position of the block? (c) Show that the block's motion is simple harmonic, and determine its period. (d) Repeat part (a) if the coefficient of kinetic friction between block and surface is 0.200.

12. A block having mass m and charge Q is connected to a spring having constant k. The block lies on a frictionless horizontal track, and the system is immersed in a uniform electric field of magnitude E, directed as shown in Figure P25.11. If the block is released from rest when the spring is unstretched (at $x = 0$), (a) by what maximum amount does the spring expand? (b) What is the equilibrium position of the block? (c) Show that the block's motion is simple harmonic, and determine its period. (d) Repeat part (a) if the coefficient of kinetic friction between block and surface is μ_k.

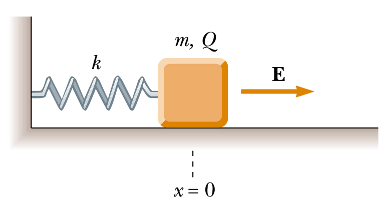


Figure P25.11 Problems 11 and 12.

13. On planet Tehar, the acceleration due to gravity is the same as that on Earth but there is also a strong downward electric field with the field being uniform close to the planet's surface. A 2.00-kg ball having a charge of 5.00 μC is thrown upward at a speed of 20.1 m/s and it hits the ground after an interval of 4.10 s. What is the potential difference between the starting point and the top point of the trajectory?

14. An insulating rod having linear charge density $\lambda =$ 40.0 μC/m and linear mass density $\mu = 0.100$ kg/m is released from rest in a uniform electric field $E =$ 100 V/m directed perpendicular to the rod (Fig. P25.14). (a) Determine the speed of the rod after it has traveled 2.00 m. (b) How does your answer to part (a) change if the electric field is not perpendicular to the rod? Explain.

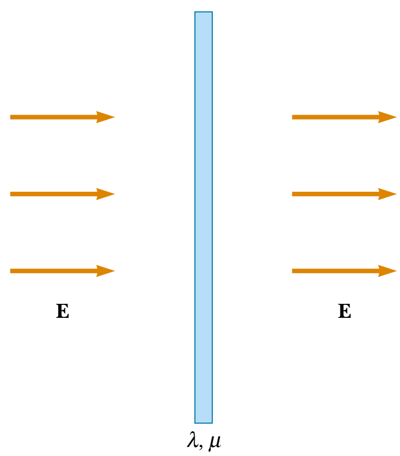


Figure P25.14

15. A particle having charge $q = +2.00$ μC and mass $m =$ 0.010 0 kg is connected to a string that is $L = 1.50$ m long and is tied to the pivot point P in Figure P25.15. The particle, string, and pivot point all lie on a horizontal table. The particle is released from rest when the string makes an angle $\theta = 60.0°$ with a uniform electric field of magnitude $E = 300$ V/m. Determine the speed of the particle when the string is parallel to the electric field (point a in Fig. P25.15).

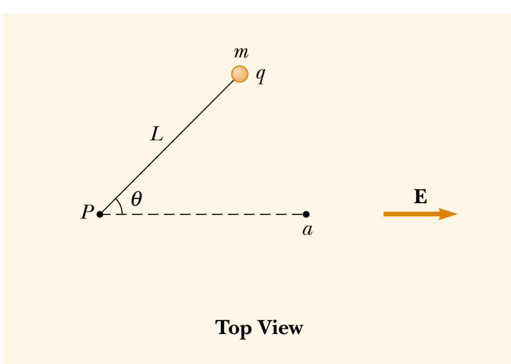


Figure P25.15

Section 25.3 Electric Potential and Potential Energy Due to Point Charges

Note: Unless stated otherwise, assume a reference level of potential $V = 0$ at $r = \infty$.

16. (a) Find the potential at a distance of 1.00 cm from a proton. (b) What is the potential difference between two points that are 1.00 cm and 2.00 cm from a proton? (c) Repeat parts (a) and (b) for an electron.

17. Given two 2.00-μC charges, as shown in Figure P25.17, and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin, (a) what is the net force exerted on q by the two 2.00-μC charges? (b) What is the electric field at the origin due to the two 2.00-μC charges? (c) What is the electric potential at the origin due to the two 2.00-μC charges?

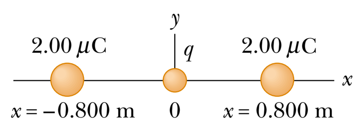


Figure P25.17

18. A charge $+q$ is at the origin. A charge $-2q$ is at $x =$ 2.00 m on the x axis. For what finite value(s) of x is (a) the electric field zero? (b) the electric potential zero?

19. The Bohr model of the hydrogen atom states that the single electron can exist only in certain allowed orbits around the proton. The radius of each Bohr orbit is $r = n^2$ (0.052 9 nm) where $n = 1, 2, 3, \ldots$. Calculate the electric potential energy of a hydrogen atom when the electron is in the (a) first allowed orbit, $n = 1$; (b) second allowed orbit, $n = 2$; and (c) when the electron has escaped from the atom ($r = \infty$). Express your answers in electron volts.

20. Two point charges $Q_1 = +5.00$ nC and $Q_2 = -3.00$ nC are separated by 35.0 cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?

21. The three charges in Figure P25.21 are at the vertices of an isosceles triangle. Calculate the electric potential at the midpoint of the base, taking $q = 7.00$ μC.

22. *Compare this problem with Problem 55 in Chapter 23.* Four identical point charges ($q = +10.0$ μC) are located on the corners of a rectangle, as shown in Figure P23.55. The dimensions of the rectangle are $L = 60.0$ cm and $W = 15.0$ cm. Calculate the electric potential energy of the charge at the lower left corner due to the other three charges.

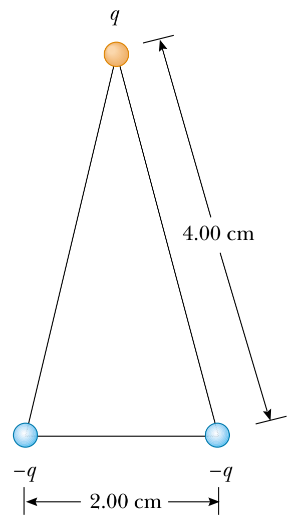


Figure P25.21

WEB **23.** Show that the amount of work required to assemble four identical point charges of magnitude Q at the corners of a square of side s is $5.41\,k_eQ^2/s$.

24. *Compare this problem with Problem 18 in Chapter 23.* Two point charges each of magnitude 2.00 μC are located on the x axis. One is at $x = 1.00$ m, and the other is at $x = -1.00$ m. (a) Determine the electric potential on the y axis at $y = 0.500$ m. (b) Calculate the electric potential energy of a third charge, of -3.00 μC, placed on the y axis at $y = 0.500$ m.

25. *Compare this problem with Problem 22 in Chapter 23.* Five equal negative point charges $-q$ are placed symmetrically around a circle of radius R. Calculate the electric potential at the center of the circle.

26. *Compare this problem with Problem 17 in Chapter 23.* Three equal positive charges q are at the corners of an equilateral triangle of side a, as shown in Figure P23.17. (a) At what point, if any, in the plane of the charges is the electric potential zero? (b) What is the electric potential at the point P due to the two charges at the base of the triangle?

27. Review Problem. Two insulating spheres having radii 0.300 cm and 0.500 cm, masses 0.100 kg and 0.700 kg, and charges -2.00 μC and 3.00 μC are released from rest when their centers are separated by 1.00 m. (a) How fast will each be moving when they collide? (*Hint:* Consider conservation of energy and linear momentum.) (b) If the spheres were conductors would the speeds be larger or smaller than those calculated in part (a)? Explain.

28. Review Problem. Two insulating spheres having radii r_1 and r_2, masses m_1 and m_2, and charges $-q_1$ and q_2 are released from rest when their centers are separated by a distance d. (a) How fast is each moving when they collide? (*Hint:* Consider conservation of energy and conservation of linear momentum.) (b) If the spheres were conductors, would the speeds be greater or less than those calculated in part (a)?

29. A small spherical object carries a charge of 8.00 nC. At what distance from the center of the object is the potential equal to 100 V? 50.0 V? 25.0 V? Is the spacing of the equipotentials proportional to the change in potential?

30. Two point charges of equal magnitude are located along the y axis equal distances above and below the x axis, as shown in Figure P25.30. (a) Plot a graph of the potential at points along the x axis over the interval $-3a < x < 3a$. You should plot the potential in units of k_eQ/a. (b) Let the charge located at $-a$ be negative and plot the potential along the y axis over the interval $-4a < y < 4a$.

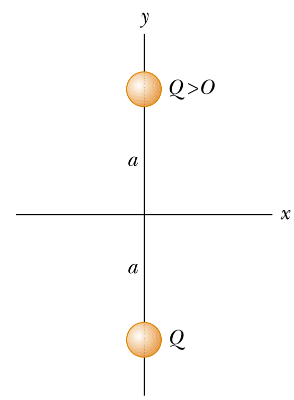


Figure P25.30

31. In Rutherford's famous scattering experiments that led to the planetary model of the atom, alpha particles (charge $+2e$, mass $= 6.64 \times 10^{-27}$ kg) were fired at a gold nucleus (charge $+79e$). An alpha particle, initially very far from the gold nucleus, is fired with a velocity of 2.00×10^7 m/s directly toward the center of the nucleus. How close does the alpha particle get to this center before turning around? Assume the gold nucleus remains stationary.

32. An electron starts from rest 3.00 cm from the center of a uniformly charged insulating sphere of radius 2.00 cm and total charge 1.00 nC. What is the speed of the electron when it reaches the surface of the sphere?

33. Calculate the energy required to assemble the array of charges shown in Figure P25.33, where $a = 0.200$ m, $b = 0.400$ m, and $q = 6.00$ μC.

34. Four identical particles each have charge q and mass m. They are released from rest at the vertices of a square of side L. How fast is each charge moving when their distance from the center of the square doubles?

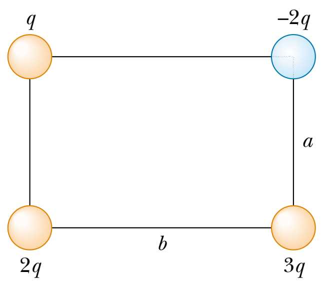


Figure P25.33

35. How much work is required to assemble eight identical point charges, each of magnitude q, at the corners of a cube of side s?

Section 25.4 Obtaining the Value of the Electric Field from the Electric Potential

36. The potential in a region between $x = 0$ and $x = 6.00$ m is $V = a + bx$ where $a = 10.0$ V and $b = -7.00$ V/m. Determine (a) the potential at $x = 0$, 3.00 m, and 6.00 m and (b) the magnitude and direction of the electric field at $x = 0$, 3.00 m, and 6.00 m.

WEB 37. Over a certain region of space, the electric potential is $V = 5x - 3x^2y + 2yz^2$. Find the expressions for the x, y, and z components of the electric field over this region. What is the magnitude of the field at the point P, which has coordinates $(1, 0, -2)$ m?

38. The electric potential inside a charged spherical conductor of radius R is given by $V = k_eQ/R$ and outside the conductor is given by $V = k_eQ/r$. Using $E_r = -dV/dr$, derive the electric field (a) inside and (b) outside this charge distribution.

39. It is shown in Example 25.7 that the potential at a point P a distance a above one end of a uniformly charged rod of length ℓ lying along the x axis is

$$V = \frac{k_eQ}{\ell}\ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right)$$

Use this result to derive an expression for the y component of the electric field at P. (*Hint:* Replace a with y.)

40. When an uncharged conducting sphere of radius a is placed at the origin of an xyz coordinate system that lies in an initially uniform electric field $\mathbf{E} = E_0\mathbf{k}$, the resulting electric potential is

$$V(x, y, z) = V_0 - E_0z + \frac{E_0a^3z}{(x^2 + y^2 + z^2)^{3/2}}$$

for points outside the sphere, where V_0 is the (constant) electric potential on the conductor. Use this equation to determine the x, y, and z components of the resulting electric field.

Section 25.5 Electric Potential Due to Continuous Charge Distributions

41. Consider a ring of radius R with the total charge Q spread uniformly over its perimeter. What is the potential difference between the point at the center of the ring and a point on its axis a distance $2R$ from the center?

42. *Compare this problem with Problem 33 in Chapter 23.* A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle, as shown in Figure P23.33. If the rod has a total charge of $-7.50\ \mu$C, find the electric potential at O, the center of the semicircle.

43. A rod of length L (Fig. P25.43) lies along the x axis with its left end at the origin and has a nonuniform charge density $\lambda = \alpha x$ (where α is a positive constant). (a) What are the units of α? (b) Calculate the electric potential at A.

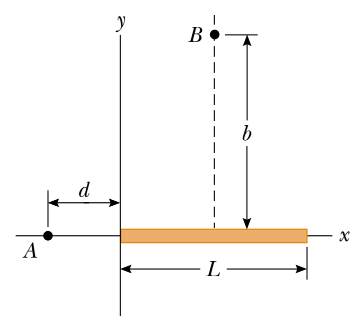


Figure P25.43 Problems 43 and 44.

44. For the arrangement described in the previous problem, calculate the electric potential at point B that lies on the perpendicular bisector of the rod a distance b above the x axis.

45. Calculate the electric potential at point P on the axis of the annulus shown in Figure P25.45, which has a uniform charge density σ.

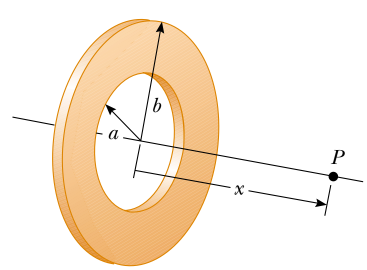


Figure P25.45

46. A wire of finite length that has a uniform linear charge density λ is bent into the shape shown in Figure P25.46. Find the electric potential at point O.

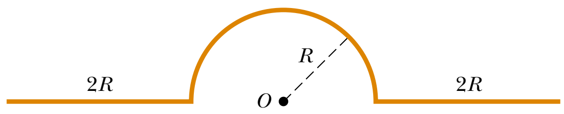


Figure P25.46

Section 25.6 Electric Potential Due to a Charged Conductor

47. How many electrons should be removed from an initially uncharged spherical conductor of radius 0.300 m to produce a potential of 7.50 kV at the surface?

48. Two charged spherical conductors are connected by a long conducting wire, and a charge of 20.0 μC is placed on the combination. (a) If one sphere has a radius of 4.00 cm and the other has a radius of 6.00 cm, what is the electric field near the surface of each sphere? (b) What is the electric potential of each sphere?

WEB **49.** A spherical conductor has a radius of 14.0 cm and charge of 26.0 μC. Calculate the electric field and the electric potential at (a) $r = 10.0$ cm, (b) $r = 20.0$ cm, and (c) $r = 14.0$ cm from the center.

50. Two concentric spherical conducting shells of radii $a = 0.400$ m and $b = 0.500$ m are connected by a thin wire, as shown in Figure P25.50. If a total charge $Q = 10.0\ \mu$C is placed on the system, how much charge settles on each sphere?

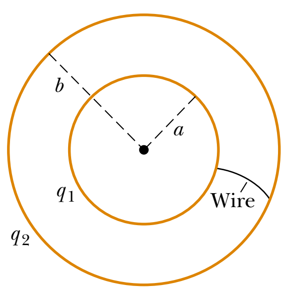


Figure P25.50

(Optional)
Section 25.7 The Millikan Oil-Drop Experiment

(Optional)
Section 25.8 Applications of Electrostatics

51. Consider a Van de Graaff generator with a 30.0-cm-diameter dome operating in dry air. (a) What is the maximum potential of the dome? (b) What is the maximum charge on the dome?

52. The spherical dome of a Van de Graaff generator can be raised to a maximum potential of 600 kV; then additional charge leaks off in sparks, by producing breakdown of the surrounding dry air. Determine (a) the charge on the dome and (b) the radius of the dome.

ADDITIONAL PROBLEMS

53. The liquid-drop model of the nucleus suggests that high-energy oscillations of certain nuclei can split the nucleus into two unequal fragments plus a few neutrons. The fragments acquire kinetic energy from their mutual Coulomb repulsion. Calculate the electric potential energy (in electron volts) of two spherical fragments from a uranium nucleus having the following charges and radii: $38e$ and 5.50×10^{-15} m; $54e$ and 6.20×10^{-15} m. Assume that the charge is distributed uniformly throughout the volume of each spherical fragment and that their surfaces are initially in contact at rest. (The electrons surrounding the nucleus can be neglected.)

54. On a dry winter day you scuff your leather-soled shoes across a carpet and get a shock when you extend the tip of one finger toward a metal doorknob. In a dark room you see a spark perhaps 5 mm long. Make order-of-magnitude estimates of (a) your electric potential and (b) the charge on your body before you touch the doorknob. Explain your reasoning.

55. The charge distribution shown in Figure P25.55 is referred to as a linear quadrupole. (a) Show that the potential at a point on the x axis where $x > a$ is

$$V = \frac{2k_eQa^2}{x^3 - xa^2}$$

(b) Show that the expression obtained in part (a) when $x \gg a$ reduces to

$$V = \frac{2k_eQa^2}{x^3}$$

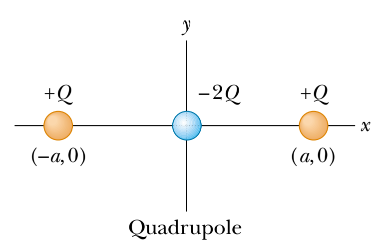


Figure P25.55

56. (a) Use the exact result from Problem 55 to find the electric field at any point along the axis of the linear quadrupole for $x > a$. (b) Evaluate E at $x = 3a$ if $a = 2.00$ mm and $Q = 3.00\ \mu$C.

57. At a certain distance from a point charge, the magnitude of the electric field is 500 V/m and the electric potential is -3.00 kV. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

58. An electron is released from rest on the axis of a uniform positively charged ring, 0.100 m from the ring's center. If the linear charge density of the ring is $+0.100\ \mu C/m$ and the radius of the ring is 0.200 m, how fast will the electron be moving when it reaches the center of the ring?

59. (a) Consider a uniformly charged cylindrical shell having total charge Q, radius R, and height h. Determine the electrostatic potential at a point a distance d from the right side of the cylinder, as shown in Figure P25.59. (*Hint:* Use the result of Example 25.5 by treating the cylinder as a collection of ring charges.) (b) Use the result of Example 25.6 to solve the same problem for a solid cylinder.

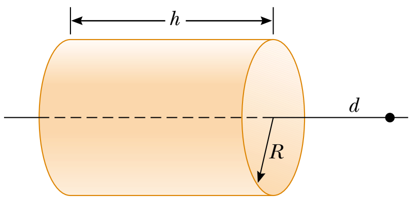


Figure P25.59

60. Two parallel plates having charges of equal magnitude but opposite sign are separated by 12.0 cm. Each plate has a surface charge density of 36.0 nC/m^2. A proton is released from rest at the positive plate. Determine (a) the potential difference between the plates, (b) the energy of the proton when it reaches the negative plate, (c) the speed of the proton just before it strikes the negative plate, (d) the acceleration of the proton, and (e) the force on the proton. (f) From the force, find the magnitude of the electric field and show that it is equal to that found from the charge densities on the plates.

61. Calculate the work that must be done to charge a spherical shell of radius R to a total charge Q.

62. A Geiger–Müller counter is a radiation detector that essentially consists of a hollow cylinder (the cathode) of inner radius r_a and a coaxial cylindrical wire (the anode) of radius r_b (Fig. P25.62). The charge per unit length on the anode is λ, while the charge per unit length on the cathode is $-\lambda$. (a) Show that the magnitude of the potential difference between the wire and the cylinder in the sensitive region of the detector is

$$\Delta V = 2k_e \lambda \ln\left(\frac{r_a}{r_b}\right)$$

(b) Show that the magnitude of the electric field over that region is given by

$$E = \frac{\Delta V}{\ln(r_a/r_b)}\left(\frac{1}{r}\right)$$

where r is the distance from the center of the anode to the point where the field is to be calculated.

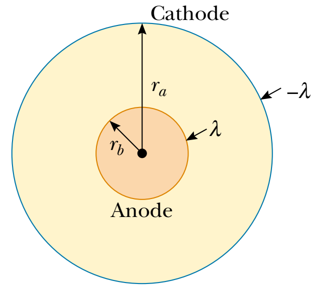


Figure P25.62

WEB **63.** From Gauss's law, the electric field set up by a uniform line of charge is

$$\mathbf{E} = \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector pointing radially away from the line and λ is the charge per unit length along the line. Derive an expression for the potential difference between $r = r_1$ and $r = r_2$.

64. A point charge q is located at $x = -R$, and a point charge $-2q$ is located at the origin. Prove that the equipotential surface that has zero potential is a sphere centered at $(-4R/3, 0, 0)$ and having a radius $r = 2R/3$.

65. Consider two thin, conducting, spherical shells as shown in cross-section in Figure P25.65. The inner shell has a radius $r_1 = 15.0$ cm and a charge of 10.0 nC. The outer shell has a radius $r_2 = 30.0$ cm and a charge of -15.0 nC. Find (a) the electric field $\mathbf{E}$ and (b) the electric potential V in regions A, B, and C, with $V = 0$ at $r = \infty$.

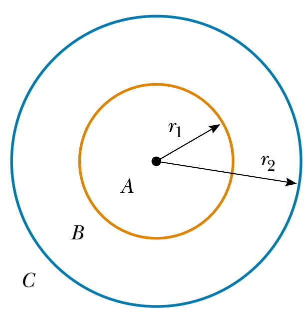


Figure P25.65

66. The x axis is the symmetry axis of a uniformly charged ring of radius R and charge Q (Fig. P25.66). A point charge Q of mass M is located at the center of the ring. When it is displaced slightly, the point charge accelerates along the x axis to infinity. Show that the ultimate speed of the point charge is

$$v = \left(\frac{2k_eQ^2}{MR}\right)^{1/2}$$

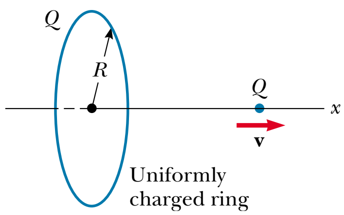


Figure P25.66

67. An infinite sheet of charge that has a surface charge density of 25.0 nC/m^2 lies in the yz plane, passes through the origin, and is at a potential of 1.00 kV at the point $y = 0$, $z = 0$. A long wire having a linear charge density of 80.0 nC/m lies parallel to the y axis and intersects the x axis at $x = 3.00$ m. (a) Determine, as a function of x, the potential along the x axis between wire and sheet. (b) What is the potential energy of a 2.00-nC charge placed at $x = 0.800$ m?

68. The thin, uniformly charged rod shown in Figure P25.68 has a linear charge density λ. Find an expression for the electric potential at P.

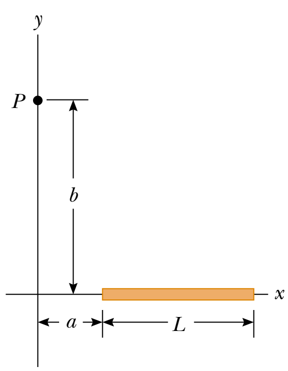


Figure P25.68

69. A dipole is located along the y axis as shown in Figure P25.69. (a) At a point P, which is far from the dipole ($r \gg a$), the electric potential is

$$V = k_e \frac{p\cos\theta}{r^2}$$

where $p = 2qa$. Calculate the radial component E_r and the perpendicular component E_θ of the associated electric field. Note that $E_\theta = -(1/r)(\partial V/\partial\theta)$. Do these results seem reasonable for $\theta = 90°$ and $0°$? for $r = 0$?

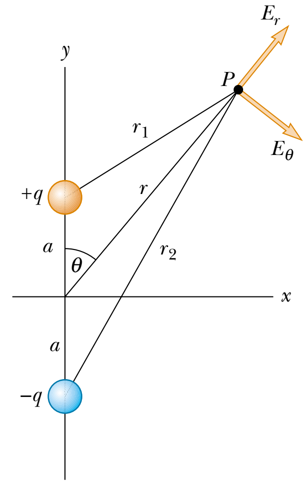


Figure P25.69

(b) For the dipole arrangement shown, express V in terms of cartesian coordinates using $r = (x^2 + y^2)^{1/2}$ and

$$\cos\theta = \frac{y}{(x^2 + y^2)^{1/2}}$$

Using these results and taking $r \gg a$, calculate the field components E_x and E_y.

70. Figure P25.70 shows several equipotential lines each labeled by its potential in volts. The distance between the lines of the square grid represents 1.00 cm. (a) Is the magnitude of the field bigger at A or at B? Why? (b) What is $\mathbf{E}$ at B? (c) Represent what the field looks like by drawing at least eight field lines.

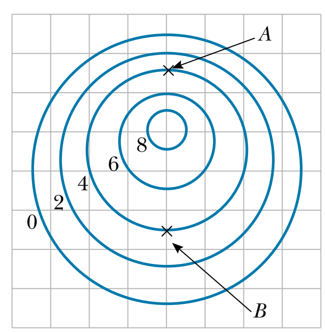


Figure P25.70

71. A disk of radius R has a nonuniform surface charge density $\sigma = Cr$, where C is a constant and r is measured from the center of the disk (Fig. P25.71). Find (by direct integration) the potential at P.

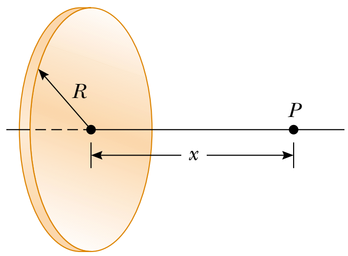


Figure P25.71

72. A solid sphere of radius R has a uniform charge density ρ and total charge Q. Derive an expression for its total electric potential energy. (*Hint:* Imagine that the sphere is constructed by adding successive layers of concentric shells of charge $dq = (4\pi r^2\,dr)\rho$ and use $dU = V\,dq$.)

73. The results of Problem 62 apply also to an electrostatic precipitator (see Figs. 25.28a and P25.62). An applied voltage $\Delta V = V_a - V_b = 50.0$ kV is to produce an electric field of magnitude 5.50 MV/m at the surface of the central wire. The outer cylindrical wall has uniform radius $r_a = 0.850$ m. (a) What should be the radius r_b of the central wire? You will need to solve a transcendental equation. (b) What is the magnitude of the electric field at the outer wall?

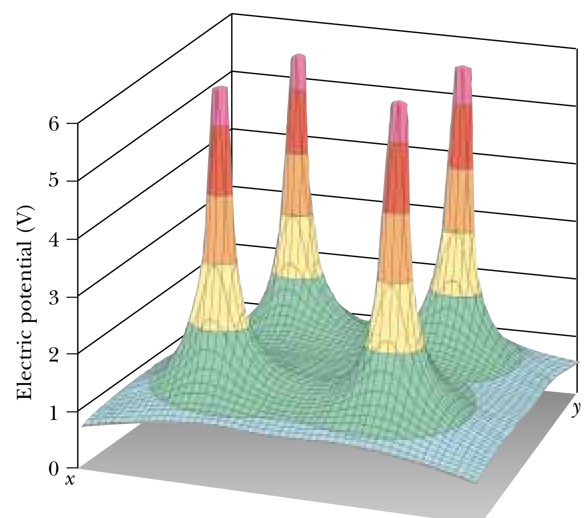
Answers to Quick Quizzes

25.1 We do if the electric field is uniform. (This is precisely what we do in the next section.) In general, however, an electric field changes from one place to another.

25.2 $B \rightarrow C$, $C \rightarrow D$, $A \rightarrow B$, $D \rightarrow E$. Moving from B to C decreases the electric potential by 2 V, so the electric field performs 2 J of work on each coulomb of charge that moves. Moving from C to D decreases the electric potential by 1 V, so 1 J of work is done by the field. It takes no work to move the charge from A to B because the electric potential does not change. Moving from D to E increases the electric potential by 1 V, and thus the field does -1 J of work, just as raising a mass to a higher elevation causes the gravitational field to do negative work on the mass.

25.3 The electric potential decreases in inverse proportion to the radius (see Eq. 25.11). The electric field magnitude decreases as the reciprocal of the radius squared (see Eq. 23.4). Because the surface area increases as r^2 while the electric field magnitude decreases as $1/r^2$, the electric flux through the surface remains constant (see Eq. 24.1).

25.4 (a) Yes. Consider four equal charges placed at the corners of a square. The electric potential graph for this situation is shown in the figure. At the center of the square, the electric field is zero because the individual fields from the four charges cancel, but the potential is not zero. This is also the situation inside a charged conductor. (b) Yes again. In Figure 25.8, for instance, the electric potential is zero at the center of the dipole, but the magnitude of the field at that point is not zero. (The two charges in a dipole are by definition of opposite sign; thus, the electric field lines created by the two charges extend from the positive to the negative charge and do not cancel anywhere.) This is the situation we presented in Example 25.4c, in which the equations we obtained give $V = 0$ and $E_x \neq 0$.



PUZZLER

Many electronic components carry a warning label like this one. What is there inside these devices that makes them so dangerous? Why wouldn't you be safe if you unplugged the equipment before opening the case? *(George Semple)*

chapter

26

Capacitance and Dielectrics

Chapter Outline

In this chapter, we discuss *capacitors*—devices that store electric charge. Capacitors are commonly used in a variety of electric circuits. For instance, they are used to tune the frequency of radio receivers, as filters in power supplies, to eliminate sparking in automobile ignition systems, and as energy-storing devices in electronic flash units.

A capacitor consists of two conductors separated by an insulator. We shall see that the capacitance of a given capacitor depends on its geometry and on the material—called a *dielectric*—that separates the conductors.

26.1 DEFINITION OF CAPACITANCE

13.5

Consider two conductors carrying charges of equal magnitude but of opposite sign, as shown in Figure 26.1. Such a combination of two conductors is called a **capacitor.** The conductors are called *plates.* A potential difference ΔV exists between the conductors due to the presence of the charges. Because the unit of potential difference is the volt, a potential difference is often called a **voltage.** We shall use this term to describe the potential difference across a circuit element or between two points in space.

What determines how much charge is on the plates of a capacitor for a given voltage? In other words, what is the *capacity* of the device for storing charge at a particular value of ΔV? Experiments show that the quantity of charge Q on a capacitor[1] is linearly proportional to the potential difference between the conductors; that is, $Q \propto \Delta V$. The proportionality constant depends on the shape and separation of the conductors.[2] We can write this relationship as $Q = C\Delta V$ if we define capacitance as follows:

Definition of capacitance

The **capacitance** C of a capacitor is the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them:

$$C \equiv \frac{Q}{\Delta V} \tag{26.1}$$

Note that by definition *capacitance is always a positive quantity.* Furthermore, the potential difference ΔV is always expressed in Equation 26.1 as a positive quantity. Because the potential difference increases linearly with the stored charge, the ratio $Q/\Delta V$ is constant for a given capacitor. Therefore, capacitance is a measure of a capacitor's ability to store charge and electric potential energy.

From Equation 26.1, we see that capacitance has SI units of coulombs per volt. The SI unit of capacitance is the **farad** (F), which was named in honor of Michael Faraday:

$$1 \text{ F} = 1 \text{ C/V}$$

The farad is a very large unit of capacitance. In practice, typical devices have capacitances ranging from microfarads (10^{-6} F) to picofarads (10^{-12} F). For practical purposes, capacitors often are labeled "mF" for microfarads and "mmF" for micromicrofarads or, equivalently, "pF" for picofarads.

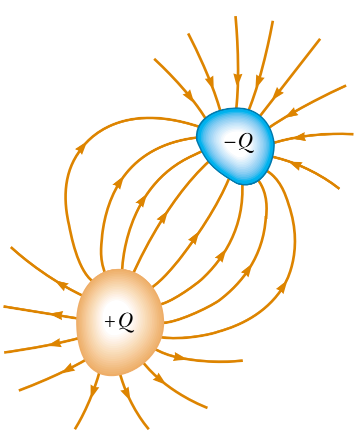


Figure 26.1 A capacitor consists of two conductors carrying charges of equal magnitude but opposite sign.

[1] Although the total charge on the capacitor is zero (because there is as much excess positive charge on one conductor as there is excess negative charge on the other), it is common practice to refer to the magnitude of the charge on either conductor as "the charge on the capacitor."

[2] The proportionality between ΔV and Q can be proved from Coulomb's law or by experiment.

A collection of capacitors used in a variety of applications.

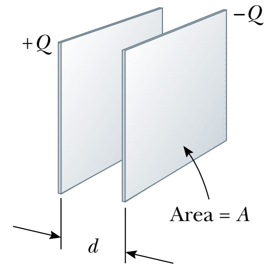


Figure 26.2 A parallel-plate capacitor consists of two parallel conducting plates, each of area A, separated by a distance d. When the capacitor is charged, the plates carry equal amounts of charge. One plate carries positive charge, and the other carries negative charge.

Let us consider a capacitor formed from a pair of parallel plates, as shown in Figure 26.2. Each plate is connected to one terminal of a battery (not shown in Fig. 26.2), which acts as a source of potential difference. If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires when the connections are made. Let us focus on the plate connected to the negative terminal of the battery. The electric field applies a force on electrons in the wire just outside this plate; this force causes the electrons to move onto the plate. This movement continues until the plate, the wire, and the terminal are all at the same electric potential. Once this equilibrium point is attained, a potential difference no longer exists between the terminal and the plate, and as a result no electric field is present in the wire, and the movement of electrons stops. The plate now carries a negative charge. A similar process occurs at the other capacitor plate, with electrons moving from the plate to the wire, leaving the plate positively charged. In this final configuration, the potential difference across the capacitor plates is the same as that between the terminals of the battery.

Suppose that we have a capacitor rated at 4 pF. This rating means that the capacitor can store 4 pC of charge for each volt of potential difference between the two conductors. If a 9-V battery is connected across this capacitor, one of the conductors ends up with a net charge of -36 pC and the other ends up with a net charge of $+36$ pC.

26.2 CALCULATING CAPACITANCE

We can calculate the capacitance of a pair of oppositely charged conductors in the following manner: We assume a charge of magnitude Q, and we calculate the potential difference using the techniques described in the preceding chapter. We then use the expression $C = Q/\Delta V$ to evaluate the capacitance. As we might expect, we can perform this calculation relatively easily if the geometry of the capacitor is simple.

We can calculate the capacitance of an isolated spherical conductor of radius R and charge Q if we assume that the second conductor making up the capacitor is a concentric hollow sphere of infinite radius. The electric potential of the sphere of radius R is simply k_eQ/R, and setting $V = 0$ at infinity as usual, we have

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_eQ/R} = \frac{R}{k_e} = 4\pi\epsilon_0 R \qquad \textbf{(26.2)}$$

This expression shows that the capacitance of an isolated charged sphere is proportional to its radius and is independent of both the charge on the sphere and the potential difference.

QuickLab

Roll some socks into balls and stuff them into a shoebox. What determines how many socks fit in the box? Relate how hard you push on the socks to ΔV for a capacitor. How does the size of the box influence its "sock capacity"?

The capacitance of a pair of conductors depends on the geometry of the conductors. Let us illustrate this with three familiar geometries, namely, parallel plates, concentric cylinders, and concentric spheres. In these examples, we assume that the charged conductors are separated by a vacuum. The effect of a dielectric material placed between the conductors is treated in Section 26.5.

Parallel-Plate Capacitors

Two parallel metallic plates of equal area A are separated by a distance d, as shown in Figure 26.2. One plate carries a charge Q, and the other carries a charge $-Q$. Let us consider how the geometry of these conductors influences the capacity of the combination to store charge. Recall that charges of like sign repel one another. As a capacitor is being charged by a battery, electrons flow into the negative plate and out of the positive plate. If the capacitor plates are large, the accumulated charges are able to distribute themselves over a substantial area, and the amount of charge that can be stored on a plate for a given potential difference increases as the plate area is increased. Thus, we expect the capacitance to be proportional to the plate area A.

Now let us consider the region that separates the plates. If the battery has a constant potential difference between its terminals, then the electric field between the plates must increase as d is decreased. Let us imagine that we move the plates closer together and consider the situation before any charges have had a chance to move in response to this change. Because no charges have moved, the electric field between the plates has the same value but extends over a shorter distance. Thus, the magnitude of the potential difference between the plates $\Delta V = Ed$ (Eq. 25.6) is now smaller. The difference between this new capacitor voltage and the terminal voltage of the battery now exists as a potential difference across the wires connecting the battery to the capacitor. This potential difference results in an electric field in the wires that drives more charge onto the plates, increasing the potential difference between the plates. When the potential difference between the plates again matches that of the battery, the potential difference across the wires falls back to zero, and the flow of charge stops. Thus, moving the plates closer together causes the charge on the capacitor to increase. If d is increased, the charge decreases. As a result, we expect the device's capacitance to be inversely proportional to d.

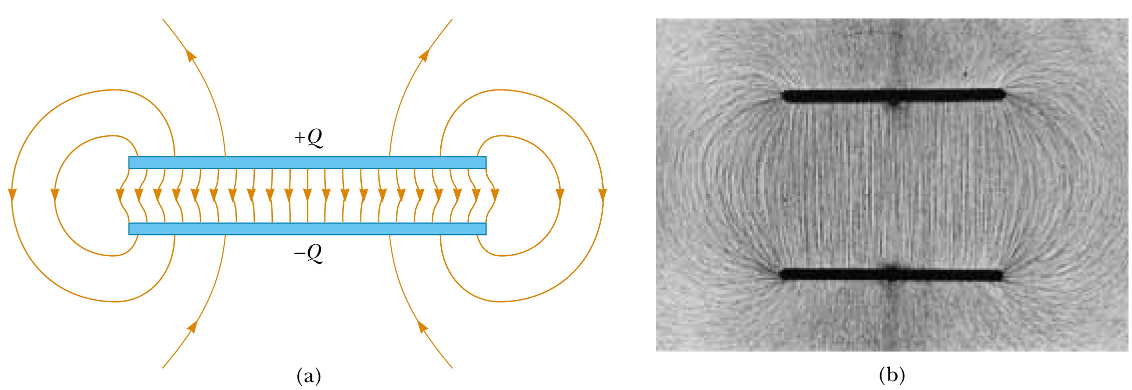


Figure 26.3 (a) The electric field between the plates of a parallel-plate capacitor is uniform near the center but nonuniform near the edges. (b) Electric field pattern of two oppositely charged conducting parallel plates. Small pieces of thread on an oil surface align with the electric field.

We can verify these physical arguments with the following derivation. The surface charge density on either plate is $\sigma = Q/A$. If the plates are very close together (in comparison with their length and width), we can assume that the electric field is uniform between the plates and is zero elsewhere. According to the last paragraph of Example 24.8, the value of the electric field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals Ed (see Eq. 25.6); therefore,

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

Substituting this result into Equation 26.1, we find that the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d} \quad \textbf{(26.3)}$$

That is, **the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation,** just as we expect from our conceptual argument.

A careful inspection of the electric field lines for a parallel-plate capacitor reveals that the field is uniform in the central region between the plates, as shown in Figure 26.3a. However, the field is nonuniform at the edges of the plates. Figure 26.3b is a photograph of the electric field pattern of a parallel-plate capacitor. Note the nonuniform nature of the electric field at the ends of the plates. Such end effects can be neglected if the plate separation is small compared with the length of the plates.

Quick Quiz 26.1

Many computer keyboard buttons are constructed of capacitors, as shown in Figure 26.4. When a key is pushed down, the soft insulator between the movable plate and the fixed plate is compressed. When the key is pressed, the capacitance (a) increases, (b) decreases, or (c) changes in a way that we cannot determine because the complicated electric circuit connected to the keyboard button may cause a change in ΔV.

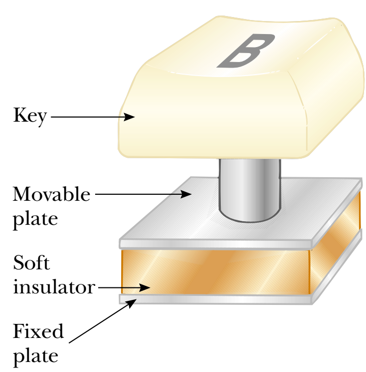


Figure 26.4 One type of computer keyboard button.

EXAMPLE 26.1 Parallel-Plate Capacitor

A parallel-plate capacitor has an area $A = 2.00 \times 10^{-4}\ \text{m}^2$ and a plate separation $d = 1.00$ mm. Find its capacitance.

Solution From Equation 26.3, we find that

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\left(\frac{2.00 \times 10^{-4}\ \text{m}^2}{1.00 \times 10^{-3}\ \text{m}}\right)$$

$$= 1.77 \times 10^{-12}\ \text{F} = \boxed{1.77\ \text{pF}}$$

Exercise What is the capacitance for a plate separation of 3.00 mm?

Answer 0.590 pF.

Cylindrical and Spherical Capacitors

From the definition of capacitance, we can, in principle, find the capacitance of any geometric arrangement of conductors. The following examples demonstrate the use of this definition to calculate the capacitance of the other familiar geometries that we mentioned: cylinders and spheres.

EXAMPLE 26.2 The Cylindrical Capacitor

A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of negligible thickness, radius $b > a$, and charge $-Q$ (Fig. 26.5a). Find the capacitance of this cylindrical capacitor if its length is ℓ.

Solution It is difficult to apply physical arguments to this configuration, although we can reasonably expect the capacitance to be proportional to the cylinder length ℓ for the same reason that parallel-plate capacitance is proportional to plate area: Stored charges have more room in which to be distributed. If we assume that ℓ is much greater than a and b, we can neglect end effects. In this case, the electric field is perpendicular to the long axis of the cylinders and is confined to the region between them (Fig. 26.5b). We must first calculate the potential difference between the two cylinders, which is given in general by

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s}$$

where $\mathbf{E}$ is the electric field in the region $a < r < b$. In Chapter 24, we showed using Gauss's law that the magnitude of the electric field of a cylindrical charge distribution having linear charge density λ is $E_r = 2k_e\lambda/r$ (Eq. 24.7). The same result applies here because, according to Gauss's law, the charge on the outer cylinder does not contribute to the electric field inside it. Using this result and noting from Figure 26.5b that $\mathbf{E}$ is along r, we find that

$$V_b - V_a = -\int_a^b E_r\,dr = -2k_e\lambda \int_a^b \frac{dr}{r} = -2k_e\lambda \ln\left(\frac{b}{a}\right)$$

Substituting this result into Equation 26.1 and using the fact that $\lambda = Q/\ell$, we obtain

$$C = \frac{Q}{\Delta V} = \frac{Q}{\dfrac{2k_eQ}{\ell}\ln\left(\dfrac{b}{a}\right)} = \boxed{\frac{\ell}{2k_e\ln\left(\dfrac{b}{a}\right)}} \quad \textbf{(26.4)}$$

where ΔV is the magnitude of the potential difference, given by $\Delta V = |V_b - V_a| = 2k_e\lambda \ln\,(b/a)$, a positive quantity. As predicted, the capacitance is proportional to the length of the cylinders. As we might expect, the capacitance also depends on the radii of the two cylindrical conductors. From Equation 26.4, we see that the capacitance per unit length of a combination of concentric cylindrical conductors is

$$\frac{C}{\ell} = \frac{1}{2k_e\ln\left(\dfrac{b}{a}\right)} \quad \textbf{(26.5)}$$

An example of this type of geometric arrangement is a *coaxial cable,* which consists of two concentric cylindrical conductors separated by an insulator. The cable carries electrical signals in the inner and outer conductors. Such a geometry is especially useful for shielding the signals from any possible external influences.

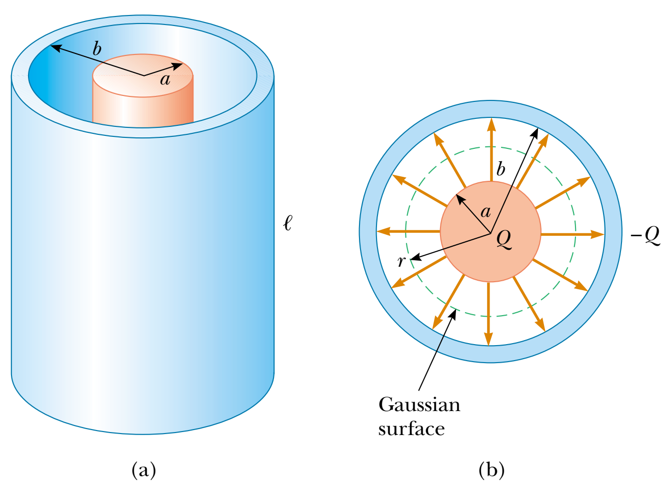


Figure 26.5 (a) A cylindrical capacitor consists of a solid cylindrical conductor of radius a and length ℓ surrounded by a coaxial cylindrical shell of radius b. (b) End view. The dashed line represents the end of the cylindrical gaussian surface of radius r and length ℓ.

EXAMPLE 26.3 The Spherical Capacitor

A spherical capacitor consists of a spherical conducting shell of radius b and charge $-Q$ concentric with a smaller conducting sphere of radius a and charge Q (Fig. 26.6). Find the capacitance of this device.

Solution As we showed in Chapter 24, the field outside a spherically symmetric charge distribution is radial and given by the expression k_eQ/r^2. In this case, this result applies to the field between the spheres ($a < r < b$). From

Gauss's law we see that only the inner sphere contributes to this field. Thus, the potential difference between the spheres is

$$V_b - V_a = -\int_a^b E_r\,dr = -k_eQ\int_a^b \frac{dr}{r^2} = k_eQ\left[\frac{1}{r}\right]_a^b$$
$$= k_eQ\left(\frac{1}{b} - \frac{1}{a}\right)$$

The magnitude of the potential difference is

$$\Delta V = |V_b - V_a| = k_eQ\frac{(b-a)}{ab}$$

Substituting this value for ΔV into Equation 26.1, we obtain

$$C = \frac{Q}{\Delta V} = \boxed{\frac{ab}{k_e(b-a)}} \qquad \textbf{(26.6)}$$

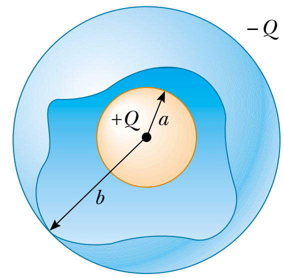


Figure 26.6 A spherical capacitor consists of an inner sphere of radius a surrounded by a concentric spherical shell of radius b. The electric field between the spheres is directed radially outward when the inner sphere is positively charged.

Exercise Show that as the radius b of the outer sphere approaches infinity, the capacitance approaches the value $a/k_e = 4\pi\epsilon_0 a$.

Quick Quiz 26.2

What is the magnitude of the electric field in the region outside the spherical capacitor described in Example 26.3?

26.3 COMBINATIONS OF CAPACITORS

13.5 Two or more capacitors often are combined in electric circuits. We can calculate the equivalent capacitance of certain combinations using methods described in this section. The circuit symbols for capacitors and batteries, as well as the color codes used for them in this text, are given in Figure 26.7. The symbol for the capacitor reflects the geometry of the most common model for a capacitor—a pair of parallel plates. The positive terminal of the battery is at the higher potential and is represented in the circuit symbol by the longer vertical line.

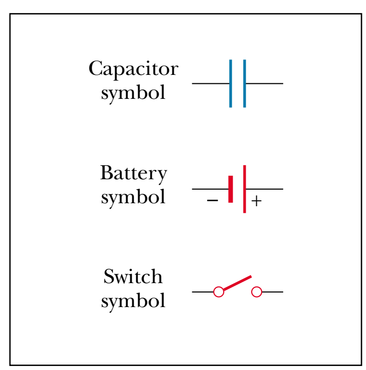


Figure 26.7 Circuit symbols for capacitors, batteries, and switches. Note that capacitors are in blue and batteries and switches are in red.

Parallel Combination

Two capacitors connected as shown in Figure 26.8a are known as a *parallel combination* of capacitors. Figure 26.8b shows a circuit diagram for this combination of capacitors. The left plates of the capacitors are connected by a conducting wire to the positive terminal of the battery and are therefore both at the same electric potential as the positive terminal. Likewise, the right plates are connected to the negative terminal and are therefore both at the same potential as the negative terminal. Thus, **the individual potential differences across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.**

In a circuit such as that shown in Figure 26.8, the voltage applied across the combination is the terminal voltage of the battery. Situations can occur in which

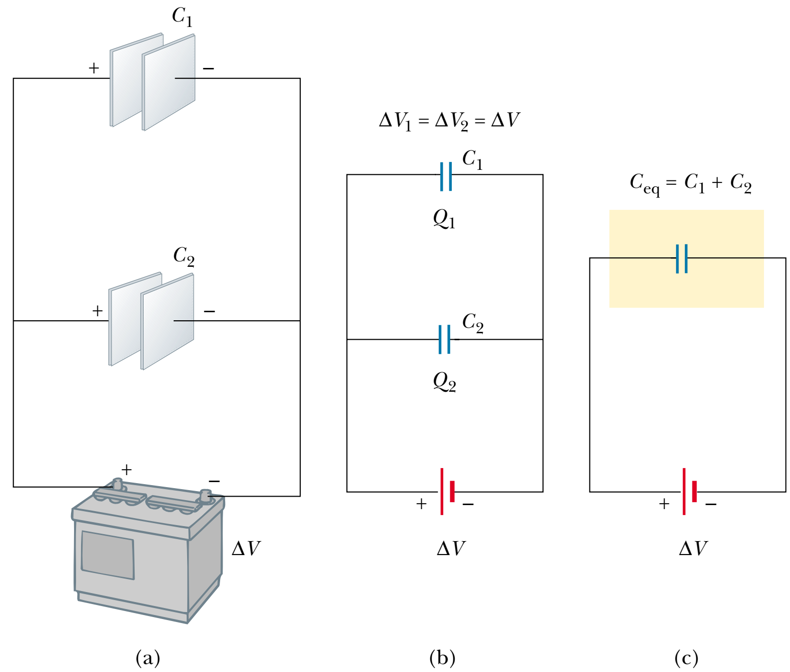


Figure 26.8 (a) A parallel combination of two capacitors in an electric circuit in which the potential difference across the battery terminals is ΔV. (b) The circuit diagram for the parallel combination. (c) The equivalent capacitance is $C_{eq} = C_1 + C_2$.

the parallel combination is in a circuit with other circuit elements; in such situations, we must determine the potential difference across the combination by analyzing the entire circuit.

When the capacitors are first connected in the circuit shown in Figure 26.8, electrons are transferred between the wires and the plates; this transfer leaves the left plates positively charged and the right plates negatively charged. The energy source for this charge transfer is the internal chemical energy stored in the battery, which is converted to electric potential energy associated with the charge separation. The flow of charge ceases when the voltage across the capacitors is equal to that across the battery terminals. The capacitors reach their maximum charge when the flow of charge ceases. Let us call the maximum charges on the two capacitors Q_1 and Q_2. The *total charge* Q stored by the two capacitors is

$$Q = Q_1 + Q_2 \qquad \textbf{(26.7)}$$

That is, **the total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors.** Because the voltages across the capacitors are the same, the charges that they carry are

$$Q_1 = C_1\,\Delta V \qquad Q_2 = C_2\,\Delta V$$

Suppose that we wish to replace these two capacitors by one *equivalent capacitor* having a capacitance C_{eq}, as shown in Figure 26.8c. The effect this equivalent capacitor has on the circuit must be exactly the same as the effect of the combination of the two individual capacitors. That is, the equivalent capacitor must store Q units of charge when connected to the battery. We can see from Figure 26.8c that the voltage across the equivalent capacitor also is ΔV because the equivalent capacitor is connected directly across the battery terminals. Thus, for the equivalent capacitor,

$$Q = C_{eq}\,\Delta V$$

Substituting these three relationships for charge into Equation 26.7, we have

$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \begin{pmatrix}\text{parallel} \\ \text{combination}\end{pmatrix}$$

If we extend this treatment to three or more capacitors connected in parallel, we find the equivalent capacitance to be

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)} \tag{26.8}$$

Thus, **the equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitances.** This makes sense because we are essentially combining the areas of all the capacitor plates when we connect them with conducting wire.

Series Combination

Two capacitors connected as shown in Figure 26.9a are known as a *series combination* of capacitors. The left plate of capacitor 1 and the right plate of capacitor 2 are connected to the terminals of a battery. The other two plates are connected to each other and to nothing else; hence, they form an isolated conductor that is initially uncharged and must continue to have zero net charge. To analyze this combination, let us begin by considering the uncharged capacitors and follow what happens just after a battery is connected to the circuit. When the battery is con-

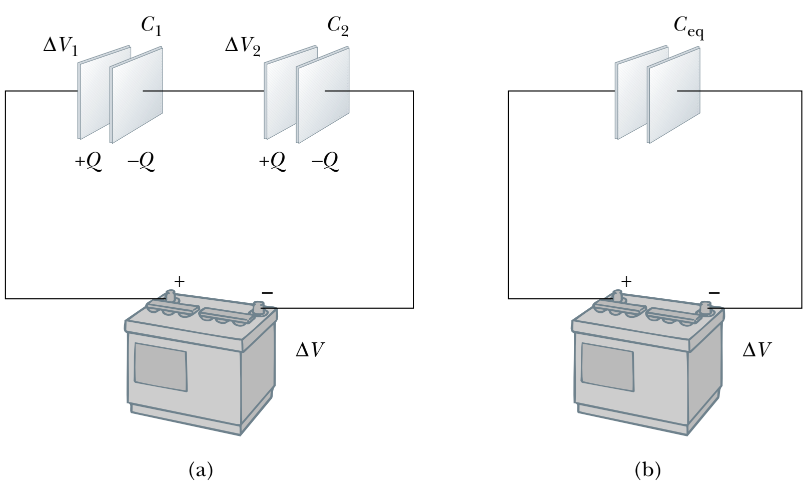


Figure 26.9 (a) A series combination of two capacitors. The charges on the two capacitors are the same. (b) The capacitors replaced by a single equivalent capacitor. The equivalent capacitance can be calculated from the relationship

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

nected, electrons are transferred out of the left plate of C_1 and into the right plate of C_2. As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is forced off the left plate of C_2, and this left plate therefore has an excess positive charge. The negative charge leaving the left plate of C_2 travels through the connecting wire and accumulates on the right plate of C_1. As a result, all the right plates end up with a charge $-Q$, and all the left plates end up with a charge $+Q$. Thus, **the charges on capacitors connected in series are the same.**

From Figure 26.9a, we see that the voltage ΔV across the battery terminals is split between the two capacitors:

$$\Delta V = \Delta V_1 + \Delta V_2 \qquad \textbf{(26.9)}$$

where ΔV_1 and ΔV_2 are the potential differences across capacitors C_1 and C_2, respectively. In general, **the total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.**

Suppose that an equivalent capacitor has the same effect on the circuit as the series combination. After it is fully charged, the equivalent capacitor must have a charge of $-Q$ on its right plate and a charge of $+Q$ on its left plate. Applying the definition of capacitance to the circuit in Figure 26.9b, we have

$$\Delta V = \frac{Q}{C_{\text{eq}}}$$

Because we can apply the expression $Q = C\,\Delta V$ to each capacitor shown in Figure 26.9a, the potential difference across each is

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

Substituting these expressions into Equation 26.9 and noting that $\Delta V = Q/C_{\text{eq}}$, we have

$$\frac{Q}{C_{\text{eq}}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Canceling Q, we arrive at the relationship

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \left(\begin{array}{l}\text{series}\\ \text{combination}\end{array}\right)$$

When this analysis is applied to three or more capacitors connected in series, the relationship for the equivalent capacitance is

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \left(\begin{array}{l}\text{series}\\ \text{combination}\end{array}\right) \qquad \textbf{(26.10)}$$

This demonstrates that **the equivalent capacitance of a series combination is always less than any individual capacitance in the combination.**

EXAMPLE 26.4 Equivalent Capacitance

Find the equivalent capacitance between a and b for the combination of capacitors shown in Figure 26.10a. All capacitances are in microfarads.

Solution Using Equations 26.8 and 26.10, we reduce the combination step by step as indicated in the figure. The 1.0-μF and 3.0-μF capacitors are in parallel and combine according to the expression $C_{eq} = C_1 + C_2 = 4.0\ \mu F$. The $2.0\text{-}\mu F$ and $6.0\text{-}\mu F$ capacitors also are in parallel and have an equivalent capacitance of $8.0\ \mu F$. Thus, the upper branch in Figure 26.10b consists of two $4.0\text{-}\mu F$ capacitors in series, which combine as follows:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu F} + \frac{1}{4.0\ \mu F} = \frac{1}{2.0\ \mu F}$$

$$C_{eq} = \frac{1}{1/2.0\ \mu F} = 2.0\ \mu F$$

The lower branch in Figure 26.10b consists of two $8.0\text{-}\mu F$ capacitors in series, which combine to yield an equivalent capacitance of $4.0\ \mu F$. Finally, the $2.0\text{-}\mu F$ and $4.0\text{-}\mu F$ capacitors in Figure 26.10c are in parallel and thus have an equivalent capacitance of $6.0\ \mu F$.

Exercise Consider three capacitors having capacitances of $3.0\ \mu F$, $6.0\ \mu F$, and $12\ \mu F$. Find their equivalent capacitance when they are connected (a) in parallel and (b) in series.

Answer (a) $21\ \mu F$; (b) $1.7\ \mu F$.

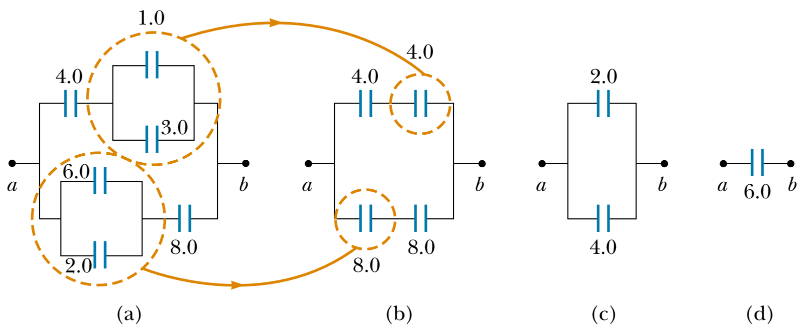


Figure 26.10 To find the equivalent capacitance of the capacitors in part (a), we reduce the various combinations in steps as indicated in parts (b), (c), and (d), using the series and parallel rules described in the text.

26.4 ENERGY STORED IN A CHARGED CAPACITOR

13.5

Almost everyone who works with electronic equipment has at some time verified that a capacitor can store energy. If the plates of a charged capacitor are connected by a conductor, such as a wire, charge moves between the plates and the connecting wire until the capacitor is uncharged. The discharge can often be observed as a visible spark. If you should accidentally touch the opposite plates of a charged capacitor, your fingers act as a pathway for discharge, and the result is an electric shock. The degree of shock you receive depends on the capacitance and on the voltage applied to the capacitor. Such a shock could be fatal if high voltages are present, such as in the power supply of a television set. Because the charges can be stored in a capacitor even when the set is turned off, unplugging the television does not make it safe to open the case and touch the components inside.

Consider a parallel-plate capacitor that is initially uncharged, such that the initial potential difference across the plates is zero. Now imagine that the capacitor is connected to a battery and develops a maximum charge Q. (We assume that the capacitor is charged slowly so that the problem can be considered as an electrostatic system.) When the capacitor is connected to the battery, electrons in the wire just outside the plate connected to the negative terminal move into the plate to give it a negative charge. Electrons in the plate connected to the positive terminal move out of the plate into the wire to give the plate a positive charge. Thus, charges move only a small distance in the wires.

To calculate the energy of the capacitor, we shall assume a different process—one that does not actually occur but gives the same final result. We can make this

QuickLab

Here's how to find out whether your calculator has a capacitor to protect values or programs during battery changes: Store a number in your calculator's memory, remove the calculator battery for a moment, and then quickly replace it. Was the number that you stored preserved while the battery was out of the calculator? (You may want to write down any critical numbers or programs that are stored in the calculator before trying this!)

assumption because the energy in the final configuration does not depend on the actual charge-transfer process. We imagine that we reach in and grab a small amount of positive charge on the plate connected to the negative terminal and apply a force that causes this positive charge to move over to the plate connected to the positive terminal. Thus, we do work on the charge as we transfer it from one plate to the other. At first, no work is required to transfer a small amount of charge dq from one plate to the other.[3] However, once this charge has been transferred, a small potential difference exists between the plates. Therefore, work must be done to move additional charge through this potential difference. As more and more charge is transferred from one plate to the other, the potential difference increases in proportion, and more work is required.

Suppose that q is the charge on the capacitor at some instant during the charging process. At the same instant, the potential difference across the capacitor is $\Delta V = q/C$. From Section 25.2, we know that the work necessary to transfer an increment of charge dq from the plate carrying charge $-q$ to the plate carrying charge q (which is at the higher electric potential) is

$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

This is illustrated in Figure 26.11. The total work required to charge the capacitor from $q = 0$ to some final charge $q = Q$ is

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

The work done in charging the capacitor appears as electric potential energy U stored in the capacitor. Therefore, we can express the potential energy stored in a charged capacitor in the following forms:

Energy stored in a charged capacitor

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2 \qquad \textbf{(26.11)}$$

This result applies to any capacitor, regardless of its geometry. We see that for a given capacitance, the stored energy increases as the charge increases and as the potential difference increases. In practice, there is a limit to the maximum energy

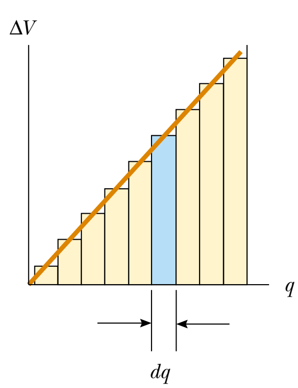


Figure 26.11 A plot of potential difference versus charge for a capacitor is a straight line having a slope $1/C$. The work required to move charge dq through the potential difference ΔV across the capacitor plates is given by the area of the shaded rectangle. The total work required to charge the capacitor to a final charge Q is the triangular area under the straight line, $W = \frac{1}{2}Q\,\Delta V$. (Don't forget that 1 V = 1 J/C; hence, the unit for the area is the joule.)

[3] We shall use lowercase q for the varying charge on the capacitor while it is charging, to distinguish it from uppercase Q, which is the total charge on the capacitor after it is completely charged.

(or charge) that can be stored because, at a sufficiently great value of ΔV, discharge ultimately occurs between the plates. For this reason, capacitors are usually labeled with a maximum operating voltage.

Quick Quiz 26.3

You have three capacitors and a battery. How should you combine the capacitors and the battery in one circuit so that the capacitors will store the maximum possible energy?

We can consider the energy stored in a capacitor as being stored in the electric field created between the plates as the capacitor is charged. This description is reasonable in view of the fact that the electric field is proportional to the charge on the capacitor. For a parallel-plate capacitor, the potential difference is related to the electric field through the relationship $\Delta V = Ed$. Furthermore, its capacitance is $C = \epsilon_0 A/d$ (Eq. 26.3). Substituting these expressions into Equation 26.11, we obtain

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \frac{1}{2}(\epsilon_0 A d)E^2 \tag{26.12}$$

Energy stored in a parallel-plate capacitor

Because the volume V (volume, not voltage!) occupied by the electric field is Ad, the *energy per unit volume* $u_E = U/V = U/Ad$, known as the *energy density*, is

$$u_E = \tfrac{1}{2}\epsilon_0 E^2 \tag{26.13}$$

Energy density in an electric field

Although Equation 26.13 was derived for a parallel-plate capacitor, the expression is generally valid. That is, the **energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.**

This bank of capacitors stores electrical energy for use in the particle accelerator at FermiLab, located outside Chicago. Because the electric utility company cannot provide a large enough burst of energy to operate the equipment, these capacitors are slowly charged up, and then the energy is rapidly "dumped" into the accelerator. In this sense, the setup is much like a fire-protection water tank on top of a building. The tank collects water and stores it for situations in which a lot of water is needed in a short time.

EXAMPLE 26.5 Rewiring Two Charged Capacitors

Two capacitors C_1 and C_2 (where $C_1 > C_2$) are charged to the same initial potential difference ΔV_i, but with opposite polarity. The charged capacitors are removed from the battery, and their plates are connected as shown in Figure 26.12a. The switches S_1 and S_2 are then closed, as shown in Figure 26.12b. (a) Find the final potential difference ΔV_f between a and b after the switches are closed.

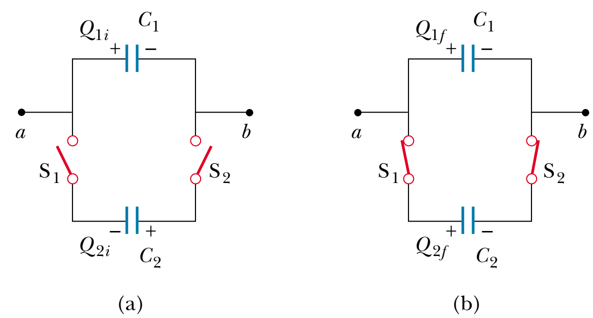


Figure 26.12

Solution Let us identify the left-hand plates of the capacitors as an isolated system because they are not connected to the right-hand plates by conductors. The charges on the left-hand plates before the switches are closed are

$$Q_{1i} = C_1 \Delta V_i \qquad \text{and} \qquad Q_{2i} = -C_2 \Delta V_i$$

The negative sign for Q_{2i} is necessary because the charge on the left plate of capacitor C_2 is negative. The total charge Q in the system is

$$(1) \qquad Q = Q_{1i} + Q_{2i} = (C_1 - C_2)\Delta V_i$$

After the switches are closed, the total charge in the system remains the same:

$$(2) \qquad Q = Q_{1f} + Q_{2f}$$

The charges redistribute until the entire system is at the same potential ΔV_f. Thus, the final potential difference across C_1 must be the same as the final potential difference across C_2. To satisfy this requirement, the charges on the capacitors after the switches are closed are

$$Q_{1f} = C_1 \Delta V_f \qquad \text{and} \qquad Q_{2f} = C_2 \Delta V_f$$

Dividing the first equation by the second, we have

$$\frac{Q_{1f}}{Q_{2f}} = \frac{C_1 \Delta V_f}{C_2 \Delta V_f} = \frac{C_1}{C_2}$$

$$(3) \qquad Q_{1f} = \frac{C_1}{C_2} Q_{2f}$$

Combining Equations (2) and (3), we obtain

$$Q = Q_{1f} + Q_{2f} = \frac{C_1}{C_2} Q_{2f} + Q_{2f} = Q_{2f}\left(1 + \frac{C_1}{C_2}\right)$$

$$Q_{2f} = Q\left(\frac{C_2}{C_1 + C_2}\right)$$

Using Equation (3) to find Q_{1f} in terms of Q, we have

$$Q_{1f} = \frac{C_1}{C_2} Q_{2f} = \frac{C_1}{C_2} Q\left(\frac{C_2}{C_1 + C_2}\right) = Q\left(\frac{C_1}{C_1 + C_2}\right)$$

Finally, using Equation 26.1 to find the voltage across each capacitor, we find that

$$\Delta V_{1f} = \frac{Q_{1f}}{C_1} = \frac{Q\left(\frac{C_1}{C_1 + C_2}\right)}{C_1} = \frac{Q}{C_1 + C_2}$$

$$\Delta V_{2f} = \frac{Q_{2f}}{C_2} = \frac{Q\left(\frac{C_2}{C_1 + C_2}\right)}{C_2} = \frac{Q}{C_1 + C_2}$$

As noted earlier, $\Delta V_{1f} = \Delta V_{2f} = \Delta V_f$.

To express ΔV_f in terms of the given quantities C_1, C_2, and ΔV_i, we substitute the value of Q from Equation (1) to obtain

$$\boxed{\Delta V_f = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i}$$

(b) Find the total energy stored in the capacitors before and after the switches are closed and the ratio of the final energy to the initial energy.

Solution Before the switches are closed, the total energy stored in the capacitors is

$$U_i = \tfrac{1}{2}C_1(\Delta V_i)^2 + \tfrac{1}{2}C_2(\Delta V_i)^2 = \boxed{\tfrac{1}{2}(C_1 + C_2)(\Delta V_i)^2}$$

After the switches are closed, the total energy stored in the capacitors is

$$U_f = \tfrac{1}{2}C_1(\Delta V_f)^2 + \tfrac{1}{2}C_2(\Delta V_f)^2 = \tfrac{1}{2}(C_1 + C_2)(\Delta V_f)^2$$

$$= \frac{1}{2}(C_1 + C_2)\left(\frac{Q}{C_1 + C_2}\right)^2 = \frac{1}{2}\frac{Q^2}{C_1 + C_2}$$

Using Equation (1), we can express this as

$$U_f = \frac{1}{2}\frac{Q^2}{(C_1 + C_2)} = \boxed{\frac{1}{2}\frac{(C_1 - C_2)^2(\Delta V_i)^2}{(C_1 + C_2)}}$$

Therefore, the ratio of the final energy stored to the initial energy stored is

$$\frac{U_f}{U_i} = \frac{\frac{1}{2}\frac{(C_1 - C_2)^2(\Delta V_i)^2}{(C_1 + C_2)}}{\frac{1}{2}(C_1 + C_2)(\Delta V_i)^2} = \boxed{\left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2}$$

This ratio is less than unity, indicating that the final energy is less than the initial energy. At first, you might think that the law of energy conservation has been violated, but this is not the case. The “missing” energy is radiated away in the form of electromagnetic waves, as we shall see in Chapter 34.

Quick Quiz 26.4

You charge a parallel-plate capacitor, remove it from the battery, and prevent the wires connected to the plates from touching each other. When you pull the plates apart, do the following quantities increase, decrease, or stay the same? (a) C; (b) Q; (c) E between the plates; (d) ΔV; (e) energy stored in the capacitor.

Quick Quiz 26.5

Repeat Quick Quiz 26.4, but this time answer the questions for the situation in which the battery remains connected to the capacitor while you pull the plates apart.

One device in which capacitors have an important role is the *defibrillator* (Fig. 26.13). Up to 360 J is stored in the electric field of a large capacitor in a defibrillator when it is fully charged. The defibrillator can deliver all this energy to a patient in about 2 ms. (This is roughly equivalent to 3 000 times the power output of a 60-W lightbulb!) The sudden electric shock stops the fibrillation (random contractions) of the heart that often accompanies heart attacks and helps to restore the correct rhythm.

web

To learn more about defibrillators, visit **www.physiocontrol.com**

A camera’s flash unit also uses a capacitor, although the total amount of energy stored is much less than that stored in a defibrillator. After the flash unit’s capacitor is charged, tripping the camera’s shutter causes the stored energy to be sent through a special lightbulb that briefly illuminates the subject being photographed.

Figure 26.13 In a hospital or at an emergency scene, you might see a patient being revived with a defibrillator. The defibrillator’s paddles are applied to the patient’s chest, and an electric shock is sent through the chest cavity. The aim of this technique is to restore the heart’s normal rhythm pattern.

26.5 CAPACITORS WITH DIELECTRICS

A **dielectric** is a nonconducting material, such as rubber, glass, or waxed paper. When a dielectric is inserted between the plates of a capacitor, the capacitance increases. If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor κ, which is called the **dielectric constant.** The dielectric constant is a property of a material and varies from one material to another. In this section, we analyze this change in capacitance in terms of electrical parameters such as electric charge, electric field, and potential difference; in Section 26.7, we shall discuss the microscopic origin of these changes.

We can perform the following experiment to illustrate the effect of a dielectric in a capacitor: Consider a parallel-plate capacitor that without a dielectric has a charge Q_0 and a capacitance C_0. The potential difference across the capacitor is $\Delta V_0 = Q_0/C_0$. Figure 26.14a illustrates this situation. The potential difference is measured by a *voltmeter,* which we shall study in greater detail in Chapter 28. Note that no battery is shown in the figure; also, we must assume that no charge can flow through an ideal voltmeter, as we shall learn in Section 28.5. Hence, there is no path by which charge can flow and alter the charge on the capacitor. If a dielectric is now inserted between the plates, as shown in Figure 26.14b, the voltmeter indicates that the voltage between the plates decreases to a value ΔV. The voltages with and without the dielectric are related by the factor κ as follows:

$$\Delta V = \frac{\Delta V_0}{\kappa}$$

Because $\Delta V < \Delta V_0$, we see that $\kappa > 1$.

Because the charge Q_0 on the capacitor does not change, we conclude that the capacitance must change to the value

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

The capacitance of a filled capacitor is greater than that of an empty one by a factor κ.

$$C = \kappa C_0 \tag{26.14}$$

That is, the capacitance *increases* by the factor κ when the dielectric completely fills the region between the plates.[4] For a parallel-plate capacitor, where $C_0 = \epsilon_0 A/d$ (Eq. 26.3), we can express the capacitance when the capacitor is filled with a dielectric as

$$C = \kappa \frac{\epsilon_0 A}{d} \tag{26.15}$$

From Equations 26.3 and 26.15, it would appear that we could make the capacitance very large by decreasing d, the distance between the plates. In practice, the lowest value of d is limited by the electric discharge that could occur through the dielectric medium separating the plates. For any given separation d, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** (maximum electric field) of the dielectric. If the magnitude of the electric field in the dielectric exceeds the dielectric strength, then the insulating properties break down and the dielectric begins to conduct. Insulating materials have values of κ greater than unity and dielectric strengths

[4] If the dielectric is introduced while the potential difference is being maintained constant by a battery, the charge increases to a value $Q = \kappa Q_0$. The additional charge is supplied by the battery, and the capacitance again increases by the factor κ.

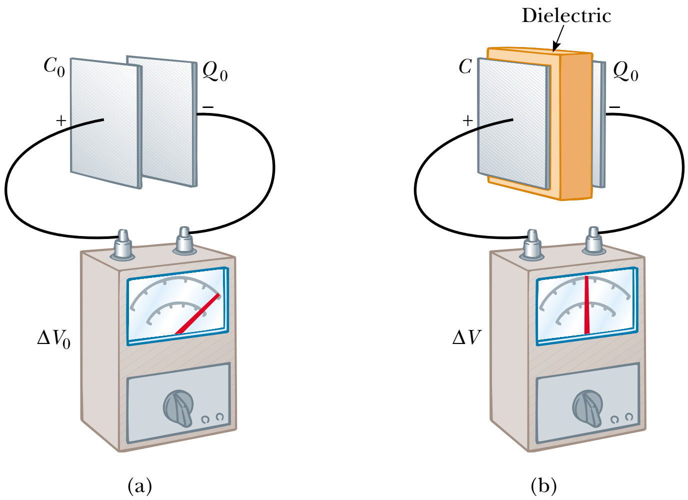


Figure 26.14 A charged capacitor (a) before and (b) after insertion of a dielectric between the plates. The charge on the plates remains unchanged, but the potential difference decreases from ΔV_0 to $\Delta V = \Delta V_0/\kappa$. Thus, the capacitance *increases* from C_0 to κC_0.

greater than that of air, as Table 26.1 indicates. Thus, we see that a dielectric provides the following advantages:

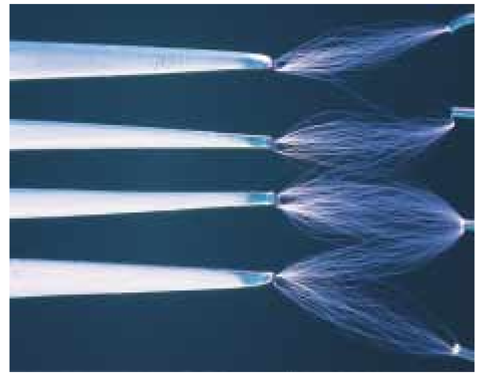
- Increase in capacitance
- Increase in maximum operating voltage
- Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C

TABLE 26.1 Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (V/m)
Air (dry)	1.000 59	3×10^6
Bakelite	4.9	24×10^6
Fused quartz	3.78	8×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Polystyrene	2.56	24×10^6
Polyvinyl chloride	3.4	40×10^6
Porcelain	6	12×10^6
Pyrex glass	5.6	14×10^6
Silicone oil	2.5	15×10^6
Strontium titanate	233	8×10^6
Teflon	2.1	60×10^6
Vacuum	1.000 00	—
Water	80	—

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.

(a)



(b)

(a) Kirlian photograph created by dropping a steel ball into a high-energy electric field. Kirlian photography is also known as *electrophotography.* (b) Sparks from static electricity discharge between a fork and four electrodes. Many sparks were used to create this image because only one spark forms for a given discharge. Note that the bottom prong discharges to both electrodes at the bottom right. The light of each spark is created by the excitation of gas atoms along its path.

Types of Capacitors

Commercial capacitors are often made from metallic foil interlaced with thin sheets of either paraffin-impregnated paper or Mylar as the dielectric material. These alternate layers of metallic foil and dielectric are rolled into a cylinder to form a small package (Fig. 26.15a). High-voltage capacitors commonly consist of a number of interwoven metallic plates immersed in silicone oil (Fig. 26.15b). Small capacitors are often constructed from ceramic materials. Variable capacitors (typically 10 to 500 pF) usually consist of two interwoven sets of metallic plates, one fixed and the other movable, and contain air as the dielectric.

Often, an *electrolytic capacitor* is used to store large amounts of charge at relatively low voltages. This device, shown in Figure 26.15c, consists of a metallic foil in contact with an *electrolyte*—a solution that conducts electricity by virtue of the motion of ions contained in the solution. When a voltage is applied between the foil and the electrolyte, a thin layer of metal oxide (an insulator) is formed on the foil,

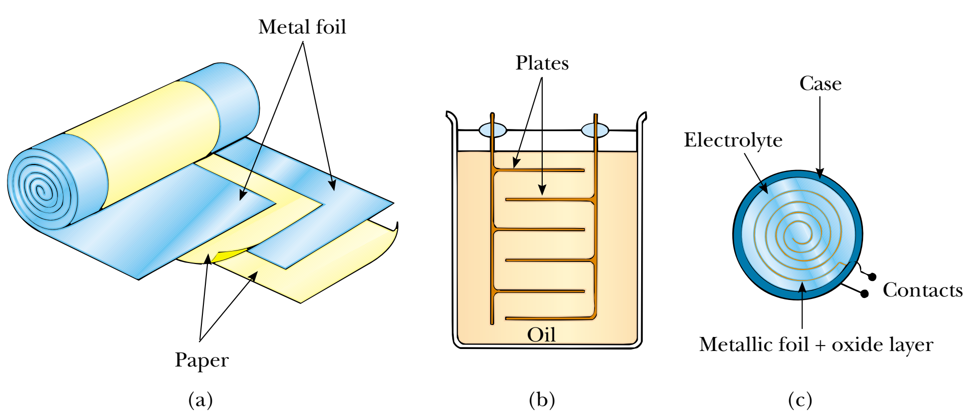


Figure 26.15 Three commercial capacitor designs. (a) A tubular capacitor, whose plates are separated by paper and then rolled into a cylinder. (b) A high-voltage capacitor consisting of many parallel plates separated by insulating oil. (c) An electrolytic capacitor.

and this layer serves as the dielectric. Very large values of capacitance can be obtained in an electrolytic capacitor because the dielectric layer is very thin, and thus the plate separation is very small.

Electrolytic capacitors are not reversible as are many other capacitors—they have a polarity, which is indicated by positive and negative signs marked on the device. When electrolytic capacitors are used in circuits, the polarity must be aligned properly. If the polarity of the applied voltage is opposite that which is intended, the oxide layer is removed and the capacitor conducts electricity instead of storing charge.

Quick Quiz 26.6

If you have ever tried to hang a picture, you know it can be difficult to locate a wooden stud in which to anchor your nail or screw. A carpenter's stud-finder is basically a capacitor with its plates arranged side by side instead of facing one another, as shown in Figure 26.16. When the device is moved over a stud, does the capacitance increase or decrease?

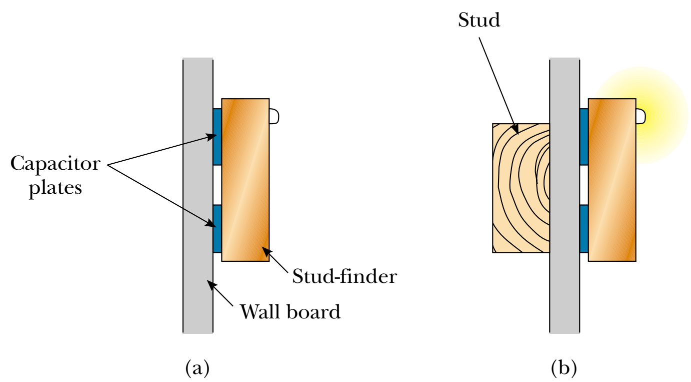


Figure 26.16 A stud-finder. (a) The materials between the plates of the capacitor are the wallboard and air. (b) When the capacitor moves across a stud in the wall, the materials between the plates are the wallboard and the wood. The change in the dielectric constant causes a signal light to illuminate.

Example 26.6 A Paper-Filled Capacitor

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0-mm thickness of paper. (a) Find its capacitance.

Solution Because $\kappa = 3.7$ for paper (see Table 26.1), we have

$$C = \kappa \frac{\epsilon_0 A}{d} = 3.7(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\left(\frac{6.0 \times 10^{-4}\ \text{m}^2}{1.0 \times 10^{-3}\ \text{m}}\right)$$

$$= 20 \times 10^{-12}\ \text{F} = \boxed{20\ \text{pF}}$$

(b) What is the maximum charge that can be placed on the capacitor?

Solution From Table 26.1 we see that the dielectric strength of paper is 16×10^6 V/m. Because the thickness of the paper is 1.0 mm, the maximum voltage that can be applied before breakdown is

$$\Delta V_{\text{max}} = E_{\text{max}} d = (16 \times 10^6\ \text{V/m})(1.0 \times 10^{-3}\ \text{m})$$
$$= 16 \times 10^3\ \text{V}$$

Hence, the maximum charge is

$$Q_{\text{max}} = C\Delta V_{\text{max}} = (20 \times 10^{-12}\ \text{F})(16 \times 10^3\ \text{V}) = \boxed{0.32\ \mu\text{C}}$$

Exercise What is the maximum energy that can be stored in the capacitor?

Answer 2.6×10^{-3} J.

EXAMPLE 26.7 Energy Stored Before and After

A parallel-plate capacitor is charged with a battery to a charge Q_0, as shown in Figure 26.17a. The battery is then removed, and a slab of material that has a dielectric constant κ is inserted between the plates, as shown in Figure 26.17b. Find the energy stored in the capacitor before and after the dielectric is inserted.

Solution The energy stored in the absence of the dielectric is (see Eq. 26.11):

$$U_0 = \frac{Q_0^2}{2C_0}$$

After the battery is removed and the dielectric inserted, the *charge on the capacitor remains the same*. Hence, the energy stored in the presence of the dielectric is

$$U = \frac{Q_0^2}{2C}$$

But the capacitance in the presence of the dielectric is $C = \kappa C_0$, so U becomes

$$U = \frac{Q_0^2}{2\kappa C_0} = \frac{U_0}{\kappa}$$

Because $\kappa > 1$, the final energy is less than the initial energy. We can account for the "missing" energy by noting that the dielectric, when inserted, gets pulled into the device (see the following discussion and Figure 26.18). An external agent must do negative work to keep the dielectric from accelerating. This work is simply the difference $U - U_0$. (Alternatively, the positive work done by the system on the external agent is $U_0 - U$.)

Exercise Suppose that the capacitance in the absence of a dielectric is 8.50 pF and that the capacitor is charged to a potential difference of 12.0 V. If the battery is disconnected and a slab of polystyrene is inserted between the plates, what is $U_0 - U$?

Answer 373 pJ.

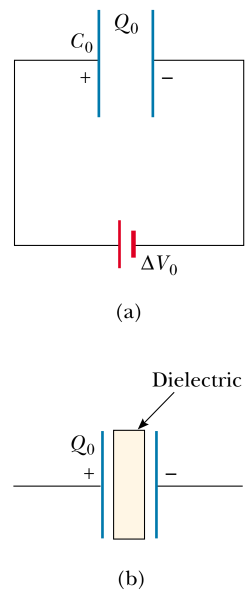


Figure 26.17

As we have seen, the energy of a capacitor not connected to a battery is lowered when a dielectric is inserted between the plates; this means that negative work is done on the dielectric by the external agent inserting the dielectric into the capacitor. This, in turn, implies that a force that draws it into the capacitor must be acting on the dielectric. This force originates from the nonuniform nature of the electric field of the capacitor near its edges, as indicated in Figure 26.18. The horizontal component of this *fringe field* acts on the induced charges on the surface of the dielectric, producing a net horizontal force directed into the space between the capacitor plates.

Quick Quiz 26.7

A fully charged parallel-plate capacitor remains connected to a battery while you slide a dielectric between the plates. Do the following quantities increase, decrease, or stay the same? (a) C; (b) Q; (c) E between the plates; (d) ΔV; (e) energy stored in the capacitor.

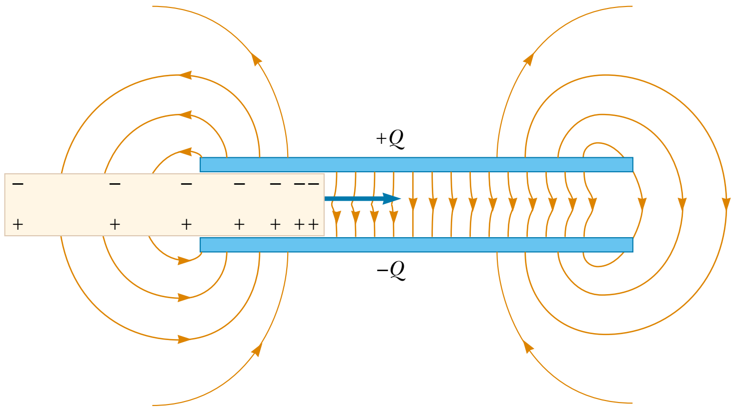


Figure 26.18 The nonuniform electric field near the edges of a parallel-plate capacitor causes a dielectric to be pulled into the capacitor. Note that the field acts on the induced surface charges on the dielectric, which are nonuniformly distributed.

Optional Section

26.6 ELECTRIC DIPOLE IN AN ELECTRIC FIELD

We have discussed the effect on the capacitance of placing a dielectric between the plates of a capacitor. In Section 26.7, we shall describe the microscopic origin of this effect. Before we can do so, however, we need to expand upon the discussion of the electric dipole that we began in Section 23.4 (see Example 23.6). The electric dipole consists of two charges of equal magnitude but opposite sign separated by a distance $2a$, as shown in Figure 26.19. The **electric dipole moment** of this configuration is defined as the vector $\mathbf{p}$ directed from $-q$ to $+q$ along the line joining the charges and having magnitude $2aq$:

$$p \equiv 2aq \qquad \textbf{(26.16)}$$

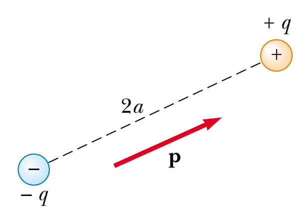


Figure 26.19 An electric dipole consists of two charges of equal magnitude but opposite sign separated by a distance of $2a$. The electric dipole moment $\mathbf{p}$ is directed from $-q$ to $+q$.

Now suppose that an electric dipole is placed in a uniform electric field $\mathbf{E}$, as shown in Figure 26.20. We identify $\mathbf{E}$ as the field *external* to the dipole, distinguishing it from the field *due to* the dipole, which we discussed in Section 23.4. The field $\mathbf{E}$ is established by some other charge distribution, and we place the dipole into this field. Let us imagine that the dipole moment makes an angle θ with the field.

The electric forces acting on the two charges are equal in magnitude but opposite in direction as shown in Figure 26.20 (each has a magnitude $F = qE$). Thus, the net force on the dipole is zero. However, the two forces produce a net torque on the dipole; as a result, the dipole rotates in the direction that brings the dipole moment vector into greater alignment with the field. The torque due to the force on the positive charge about an axis through O in Figure 26.20 is $Fa \sin\theta$, where $a \sin\theta$ is the moment arm of F about O. This force tends to produce a clockwise rotation. The torque about O on the negative charge also is $Fa \sin\theta$; here again, the force tends to produce a clockwise rotation. Thus, the net torque about O is

$$\tau = 2Fa \sin\theta$$

Because $F = qE$ and $p = 2aq$, we can express τ as

$$\tau = 2aqE \sin\theta = pE \sin\theta \qquad \textbf{(26.17)}$$

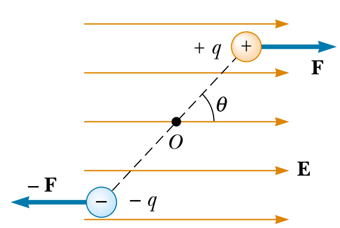


Figure 26.20 An electric dipole in a uniform external electric field. The dipole moment $\mathbf{p}$ is at an angle θ to the field, causing the dipole to experience a torque.

It is convenient to express the torque in vector form as the cross product of the vectors **p** and **E**:

Torque on an electric dipole in an external electric field

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \tag{26.18}$$

We can determine the potential energy of the system of an electric dipole in an external electric field as a function of the orientation of the dipole with respect to the field. To do this, we recognize that work must be done by an external agent to rotate the dipole through an angle so as to cause the dipole moment vector to become less aligned with the field. The work done is then stored as potential energy in the system of the dipole and the external field. The work dW required to rotate the dipole through an angle $d\theta$ is $dW = \tau\, d\theta$ (Eq. 10.22). Because $\tau = pE \sin\theta$ and because the work is transformed into potential energy U, we find that, for a rotation from θ_i to θ_f, the change in potential energy is

$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau\, d\theta = \int_{\theta_i}^{\theta_f} pE \sin\theta\, d\theta = pE \int_{\theta_i}^{\theta_f} \sin\theta\, d\theta$$

$$= pE\Big[-\cos\theta\Big]_{\theta_i}^{\theta_f} = pE(\cos\theta_i - \cos\theta_f)$$

The term that contains $\cos\theta_i$ is a constant that depends on the initial orientation of the dipole. It is convenient for us to choose $\theta_i = 90°$, so that $\cos\theta_i = \cos 90° = 0$. Furthermore, let us choose $U_i = 0$ at $\theta_i = 90°$ as our reference of potential energy. Hence, we can express a general value of $U = U_f$ as

$$U = -pE\cos\theta \tag{26.19}$$

We can write this expression for the potential energy of a dipole in an electric field as the dot product of the vectors **p** and **E**:

Potential energy of a dipole in an electric field

$$U = -\mathbf{p} \cdot \mathbf{E} \tag{26.20}$$

To develop a conceptual understanding of Equation 26.19, let us compare this expression with the expression for the potential energy of an object in the gravitational field of the Earth, $U = mgh$ (see Chapter 8). The gravitational expression includes a parameter associated with the object we place in the field—its mass m. Likewise, Equation 26.19 includes a parameter of the object in the electric field—its dipole moment p. The gravitational expression includes the magnitude of the gravitational field g. Similarly, Equation 26.19 includes the magnitude of the electric field E. So far, these two contributions to the potential energy expressions appear analogous. However, the final contribution is somewhat different in the two cases. In the gravitational expression, the potential energy depends on how high we lift the object, measured by h. In Equation 26.19, the potential energy depends on the angle θ through which we rotate the dipole. In both cases, we are making a change in the system. In the gravitational case, the change involves moving an object in a *translational* sense, whereas in the electrical case, the change involves moving an object in a *rotational* sense. In both cases, however, once the change is made, the system tends to return to the original configuration when the object is released: the object of mass m falls back to the ground, and the dipole begins to rotate back toward the configuration in which it was aligned with the field. Thus, apart from the type of motion, the expressions for potential energy in these two cases are similar.

Molecules are said to be *polarized* when a separation exists between the average position of the negative charges and the average position of the positive charges in the molecule. In some molecules, such as water, this condition is always present—such molecules are called **polar molecules.** Molecules that do not possess a permanent polarization are called **nonpolar molecules.**

We can understand the permanent polarization of water by inspecting the geometry of the water molecule. In the water molecule, the oxygen atom is bonded to the hydrogen atoms such that an angle of 105° is formed between the two bonds (Fig. 26.21). The center of the negative charge distribution is near the oxygen atom, and the center of the positive charge distribution lies at a point midway along the line joining the hydrogen atoms (the point labeled × in Fig. 26.21). We can model the water molecule and other polar molecules as dipoles because the average positions of the positive and negative charges act as point charges. As a result, we can apply our discussion of dipoles to the behavior of polar molecules.

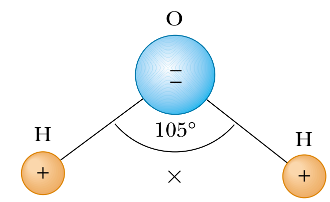


Figure 26.21 The water molecule, H_2O, has a permanent polarization resulting from its bent geometry. The center of the positive charge distribution is at the point ×.

Microwave ovens take advantage of the polar nature of the water molecule. When in operation, microwave ovens generate a rapidly changing electric field that causes the polar molecules to swing back and forth, absorbing energy from the field in the process. Because the jostling molecules collide with each other, the energy they absorb from the field is converted to internal energy, which corresponds to an increase in temperature of the food.

Another household scenario in which the dipole structure of water is exploited is washing with soap and water. Grease and oil are made up of nonpolar molecules, which are generally not attracted to water. Plain water is not very useful for removing this type of grime. Soap contains long molecules called *surfactants.* In a long molecule, the polarity characteristics of one end of the molecule can be different from those at the other end. In a surfactant molecule, one end acts like a nonpolar molecule and the other acts like a polar molecule. The nonpolar end can attach to a grease or oil molecule, and the polar end can attach to a water molecule. Thus, the soap serves as a chain, linking the dirt and water molecules together. When the water is rinsed away, the grease and oil go with it.

A symmetric molecule (Fig. 26.22a) has no permanent polarization, but polarization can be induced by placing the molecule in an electric field. A field directed to the left, as shown in Figure 26.22b, would cause the center of the positive charge distribution to shift to the left from its initial position and the center of the negative charge distribution to shift to the right. This *induced polarization* is the effect that predominates in most materials used as dielectrics in capacitors.

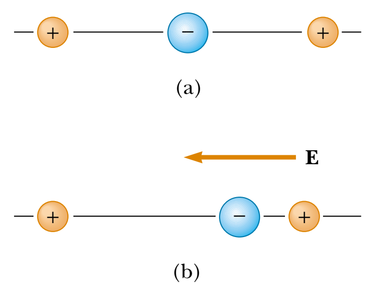


Figure 26.22 (a) A symmetric molecule has no permanent polarization. (b) An external electric field induces a polarization in the molecule.

EXAMPLE 26.8 The H_2O Molecule

The water (H_2O) molecule has an electric dipole moment of 6.3×10^{-30} C·m. A sample contains 10^{21} water molecules, with the dipole moments all oriented in the direction of an electric field of magnitude 2.5×10^5 N/C. How much work is required to rotate the dipoles from this orientation ($\theta = 0°$) to one in which all the dipole moments are perpendicular to the field ($\theta = 90°$)?

Solution The work required to rotate one molecule 90° is equal to the difference in potential energy between the 90° orientation and the 0° orientation. Using Equation 26.19, we obtain

$$\begin{aligned} W &= U_{90} - U_0 = (-pE\cos 90°) - (-pE\cos 0°) \\ &= pE = (6.3 \times 10^{-30}\ \mathrm{C\cdot m})(2.5 \times 10^5\ \mathrm{N/C}) \\ &= 1.6 \times 10^{-24}\ \mathrm{J} \end{aligned}$$

Because there are 10^{21} molecules in the sample, the *total* work required is

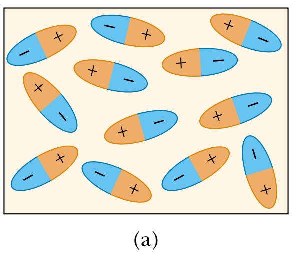
$$W_{\text{total}} = (10^{21})(1.6 \times 10^{-24}\ \mathrm{J}) = \boxed{1.6 \times 10^{-3}\ \mathrm{J}}$$

Optional Section

26.7 AN ATOMIC DESCRIPTION OF DIELECTRICS

In Section 26.5 we found that the potential difference ΔV_0 between the plates of a capacitor is reduced to $\Delta V_0/\kappa$ when a dielectric is introduced. Because the potential difference between the plates equals the product of the electric field and the separation d, the electric field is also reduced. Thus, if $\mathbf{E}_0$ is the electric field without the dielectric, the field in the presence of a dielectric is

$$\mathbf{E} = \frac{\mathbf{E}_0}{\kappa} \tag{26.21}$$



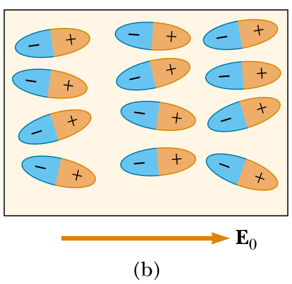


Figure 26.23 (a) Polar molecules are randomly oriented in the absence of an external electric field. (b) When an external field is applied, the molecules partially align with the field.

Let us first consider a dielectric made up of polar molecules placed in the electric field between the plates of a capacitor. The dipoles (that is, the polar molecules making up the dielectric) are randomly oriented in the absence of an electric field, as shown in Figure 26.23a. When an external field $\mathbf{E}_0$ due to charges on the capacitor plates is applied, a torque is exerted on the dipoles, causing them to partially align with the field, as shown in Figure 26.23b. We can now describe the dielectric as being polarized. The degree of alignment of the molecules with the electric field depends on temperature and on the magnitude of the field. In general, the alignment increases with decreasing temperature and with increasing electric field.

If the molecules of the dielectric are nonpolar, then the electric field due to the plates produces some charge separation and an *induced dipole moment*. These induced dipole moments tend to align with the external field, and the dielectric is polarized. Thus, we can polarize a dielectric with an external field regardless of whether the molecules are polar or nonpolar.

With these ideas in mind, consider a slab of dielectric material placed between the plates of a capacitor so that it is in a uniform electric field $\mathbf{E}_0$, as shown in Figure 26.24a. The electric field due to the plates is directed to the right and polarizes the dielectric. The net effect on the dielectric is the formation of an *induced* positive surface charge density σ_{ind} on the right face and an equal negative surface charge density $-\sigma_{\text{ind}}$ on the left face, as shown in Figure 26.24b. These induced surface charges on the dielectric give rise to an induced electric field $\mathbf{E}_{\text{ind}}$ in the direction opposite the external field $\mathbf{E}_0$. Therefore, the net electric field $\mathbf{E}$ in the

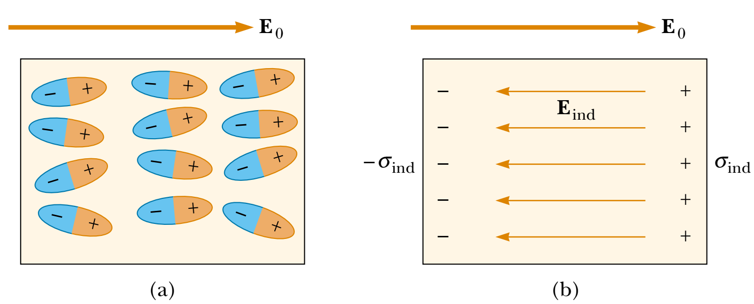


Figure 26.24 (a) When a dielectric is polarized, the dipole moments of the molecules in the dielectric are partially aligned with the external field $\mathbf{E}_0$. (b) This polarization causes an induced negative surface charge on one side of the dielectric and an equal induced positive surface charge on the opposite side. This separation of charge results in a reduction in the net electric field within the dielectric.

dielectric has a magnitude

$$E = E_0 - E_{\text{ind}} \tag{26.22}$$

In the parallel-plate capacitor shown in Figure 26.25, the external field E_0 is related to the charge density σ on the plates through the relationship $E_0 = \sigma/\epsilon_0$. The induced electric field in the dielectric is related to the induced charge density σ_{ind} through the relationship $E_{\text{ind}} = \sigma_{\text{ind}}/\epsilon_0$. Because $E = E_0/\kappa = \sigma/\kappa\epsilon_0$, substitution into Equation 26.22 gives

$$\frac{\sigma}{\kappa\epsilon_0} = \frac{\sigma}{\epsilon_0} - \frac{\sigma_{\text{ind}}}{\epsilon_0}$$

$$\sigma_{\text{ind}} = \left(\frac{\kappa - 1}{\kappa}\right)\sigma \tag{26.23}$$

Because $\kappa > 1$, this expression shows that the charge density σ_{ind} induced on the dielectric is less than the charge density σ on the plates. For instance, if $\kappa = 3$, we see that the induced charge density is two-thirds the charge density on the plates. If no dielectric is present, then $\kappa = 1$ and $\sigma_{\text{ind}} = 0$ as expected. However, if the dielectric is replaced by an electrical conductor, for which $E = 0$, then Equation 26.22 indicates that $E_0 = E_{\text{ind}}$; this corresponds to $\sigma_{\text{ind}} = \sigma$. That is, the surface charge induced on the conductor is equal in magnitude but opposite in sign to that on the plates, resulting in a net electric field of zero in the conductor.

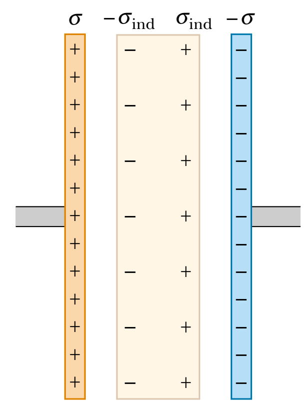


Figure 26.25 Induced charge on a dielectric placed between the plates of a charged capacitor. Note that the induced charge density on the dielectric is *less* than the charge density on the plates.

EXAMPLE 26.9 Effect of a Metallic Slab

A parallel-plate capacitor has a plate separation d and plate area A. An uncharged metallic slab of thickness a is inserted midway between the plates. (a) Find the capacitance of the device.

Solution We can solve this problem by noting that any charge that appears on one plate of the capacitor must induce a charge of equal magnitude but opposite sign on the near side of the slab, as shown in Figure 26.26a. Consequently, the net charge on the slab remains zero, and the electric field inside the slab is zero. Hence, the capacitor is equivalent to two capacitors in series, each having a plate separation $(d - a)/2$, as shown in Figure 26.26b.

Using the rule for adding two capacitors in series (Eq. 26.10), we obtain

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\dfrac{\epsilon_0 A}{(d-a)/2}} + \frac{1}{\dfrac{\epsilon_0 A}{(d-a)/2}}$$

$$\boxed{C = \frac{\epsilon_0 A}{d - a}}$$

Note that C approaches infinity as a approaches d. Why?

(b) Show that the capacitance is unaffected if the metallic slab is infinitesimally thin.

Solution In the result for part (a), we let $a \rightarrow 0$:

$$C = \lim_{a \to 0} \frac{\epsilon_0 A}{d - a} = \frac{\epsilon_0 A}{d}$$

which is the original capacitance.

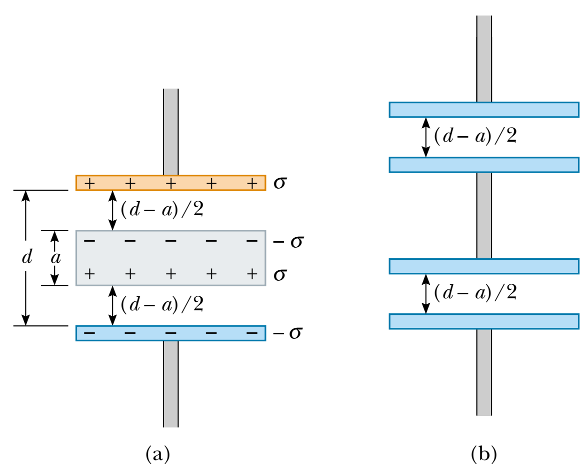


Figure 26.26 (a) A parallel-plate capacitor of plate separation d partially filled with a metallic slab of thickness a. (b) The equivalent circuit of the device in part (a) consists of two capacitors in series, each having a plate separation $(d - a)/2$.

(c) Show that the answer to part (a) does not depend on where the slab is inserted.

Solution Let us imagine that the slab in Figure 26.26a is moved upward so that the distance between the upper edge of the slab and the upper plate is b. Then, the distance between the lower edge of the slab and the lower plate is $d - b - a$. As in part (a), we find the total capacitance of the series combination:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{\epsilon_0 A}{b}} + \frac{1}{\frac{\epsilon_0 A}{d-b-a}}$$

$$= \frac{b}{\epsilon_0 A} + \frac{d-b-a}{\epsilon_0 A} = \frac{d-a}{\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d-a}$$

This is the same result as in part (a). It is independent of the value of b, so it does not matter where the slab is located.

EXAMPLE 26.10 A Partially Filled Capacitor

A parallel-plate capacitor with a plate separation d has a capacitance C_0 in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant κ and thickness $\frac{1}{3}d$ is inserted between the plates (Fig. 26.27a)?

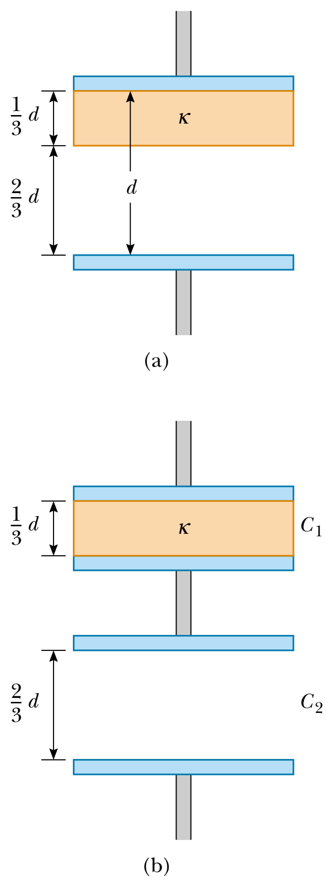


Figure 26.27 (a) A parallel-plate capacitor of plate separation d partially filled with a dielectric of thickness $d/3$. (b) The equivalent circuit of the capacitor consists of two capacitors connected in series.

Solution In Example 26.9, we found that we could insert a metallic slab between the plates of a capacitor and consider the combination as two capacitors in series. The resulting capacitance was independent of the location of the slab. Furthermore, if the thickness of the slab approaches zero, then the capacitance of the system approaches the capacitance when the slab is absent. From this, we conclude that we can insert an infinitesimally thin metallic slab anywhere between the plates of a capacitor without affecting the capacitance. Thus, let us imagine sliding an infinitesimally thin metallic slab along the bottom face of the dielectric shown in Figure 26.27a. We can then consider this system to be the series combination of the two capacitors shown in Figure 26.27b: one having a plate separation $d/3$ and filled with a dielectric, and the other having a plate separation $2d/3$ and air between its plates.

From Equations 26.15 and 26.3, the two capacitances are

$$C_1 = \frac{\kappa\epsilon_0 A}{d/3} \quad \text{and} \quad C_2 = \frac{\epsilon_0 A}{2d/3}$$

Using Equation 26.10 for two capacitors combined in series, we have

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/3}{\kappa\epsilon_0 A} + \frac{2d/3}{\epsilon_0 A}$$

$$= \frac{d}{3\epsilon_0 A}\left(\frac{1}{\kappa} + 2\right) = \frac{d}{3\epsilon_0 A}\left(\frac{1+2\kappa}{\kappa}\right)$$

$$C = \left(\frac{3\kappa}{2\kappa+1}\right)\frac{\epsilon_0 A}{d}$$

Because the capacitance without the dielectric is $C_0 = \epsilon_0 A/d$, we see that

$$\boxed{C = \left(\frac{3\kappa}{2\kappa+1}\right)C_0}$$

SUMMARY

A **capacitor** consists of two conductors carrying charges of equal magnitude but opposite sign. The **capacitance** C of any capacitor is the ratio of the charge Q on either conductor to the potential difference ΔV between them:

$$C \equiv \frac{Q}{\Delta V} \tag{26.1}$$

This relationship can be used in situations in which any two of the three variables are known. It is important to remember that this ratio is constant for a given configuration of conductors because the capacitance depends only on the geometry of the conductors and not on an external source of charge or potential difference.

The SI unit of capacitance is coulombs per volt, or the **farad** (F), and 1 F = 1 C/V.

Capacitance expressions for various geometries are summarized in Table 26.2.

If two or more capacitors are connected in parallel, then the potential difference is the same across all of them. The equivalent capacitance of a parallel combination of capacitors is

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots \tag{26.8}$$

If two or more capacitors are connected in series, the charge is the same on all of them, and the equivalent capacitance of the series combination is given by

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \tag{26.10}$$

These two equations enable you to simplify many electric circuits by replacing multiple capacitors with a single equivalent capacitance.

Work is required to charge a capacitor because the charging process is equivalent to the transfer of charges from one conductor at a lower electric potential to another conductor at a higher potential. The work done in charging the capacitor to a charge Q equals the electric potential energy U stored in the capacitor, where

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2 \tag{26.11}$$

TABLE 26.2 Capacitance and Geometry

Geometry	Capacitance	Equation
Isolated charged sphere of radius R (second charged conductor assumed at infinity)	$C = 4\pi\epsilon_0 R$	26.2
Parallel-plate capacitor of plate area A and plate separation d	$C = \epsilon_0 \dfrac{A}{d}$	26.3
Cylindrical capacitor of length ℓ and inner and outer radii a and b, respectively	$C = \dfrac{\ell}{2k_e \ln\left(\dfrac{b}{a}\right)}$	26.4
Spherical capacitor with inner and outer radii a and b, respectively	$C = \dfrac{ab}{k_e(b-a)}$	26.6

When a dielectric material is inserted between the plates of a capacitor, the capacitance increases by a dimensionless factor κ, called the **dielectric constant:**

$$C = \kappa C_0 \tag{26.14}$$

where C_0 is the capacitance in the absence of the dielectric. The increase in capacitance is due to a decrease in the magnitude of the electric field in the presence of the dielectric and to a corresponding decrease in the potential difference between the plates—if we assume that the charging battery is removed from the circuit before the dielectric is inserted. The decrease in the magnitude of **E** arises from an internal electric field produced by aligned dipoles in the dielectric. This internal field produced by the dipoles opposes the applied field due to the capacitor plates, and the result is a reduction in the net electric field.

The **electric dipole moment p** of an electric dipole has a magnitude

$$p \equiv 2aq \tag{26.16}$$

The direction of the electric dipole moment vector is from the negative charge toward the positive charge.

The torque acting on an electric dipole in a uniform electric field **E** is

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \tag{26.18}$$

The potential energy of an electric dipole in a uniform external electric field **E** is

$$U = -\mathbf{p} \cdot \mathbf{E} \tag{26.20}$$

Problem-Solving Hints

Capacitors

- Be careful with units. When you calculate capacitance in farads, make sure that distances are expressed in meters and that you use the SI value of ϵ_0. When checking consistency of units, remember that the unit for electric fields can be either N/C or V/m.
- When two or more capacitors are connected in parallel, the potential difference across each is the same. The charge on each capacitor is proportional to its capacitance; hence, the capacitances can be added directly to give the equivalent capacitance of the parallel combination. The equivalent capacitance is always larger than the individual capacitances.
- When two or more capacitors are connected in series, they carry the same charge, and the sum of the potential differences equals the total potential difference applied to the combination. The sum of the reciprocals of the capacitances equals the reciprocal of the equivalent capacitance, which is always less than the capacitance of the smallest individual capacitor.
- A dielectric increases the capacitance of a capacitor by a factor κ (the dielectric constant) over its capacitance when air is between the plates.
- For problems in which a battery is being connected or disconnected, note whether modifications to the capacitor are made while it is connected to the battery or after it has been disconnected. If the capacitor remains connected to the battery, the voltage across the capacitor remains unchanged (equal to the battery voltage), and the charge is proportional to the capaci-

tance, although it may be modified (for instance, by the insertion of a dielectric). If you disconnect the capacitor from the battery before making any modifications to the capacitor, then its charge remains fixed. In this case, as you vary the capacitance, the voltage across the plates changes according to the expression $\Delta V = Q/C$.

QUESTIONS

1. If you were asked to design a capacitor in a situation for which small size and large capacitance were required, what factors would be important in your design?
2. The plates of a capacitor are connected to a battery. What happens to the charge on the plates if the connecting wires are removed from the battery? What happens to the charge if the wires are removed from the battery and connected to each other?
3. A farad is a very large unit of capacitance. Calculate the length of one side of a square, air-filled capacitor that has a plate separation of 1 m. Assume that it has a capacitance of 1 F.
4. A pair of capacitors are connected in parallel, while an identical pair are connected in series. Which pair would be more dangerous to handle after being connected to the same voltage source? Explain.
5. If you are given three different capacitors C_1, C_2, C_3, how many different combinations of capacitance can you produce?
6. What advantage might there be in using two identical capacitors in parallel connected in series with another identical parallel pair rather than a single capacitor?
7. Is it always possible to reduce a combination of capacitors to one equivalent capacitor with the rules we have developed? Explain.
8. Because the net charge in a capacitor is always zero, what does a capacitor store?
9. Because the charges on the plates of a parallel-plate capacitor are of opposite sign, they attract each other. Hence, it would take positive work to increase the plate separation. What happens to the external work done in this process?
10. Explain why the work needed to move a charge Q through a potential difference ΔV is $W = Q\Delta V$, whereas the energy stored in a charged capacitor is $U = \frac{1}{2}Q\Delta V$. Where does the $\frac{1}{2}$ factor come from?
11. If the potential difference across a capacitor is doubled, by what factor does the stored energy change?
12. Why is it dangerous to touch the terminals of a high-voltage capacitor even after the applied voltage has been turned off? What can be done to make the capacitor safe to handle after the voltage source has been removed?
13. Describe how you can increase the maximum operating voltage of a parallel-plate capacitor for a fixed plate separation.
14. An air-filled capacitor is charged, disconnected from the power supply, and, finally, connected to a voltmeter. Explain how and why the voltage reading changes when a dielectric is inserted between the plates of the capacitor.
15. Using the polar molecule description of a dielectric, explain how a dielectric affects the electric field inside a capacitor.
16. Explain why a dielectric increases the maximum operating voltage of a capacitor even though the physical size of the capacitor does not change.
17. What is the difference between dielectric strength and the dielectric constant?
18. Explain why a water molecule is permanently polarized. What type of molecule has no permanent polarization?
19. If a dielectric-filled capacitor is heated, how does its capacitance change? (Neglect thermal expansion and assume that the dipole orientations are temperature dependent.)

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
☐ = paired numerical/symbolic problems

Section 26.1 Definition of Capacitance

1. (a) How much charge is on each plate of a 4.00-μF capacitor when it is connected to a 12.0-V battery? (b) If this same capacitor is connected to a 1.50-V battery, what charge is stored?
2. Two conductors having net charges of $+10.0\ \mu$C and $-10.0\ \mu$C have a potential difference of 10.0 V. Determine (a) the capacitance of the system and (b) the potential difference between the two conductors if the charges on each are increased to $+100\ \mu$C and $-100\ \mu$C.

Section 26.2 Calculating Capacitance

3. An isolated charged conducting sphere of radius 12.0 cm creates an electric field of 4.90×10^4 N/C at a distance 21.0 cm from its center. (a) What is its surface charge density? (b) What is its capacitance?

4. (a) If a drop of liquid has capacitance 1.00 pF, what is its radius? (b) If another drop has radius 2.00 mm, what is its capacitance? (c) What is the charge on the smaller drop if its potential is 100 V?

5. Two conducting spheres with diameters of 0.400 m and 1.00 m are separated by a distance that is large compared with the diameters. The spheres are connected by a thin wire and are charged to 7.00 μC. (a) How is this total charge shared between the spheres? (Neglect any charge on the wire.) (b) What is the potential of the system of spheres when the reference potential is taken to be $V = 0$ at $r = \infty$?

6. Regarding the Earth and a cloud layer 800 m above the Earth as the "plates" of a capacitor, calculate the capacitance if the cloud layer has an area of 1.00 km^2. Assume that the air between the cloud and the ground is pure and dry. Assume that charge builds up on the cloud and on the ground until a uniform electric field with a magnitude of 3.00×10^6 N/C throughout the space between them makes the air break down and conduct electricity as a lightning bolt. What is the maximum charge the cloud can hold?

WEB **7.** An air-filled capacitor consists of two parallel plates, each with an area of 7.60 cm^2, separated by a distance of 1.80 mm. If a 20.0-V potential difference is applied to these plates, calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate.

8. A 1-megabit computer memory chip contains many 60.0-fF capacitors. Each capacitor has a plate area of 21.0×10^{-12} m^2. Determine the plate separation of such a capacitor (assume a parallel-plate configuration). The characteristic atomic diameter is 10^{-10} m = 0.100 nm. Express the plate separation in nanometers.

9. When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of 30.0 nC/cm^2. What is the spacing between the plates?

10. A variable air capacitor used in tuning circuits is made of N semicircular plates each of radius R and positioned a distance d from each other. As shown in Figure P26.10, a second identical set of plates is enmeshed with its plates halfway between those of the first set. The second set can rotate as a unit. Determine the capacitance as a function of the angle of rotation θ, where $\theta = 0$ corresponds to the maximum capacitance.

WEB **11.** A 50.0-m length of coaxial cable has an inner conductor that has a diameter of 2.58 mm and carries a charge of 8.10 μC. The surrounding conductor has an inner diameter of 7.27 mm and a charge of -8.10 μC. (a) What is the capacitance of this cable? (b) What is the potential difference between the two conductors? Assume the region between the conductors is air.

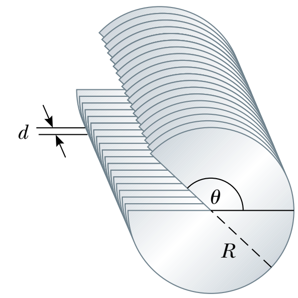


Figure P26.10

12. A 20.0-μF spherical capacitor is composed of two metallic spheres, one having a radius twice as large as the other. If the region between the spheres is a vacuum, determine the volume of this region.

13. A small object with a mass of 350 mg carries a charge of 30.0 nC and is suspended by a thread between the vertical plates of a parallel-plate capacitor. The plates are separated by 4.00 cm. If the thread makes an angle of 15.0° with the vertical, what is the potential difference between the plates?

14. A small object of mass m carries a charge q and is suspended by a thread between the vertical plates of a parallel-plate capacitor. The plate separation is d. If the thread makes an angle θ with the vertical, what is the potential difference between the plates?

15. An air-filled spherical capacitor is constructed with inner and outer shell radii of 7.00 and 14.0 cm, respectively. (a) Calculate the capacitance of the device. (b) What potential difference between the spheres results in a charge of 4.00 μC on the capacitor?

16. Find the capacitance of the Earth. (*Hint:* The outer conductor of the "spherical capacitor" may be considered as a conducting sphere at infinity where V approaches zero.)

Section 26.3 Combinations of Capacitors

17. Two capacitors $C_1 = 5.00$ μF and $C_2 = 12.0$ μF are connected in parallel, and the resulting combination is connected to a 9.00-V battery. (a) What is the value of the equivalent capacitance of the combination? What are (b) the potential difference across each capacitor and (c) the charge stored on each capacitor?

18. The two capacitors of Problem 17 are now connected in series and to a 9.00-V battery. Find (a) the value of the equivalent capacitance of the combination, (b) the voltage across each capacitor, and (c) the charge on each capacitor.

19. Two capacitors when connected in parallel give an equivalent capacitance of 9.00 pF and an equivalent capacitance of 2.00 pF when connected in series. What is the capacitance of each capacitor?

20. Two capacitors when connected in parallel give an equivalent capacitance of C_p and an equivalent capacitance of C_s when connected in series. What is the capacitance of each capacitor?

WEB **21.** Four capacitors are connected as shown in Figure P26.21. (a) Find the equivalent capacitance between points *a* and *b*. (b) Calculate the charge on each capacitor if $\Delta V_{ab} = 15.0$ V.

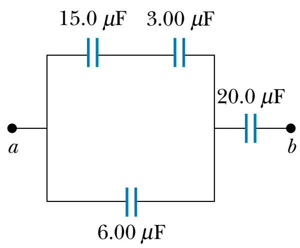


Figure P26.21

22. Evaluate the equivalent capacitance of the configuration shown in Figure P26.22. All the capacitors are identical, and each has capacitance *C*.

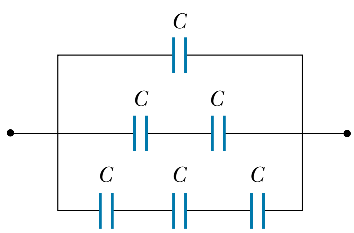


Figure P26.22

23. Consider the circuit shown in Figure P26.23, where $C_1 = 6.00\ \mu$F, $C_2 = 3.00\ \mu$F, and $\Delta V = 20.0$ V. Capacitor C_1 is first charged by the closing of switch S_1. Switch S_1 is then opened, and the charged capacitor is connected to the uncharged capacitor by the closing of S_2. Calculate the initial charge acquired by C_1 and the final charge on each.

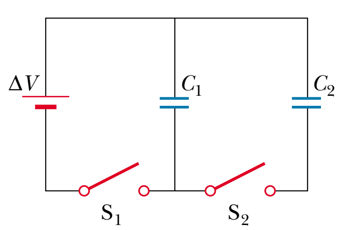


Figure P26.23

24. According to its design specification, the timer circuit delaying the closing of an elevator door is to have a capacitance of 32.0 μF between two points *A* and *B*. (a) When one circuit is being constructed, the inexpensive capacitor installed between these two points is found to have capacitance 34.8 μF. To meet the specification, one additional capacitor can be placed between the two points. Should it be in series or in parallel with the 34.8-μF capacitor? What should be its capacitance? (b) The next circuit comes down the assembly line with capacitance 29.8 μF between *A* and *B*. What additional capacitor should be installed in series or in parallel in that circuit, to meet the specification?

25. The circuit in Figure P26.25 consists of two identical parallel metallic plates connected by identical metallic springs to a 100-V battery. With the switch open, the plates are uncharged, are separated by a distance $d = 8.00$ mm, and have a capacitance $C = 2.00\ \mu$F. When the switch is closed, the distance between the plates decreases by a factor of 0.500. (a) How much charge collects on each plate and (b) what is the spring constant for each spring? (*Hint:* Use the result of Problem 35.)

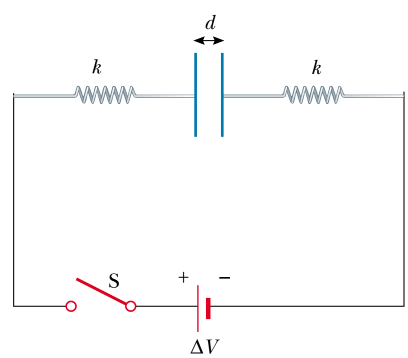


Figure P26.25

26. Figure P26.26 shows six concentric conducting spheres, A, B, C, D, E, and F having radii *R*, 2*R*, 3*R*, 4*R*, 5*R*, and 6*R*, respectively. Spheres B and C are connected by a conducting wire, as are spheres D and E. Determine the equivalent capacitance of this system.

27. A group of identical capacitors is connected first in series and then in parallel. The combined capacitance in parallel is 100 times larger than for the series connection. How many capacitors are in the group?

28. Find the equivalent capacitance between points *a* and *b* for the group of capacitors connected as shown in Figure P26.28 if $C_1 = 5.00\ \mu$F, $C_2 = 10.0\ \mu$F, and $C_3 = 2.00\ \mu$F.

29. For the network described in the previous problem if the potential difference between points *a* and *b* is 60.0 V, what charge is stored on C_3?

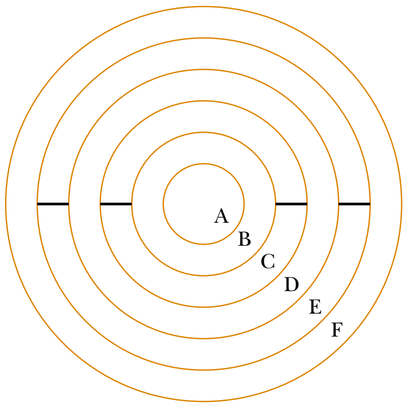


Figure P26.26

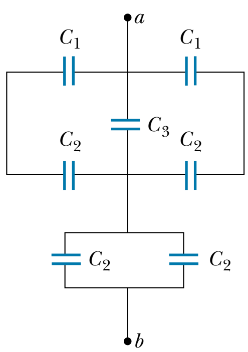


Figure P26.28 Problems 28 and 29.

30. Find the equivalent capacitance between points a and b in the combination of capacitors shown in Figure P26.30.

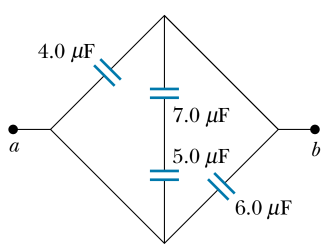


Figure P26.30

Section 26.4 Energy Stored in a Charged Capacitor

31. (a) A 3.00-μF capacitor is connected to a 12.0-V battery. How much energy is stored in the capacitor? (b) If the capacitor had been connected to a 6.00-V battery, how much energy would have been stored?

32. Two capacitors $C_1 = 25.0\ \mu$F and $C_2 = 5.00\ \mu$F are connected in parallel and charged with a 100-V power supply. (a) Draw a circuit diagram and calculate the total energy stored in the two capacitors. (b) What potential difference would be required across the same two capacitors connected in series so that the combination stores the same energy as in part (a)? Draw a circuit diagram of this circuit.

33. A parallel-plate capacitor is charged and then disconnected from a battery. By what fraction does the stored energy change (increase or decrease) when the plate separation is doubled?

34. A uniform electric field $E = 3\,000$ V/m exists within a certain region. What volume of space contains an energy equal to 1.00×10^{-7} J? Express your answer in cubic meters and in liters.

WEB **35.** A parallel-plate capacitor has a charge Q and plates of area A. Show that the force exerted on each plate by the other is $F = Q^2/2\epsilon_0 A$. (*Hint:* Let $C = \epsilon_0 A/x$ for an arbitrary plate separation x; then require that the work done in separating the two charged plates be $W = \int F\,dx$.)

36. Plate a of a parallel-plate, air-filled capacitor is connected to a spring having force constant k, and plate b is fixed. They rest on a table top as shown (top view) in Figure P26.36. If a charge $+Q$ is placed on plate a and a charge $-Q$ is placed on plate b, by how much does the spring expand?

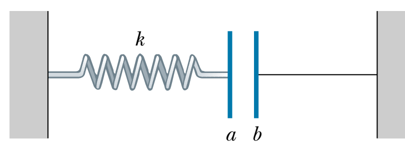


Figure P26.36

37. Review Problem. A certain storm cloud has a potential difference of 1.00×10^8 V relative to a tree. If, during a lightning storm, 50.0 C of charge is transferred through this potential difference and 1.00% of the energy is absorbed by the tree, how much water (sap in the tree) initially at 30.0°C can be boiled away? Water has a specific heat of 4 186 J/kg·°C, a boiling point of 100°C, and a heat of vaporization of 2.26×10^6 J/kg.

38. Show that the energy associated with a conducting sphere of radius R and charge Q surrounded by a vacuum is $U = k_eQ^2/2R$.

39. Einstein said that energy is associated with mass according to the famous relationship $E = mc^2$. Estimate the radius of an electron, assuming that its charge is distributed uniformly over the surface of a sphere of radius R and that the mass–energy of the electron is equal to the total energy stored in the resulting nonzero electric field between R and infinity. (See Problem 38. Experimentally, an electron nevertheless appears to be a point particle. The electric field close to the electron must be described by quantum electrodynamics, rather than the classical electrodynamics that we study.)

Section 26.5 Capacitors with Dielectrics

40. Find the capacitance of a parallel-plate capacitor that uses Bakelite as a dielectric, if each of the plates has an area of 5.00 cm^2 and the plate separation is 2.00 mm.

41. Determine (a) the capacitance and (b) the maximum voltage that can be applied to a Teflon-filled parallel-plate capacitor having a plate area of 1.75 cm^2 and plate separation of 0.040 0 mm.

42. (a) How much charge can be placed on a capacitor with air between the plates before it breaks down, if the area of each of the plates is 5.00 cm^2? (b) Find the maximum charge if polystyrene is used between the plates instead of air.

43. A commercial capacitor is constructed as shown in Figure 26.15a. This particular capacitor is rolled from two strips of aluminum separated by two strips of paraffin-coated paper. Each strip of foil and paper is 7.00 cm wide. The foil is 0.004 00 mm thick, and the paper is 0.025 0 mm thick and has a dielectric constant of 3.70. What length should the strips be if a capacitance of 9.50×10^{-8} F is desired? (Use the parallel-plate formula.)

44. The supermarket sells rolls of aluminum foil, plastic wrap, and waxed paper. Describe a capacitor made from supermarket materials. Compute order-of-magnitude estimates for its capacitance and its breakdown voltage.

45. A capacitor that has air between its plates is connected across a potential difference of 12.0 V and stores 48.0 μC of charge. It is then disconnected from the source while still charged. (a) Find the capacitance of the capacitor. (b) A piece of Teflon is inserted between the plates. Find its new capacitance. (c) Find the voltage and charge now on the capacitor.

46. A parallel-plate capacitor in air has a plate separation of 1.50 cm and a plate area of 25.0 cm^2. The plates are charged to a potential difference of 250 V and disconnected from the source. The capacitor is then immersed in distilled water. Determine (a) the charge on the plates before and after immersion, (b) the capacitance and voltage after immersion, and (c) the change in energy of the capacitor. Neglect the conductance of the liquid.

47. A conducting spherical shell has inner radius a and outer radius c. The space between these two surfaces is filled with a dielectric for which the dielectric constant is κ_1 between a and b, and κ_2 between b and c (Fig. P26.47). Determine the capacitance of this system.

48. A wafer of titanium dioxide ($\kappa = 173$) has an area of 1.00 cm^2 and a thickness of 0.100 mm. Aluminum is evaporated on the parallel faces to form a parallel-plate capacitor. (a) Calculate the capacitance. (b) When the capacitor is charged with a 12.0-V battery, what is the magnitude of charge delivered to each plate? (c) For the situation in part (b), what are the free and induced surface charge densities? (d) What is the magnitude E of the electric field?

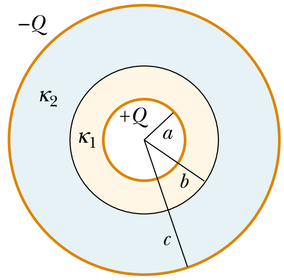


Figure P26.47

49. Each capacitor in the combination shown in Figure P26.49 has a breakdown voltage of 15.0 V. What is the breakdown voltage of the combination?

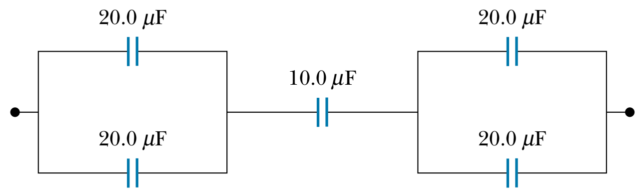


Figure P26.49

(Optional)
Section 26.6 Electric Dipole in an Electric Field

50. A small rigid object carries positive and negative 3.50-nC charges. It is oriented so that the positive charge is at the point (− 1.20 mm, 1.10 mm) and the negative charge is at the point (1.40 mm, − 1.30 mm). (a) Find the electric dipole moment of the object. The object is placed in an electric field $\mathbf{E} = (7\,800\mathbf{i} - 4\,900\mathbf{j})$ N/C. (b) Find the torque acting on the object. (c) Find the potential energy of the object in this orientation. (d) If the orientation of the object can change, find the difference between its maximum and its minimum potential energies.

51. A small object with electric dipole moment $\mathbf{p}$ is placed in a nonuniform electric field $\mathbf{E} = E(x)\,\mathbf{i}$. That is, the field is in the x direction, and its magnitude depends on the coordinate x. Let θ represent the angle between the dipole moment and the x direction. (a) Prove that the dipole experiences a net force $F = p(dE/dx)\cos\theta$ in the direction toward which the field increases. (b) Consider the field created by a spherical balloon centered at the origin. The balloon has a radius of 15.0 cm and carries a charge of 2.00 μC. Evaluate dE/dx at the point (16 cm, 0, 0). Assume that a water droplet at this point has an induced dipole moment of $(6.30\mathbf{i})$ nC·m. Find the force on it.

(Optional)
Section 26.7 An Atomic Description of Dielectrics

52. A detector of radiation called a Geiger–Muller counter consists of a closed, hollow, conducting cylinder with a fine wire along its axis. Suppose that the internal diameter of the cylinder is 2.50 cm and that the wire along the axis has a diameter of 0.200 mm. If the dielectric strength of the gas between the central wire and the cylinder is 1.20×10^6 V/m, calculate the maximum voltage that can be applied between the wire and the cylinder before breakdown occurs in the gas.

53. The general form of Gauss's law describes how a charge creates an electric field in a material, as well as in a vacuum. It is

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon}$$

where $\epsilon = \kappa\epsilon_0$ is the permittivity of the material. (a) A sheet with charge Q uniformly distributed over its area A is surrounded by a dielectric. Show that the sheet creates a uniform electric field with magnitude $E = Q/2A\epsilon$ at nearby points. (b) Two large sheets of area A carrying opposite charges of equal magnitude Q are a small distance d apart. Show that they create a uniform electric field of magnitude $E = Q/A\epsilon$ between them. (c) Assume that the negative plate is at zero potential. Show that the positive plate is at a potential $Qd/A\epsilon$. (d) Show that the capacitance of the pair of plates is $A\epsilon/d = \kappa A\epsilon_0/d$.

ADDITIONAL PROBLEMS

54. For the system of capacitors shown in Figure P26.54, find (a) the equivalent capacitance of the system, (b) the potential difference across each capacitor, (c) the charge on each capacitor, and (d) the total energy stored by the group.

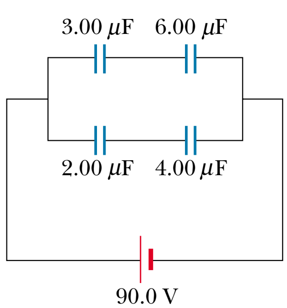


Figure P26.54

55. Consider two *long*, parallel, and oppositely charged wires of radius d with their centers separated by a distance D. Assuming the charge is distributed uniformly on the surface of each wire, show that the capacitance per unit length of this pair of wires is

$$\frac{C}{\ell} = \frac{\pi\epsilon_0}{\ln\left(\dfrac{D-d}{d}\right)}$$

56. A 2.00-nF parallel-plate capacitor is charged to an initial potential difference $\Delta V_i = 100$ V and then isolated. The dielectric material between the plates is mica ($\kappa = 5.00$). (a) How much work is required to withdraw the mica sheet? (b) What is the potential difference of the capacitor after the mica is withdrawn?

WEB 57. A parallel-plate capacitor is constructed using a dielectric material whose dielectric constant is 3.00 and whose dielectric strength is 2.00×10^8 V/m. The desired capacitance is 0.250 μF, and the capacitor must withstand a maximum potential difference of 4 000 V. Find the minimum area of the capacitor plates.

58. A parallel-plate capacitor is constructed using three dielectric materials, as shown in Figure P26.58. You may assume that $\ell \gg d$. (a) Find an expression for the capacitance of the device in terms of the plate area A and d, κ_1, κ_2, and κ_3. (b) Calculate the capacitance using the values $A = 1.00$ cm^2, $d = 2.00$ mm, $\kappa_1 = 4.90$, $\kappa_2 = 5.60$, and $\kappa_3 = 2.10$.

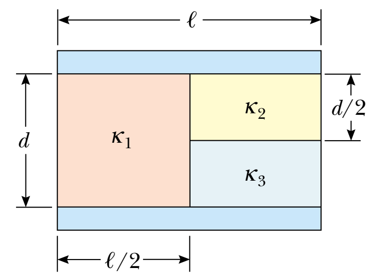


Figure P26.58

59. A conducting slab of thickness d and area A is inserted into the space between the plates of a parallel-plate capacitor with spacing s and surface area A, as shown in Figure P26.59. The slab is not necessarily halfway between the capacitor plates. What is the capacitance of the system?

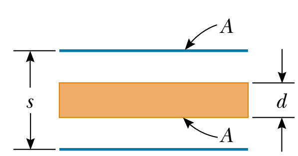


Figure P26.59

60. (a) Two spheres have radii a and b and their centers are a distance d apart. Show that the capacitance of this system is

$$C \approx \frac{4\pi\epsilon_0}{\dfrac{1}{a} + \dfrac{1}{b} - \dfrac{2}{d}}$$

provided that d is large compared with a and b. (*Hint:* Because the spheres are far apart, assume that the charge on one sphere does not perturb the charge distribution on the other sphere. Thus, the potential of each sphere is expressed as that of a symmetric charge distribution, $V = k_eQ/r$, and the total potential at each sphere is the sum of the potentials due to each sphere. (b) Show that as d approaches infinity the above result reduces to that of two isolated spheres in series.

61. When a certain air-filled parallel-plate capacitor is connected across a battery, it acquires a charge (on each plate) of q_0. While the battery connection is maintained, a dielectric slab is inserted into and fills the region between the plates. This results in the accumulation of an *additional* charge q on each plate. What is the dielectric constant of the slab?

62. A capacitor is constructed from two square plates of sides ℓ and separation d. A material of dielectric constant κ is inserted a distance x into the capacitor, as shown in Figure P26.62. (a) Find the equivalent capacitance of the device. (b) Calculate the energy stored in the capacitor if the potential difference is ΔV. (c) Find the direction and magnitude of the force exerted on the dielectric, assuming a constant potential difference ΔV. Neglect friction. (d) Obtain a numerical value for the force assuming that $\ell = 5.00$ cm, $\Delta V = 2\,000$ V, $d = 2.00$ mm, and the dielectric is glass ($\kappa = 4.50$). (*Hint:* The system can be considered as two capacitors connected in *parallel.*)

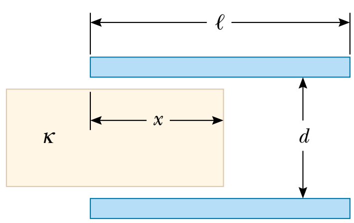


Figure P26.62 Problems 62 and 63.

63. A capacitor is constructed from two square plates of sides ℓ and separation d, as suggested in Figure P26.62. You may assume that d is much less than ℓ. The plates carry charges $+Q_0$ and $-Q_0$. A block of metal has a width ℓ, a length ℓ, and a thickness slightly less than d. It is inserted a distance x into the capacitor. The charges on the plates are not disturbed as the block slides in. In a static situation, a metal prevents an electric field from penetrating it. The metal can be thought of as a perfect dielectric, with $\kappa \to \infty$. (a) Calculate the stored energy as a function of x. (b) Find the direction and magnitude of the force that acts on the metallic block. (c) The area of the advancing front face of the block is essentially equal to ℓd. Considering the force on the block as acting on this face, find the stress (force per area) on it. (d) For comparison, express the energy density in the electric field between the capacitor plates in terms of Q_0, ℓ, d, and ϵ_0.

64. When considering the energy supply for an automobile, the energy per unit mass of the energy source is an important parameter. Using the following data, compare the energy per unit mass (J/kg) for gasoline, lead–acid batteries, and capacitors. (The ampere A will be introduced in Chapter 27 and is the SI unit of electric current. 1 A = 1 C/s.)

Gasoline: 126 000 Btu/gal; density = 670 kg/m^3
Lead–acid battery: 12.0 V; 100 A·h; mass = 16.0 kg
Capacitor: potential difference at full charge = 12.0 V; capacitance = 0.100 F; mass = 0.100 kg

65. An isolated capacitor of unknown capacitance has been charged to a potential difference of 100 V. When the charged capacitor is then connected in parallel to an uncharged 10.0-μF capacitor, the voltage across the combination is 30.0 V. Calculate the unknown capacitance.

66. A certain electronic circuit calls for a capacitor having a capacitance of 1.20 pF and a breakdown potential of 1 000 V. If you have a supply of 6.00-pF capacitors, each having a breakdown potential of 200 V, how could you meet this circuit requirement?

67. In the arrangement shown in Figure P26.67, a potential difference ΔV is applied, and C_1 is adjusted so that the voltmeter between points b and d reads zero. This "balance" occurs when $C_1 = 4.00\ \mu$F. If $C_3 = 9.00\ \mu$F and $C_4 = 12.0\ \mu$F, calculate the value of C_2.

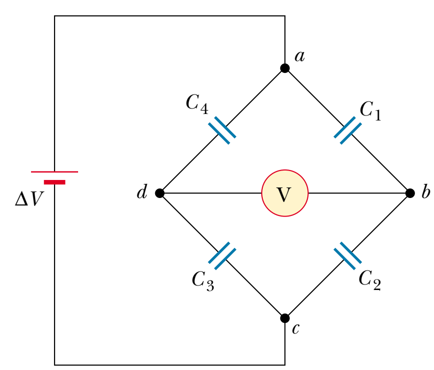


Figure P26.67

68. It is possible to obtain large potential differences by first charging a group of capacitors connected in parallel and then activating a switch arrangement that in effect disconnects the capacitors from the charging source and from each other and reconnects them in a series arrangement. The group of charged capacitors is then discharged in series. What is the maximum potential difference that can be obtained in this manner by using ten capacitors each of 500 μF and a charging source of 800 V?

69. A parallel-plate capacitor of plate separation d is charged to a potential difference ΔV_0. A dielectric slab of thickness d and dielectric constant κ is introduced between the plates *while the battery remains connected to the plates.* (a) Show that the ratio of energy stored after the dielectric is introduced to the energy stored in the empty capacitor is $U/U_0 = \kappa$. Give a physical explanation for this increase in stored energy. (b) What happens to the charge on the capacitor? (Note that this situation is not the same as Example 26.7, in which the battery was removed from the circuit before the dielectric was introduced.)

70. A parallel-plate capacitor with plates of area A and plate separation d has the region between the plates filled with two dielectric materials as in Figure P26.70. Assume that $d \ll L$ and that $d \ll W$. (a) Determine the capacitance and (b) show that when $\kappa_1 = \kappa_2 = \kappa$ your result becomes the same as that for a capacitor containing a single dielectric, $C = \kappa\epsilon_0 A/d$.

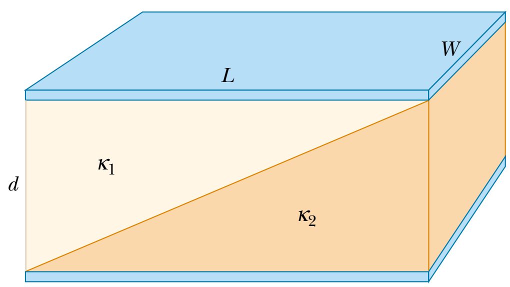


Figure P26.70

71. A vertical parallel-plate capacitor is half filled with a dielectric for which the dielectric constant is 2.00 (Fig. P26.71a). When this capacitor is positioned horizontally, what fraction of it should be filled with the same dielectric (Fig. P26.71b) so that the two capacitors have equal capacitance?

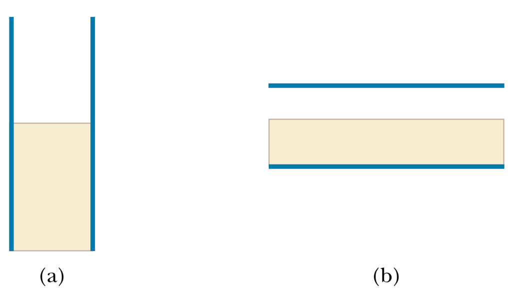


Figure P26.71

72. Capacitors $C_1 = 6.00\ \mu F$ and $C_2 = 2.00\ \mu F$ are charged as a parallel combination across a 250-V battery. The capacitors are disconnected from the battery and from each other. They are then connected positive plate to negative plate and negative plate to positive plate. Calculate the resulting charge on each capacitor.

73. The inner conductor of a coaxial cable has a radius of 0.800 mm, and the outer conductor's inside radius is 3.00 mm. The space between the conductors is filled with polyethylene, which has a dielectric constant of 2.30 and a dielectric strength of 18.0×10^6 V/m. What is the maximum potential difference that this cable can withstand?

74. You are optimizing coaxial cable design for a major manufacturer. Show that for a given outer conductor radius b, maximum potential difference capability is attained when the radius of the inner conductor is $a = b/e$ where e is the base of natural logarithms.

75. Calculate the equivalent capacitance between the points a and b in Figure P26.75. Note that this is not a simple series or parallel combination. (*Hint:* Assume a potential difference ΔV between points a and b. Write expressions for ΔV_{ab} in terms of the charges and capacitances for the various possible pathways from a to b, and require conservation of charge for those capacitor plates that are connected to each other.)

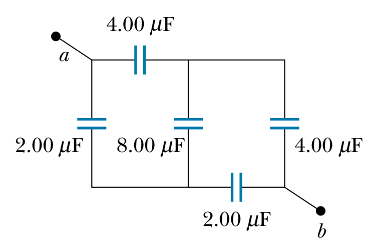


Figure P26.75

76. Determine the effective capacitance of the combination shown in Figure P26.76. (*Hint:* Consider the symmetry involved!)

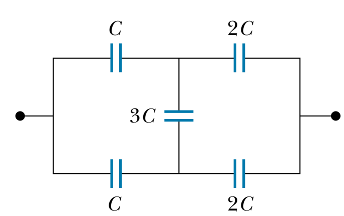


Figure P26.76

ANSWERS TO QUICK QUIZZES

26.1 (a) because the plate separation is decreased. Capacitance depends only on how a capacitor is constructed and not on the external circuit.

26.2 Zero. If you construct a spherical gaussian surface outside and concentric with the capacitor, the net charge inside the surface is zero. Applying Gauss's law to this configuration, we find that $E = 0$ at points outside the capacitor.

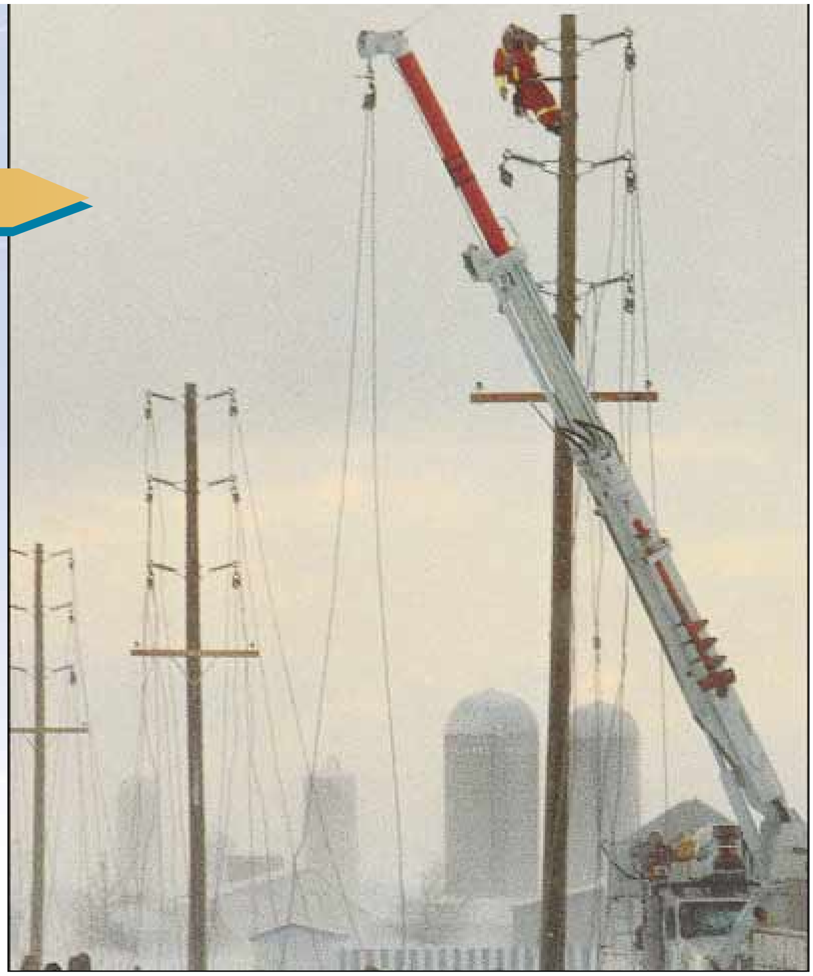
26.3 For a given voltage, the energy stored in a capacitor is proportional to C: $U = C(\Delta V)^2/2$. Thus, you want to maximize the equivalent capacitance. You do this by connecting the three capacitors in parallel, so that the capacitances add.

26.4 (a) C decreases (Eq. 26.3). (b) Q stays the same because there is no place for the charge to flow. (c) E remains constant (see Eq. 24.8 and the paragraph following it). (d) ΔV increases because $\Delta V = Q/C$, Q is constant (part b), and C decreases (part a). (e) The energy stored in the capacitor is proportional to both Q and ΔV (Eq. 26.11) and thus increases. The additional energy comes from the work you do in pulling the two plates apart.

26.5 (a) C decreases (Eq. 26.3). (b) Q decreases. The battery supplies a constant potential difference ΔV; thus, charge must flow out of the capacitor if $C = Q/\Delta V$ is to decrease. (c) E decreases because the charge density on the plates decreases. (d) ΔV remains constant because of the presence of the battery. (e) The energy stored in the capacitor decreases (Eq. 26.11).

26.6 It increases. The dielectric constant of wood (and of all other insulating materials, for that matter) is greater than 1; therefore, the capacitance increases (Eq. 26.14). This increase is sensed by the stud-finder's special circuitry, which causes an indicator on the device to light up.

26.7 (a) C increases (Eq. 26.14). (b) Q increases. Because the battery maintains a constant ΔV, Q must increase if $C\ (= Q/\Delta V)$ increases. (c) E between the plates remains constant because $\Delta V = Ed$ and neither ΔV nor d changes. The electric field due to the charges on the plates increases because more charge has flowed onto the plates. The induced surface charges on the dielectric create a field that opposes the increase in the field caused by the greater number of charges on the plates. (d) The battery maintains a constant ΔV. (e) The energy stored in the capacitor increases (Eq. 26.11). You would have to push the dielectric into the capacitor, just as you would have to do positive work to raise a mass and increase its gravitational potential energy.



PUZZLER

Electrical workers restoring power to the eastern Ontario town of St. Isadore, which was without power for several days in January 1998 because of a severe ice storm. It is very dangerous to touch fallen power transmission lines because of their high electric potential, which might be hundreds of thousands of volts relative to the ground. Why is such a high potential difference used in power transmission if it is so dangerous, and why aren't birds that perch on the wires electrocuted? *(AP/Wide World Photos/Fred Chartrand)*

chapter

27 Current and Resistance

Chapter Outline

Thus far our treatment of electrical phenomena has been confined to the study of charges at rest, or *electrostatics.* We now consider situations involving electric charges in motion. We use the term *electric current,* or simply *current,* to describe the rate of flow of charge through some region of space. Most practical applications of electricity deal with electric currents. For example, the battery in a flashlight supplies current to the filament of the bulb when the switch is turned on. A variety of home appliances operate on alternating current. In these common situations, the charges flow through a conductor, such as a copper wire. It also is possible for currents to exist outside a conductor. For instance, a beam of electrons in a television picture tube constitutes a current.

This chapter begins with the definitions of current and current density. A microscopic description of current is given, and some of the factors that contribute to the resistance to the flow of charge in conductors are discussed. A classical model is used to describe electrical conduction in metals, and some of the limitations of this model are cited.

27.1 ELECTRIC CURRENT

13.2 It is instructive to draw an analogy between water flow and current. In many localities it is common practice to install low-flow showerheads in homes as a water-conservation measure. We quantify the flow of water from these and similar devices by specifying the amount of water that emerges during a given time interval, which is often measured in liters per minute. On a grander scale, we can characterize a river current by describing the rate at which the water flows past a particular location. For example, the flow over the brink at Niagara Falls is maintained at rates between 1 400 m^3/s and 2 800 m^3/s.

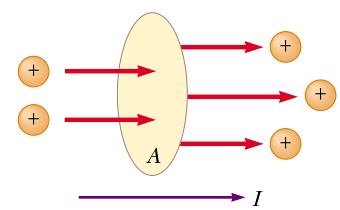


Figure 27.1 Charges in motion through an area A. The time rate at which charge flows through the area is defined as the current I. The direction of the current is the direction in which positive charges flow when free to do so.

Now consider a system of electric charges in motion. Whenever there is a net flow of charge through some region, a **current** is said to exist. To define current more precisely, suppose that the charges are moving perpendicular to a surface of area A, as shown in Figure 27.1. (This area could be the cross-sectional area of a wire, for example.) **The current is the rate at which charge flows through this surface.** If ΔQ is the amount of charge that passes through this area in a time interval Δt, the **average current** I_{av} is equal to the charge that passes through A per unit time:

$$I_{av} = \frac{\Delta Q}{\Delta t} \quad \textbf{(27.1)}$$

If the rate at which charge flows varies in time, then the current varies in time; we define the **instantaneous current** I as the differential limit of average current:

Electric current

$$I \equiv \frac{dQ}{dt} \quad \textbf{(27.2)}$$

The SI unit of current is the **ampere** (A):

$$1\ \text{A} = \frac{1\ \text{C}}{1\ \text{s}} \quad \textbf{(27.3)}$$

That is, 1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.

The direction of the current

The charges passing through the surface in Figure 27.1 can be positive or negative, or both. **It is conventional to assign to the current the same direction as the flow of positive charge.** In electrical conductors, such as copper or aluminum, the current is due to the motion of negatively charged electrons. Therefore, when we speak of current in an ordinary conductor, **the direction of the current is opposite the direction of flow of electrons.** However, if we are considering a beam of positively charged protons in an accelerator, the current is in the direction of motion of the protons. In some cases—such as those involving gases and electrolytes, for instance—the current is the result of the flow of both positive and negative charges.

If the ends of a conducting wire are connected to form a loop, all points on the loop are at the same electric potential, and hence the electric field is zero within and at the surface of the conductor. Because the electric field is zero, there is no net transport of charge through the wire, and therefore there is no current. The current in the conductor is zero even if the conductor has an excess of charge on it. However, if the ends of the conducting wire are connected to a battery, all points on the loop are not at the same potential. The battery sets up a potential difference between the ends of the loop, creating an electric field within the wire. The electric field exerts forces on the conduction electrons in the wire, causing them to move around the loop and thus creating a current.

It is common to refer to a moving charge (positive or negative) as a mobile **charge carrier.** For example, the mobile charge carriers in a metal are electrons.

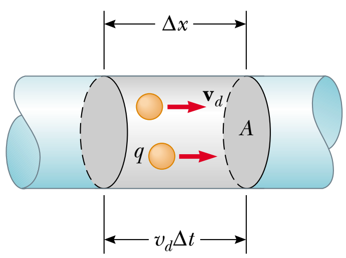


Figure 27.2 A section of a uniform conductor of cross-sectional area A. The mobile charge carriers move with a speed v_d, and the distance they travel in a time Δt is $\Delta x = v_d \Delta t$. The number of carriers in the section of length Δx is $nAv_d \Delta t$, where n is the number of carriers per unit volume.

Microscopic Model of Current

We can relate current to the motion of the charge carriers by describing a microscopic model of conduction in a metal. Consider the current in a conductor of cross-sectional area A (Fig. 27.2). The volume of a section of the conductor of length Δx (the gray region shown in Fig. 27.2) is $A\,\Delta x$. If n represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the gray section is $nA\,\Delta x$. Therefore, the charge ΔQ in this section is

$$\Delta Q = \text{number of carriers in section} \times \text{charge per carrier} = (nA\,\Delta x)q$$

where q is the charge on each carrier. If the carriers move with a speed v_d, the distance they move in a time Δt is $\Delta x = v_d\,\Delta t$. Therefore, we can write ΔQ in the form

$$\Delta Q = (nAv_d\,\Delta t)q$$

If we divide both sides of this equation by Δt, we see that the average current in the conductor is

Average current in a conductor

$$I_{\text{av}} = \frac{\Delta Q}{\Delta t} = nqv_dA \tag{27.4}$$

The speed of the charge carriers v_d is an average speed called the **drift speed.** To understand the meaning of drift speed, consider a conductor in which the charge carriers are free electrons. If the conductor is isolated—that is, the potential difference across it is zero—then these electrons undergo random motion that is analogous to the motion of gas molecules. As we discussed earlier, when a potential difference is applied across the conductor (for example, by means of a battery), an electric field is set up in the conductor; this field exerts an electric force on the electrons, producing a current. However, the electrons do not move in straight lines along the conductor. Instead, they collide repeatedly with the metal atoms, and their resultant motion is complicated and zigzag (Fig. 27.3). Despite the collisions, the electrons move slowly along the conductor (in a direction opposite that of $\mathbf{E}$) at the drift velocity $\mathbf{v}_d$.

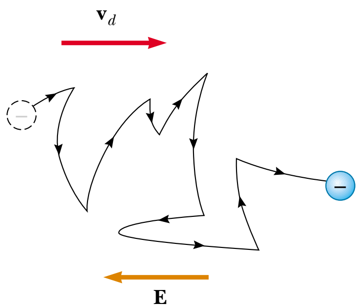


Figure 27.3 A schematic representation of the zigzag motion of an electron in a conductor. The changes in direction are the result of collisions between the electron and atoms in the conductor. Note that the net motion of the electron is opposite the direction of the electric field. Each section of the zigzag path is a parabolic segment.

We can think of the atom–electron collisions in a conductor as an effective internal friction (or drag force) similar to that experienced by the molecules of a liquid flowing through a pipe stuffed with steel wool. The energy transferred from the electrons to the metal atoms during collision causes an increase in the vibrational energy of the atoms and a corresponding increase in the temperature of the conductor.

Quick Quiz 27.1

Consider positive and negative charges moving horizontally through the four regions shown in Figure 27.4. Rank the current in these four regions, from lowest to highest.

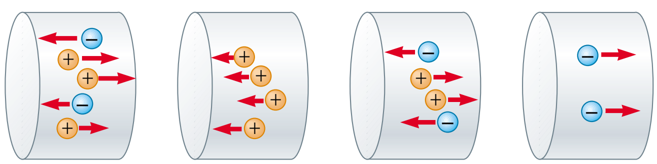


Figure 27.4

EXAMPLE 27.1 Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of 3.31×10^{-6} m^2. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is 8.95 g/cm^3.

Solution From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms (6.02×10^{23}). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (= 1 mol) of copper:

$$V = \frac{m}{\rho} = \frac{63.5\text{ g}}{8.95\text{ g/cm}^3} = 7.09\text{ cm}^3$$

Because each copper atom contributes one free electron to the current, we have

$$\begin{aligned} n &= \frac{6.02 \times 10^{23}\text{ electrons}}{7.09\text{ cm}^3}(1.00 \times 10^6\text{ cm}^3/\text{m}^3) \\ &= 8.49 \times 10^{28}\text{ electrons/m}^3 \end{aligned}$$

From Equation 27.4, we find that the drift speed is

$$v_d = \frac{I}{nqA}$$

where q is the absolute value of the charge on each electron. Thus,

$$\begin{aligned} v_d &= \frac{I}{nqA} \\ &= \frac{10.0\text{ C/s}}{(8.49 \times 10^{28}\text{ m}^{-3})(1.60 \times 10^{-19}\text{ C})(3.31 \times 10^{-6}\text{ m}^2)} \\ &= \boxed{2.22 \times 10^{-4}\text{ m/s}} \end{aligned}$$

Exercise If a copper wire carries a current of 80.0 mA, how many electrons flow past a given cross-section of the wire in 10.0 min?

Answer 3.0×10^{20} electrons.

Example 27.1 shows that typical drift speeds are very low. For instance, electrons traveling with a speed of 2.46×10^{-4} m/s would take about 68 min to travel 1 m! In view of this, you might wonder why a light turns on almost instantaneously when a switch is thrown. In a conductor, the electric field that drives the free electrons travels through the conductor with a speed close to that of light. Thus, when you flip on a light switch, the message for the electrons to start moving through the wire (the electric field) reaches them at a speed on the order of 10^8 m/s.

27.2 RESISTANCE AND OHM'S LAW

13.3

In Chapter 24 we found that no electric field can exist inside a conductor. However, this statement is true *only* if the conductor is in static equilibrium. The purpose of this section is to describe what happens when the charges in the conductor are allowed to move.

Charges moving in a conductor produce a current under the action of an electric field, which is maintained by the connection of a battery across the conductor. An electric field can exist in the conductor because the charges in this situation are in motion—that is, this is a *nonelectrostatic* situation.

Consider a conductor of cross-sectional area A carrying a current I. The **current density** J in the conductor is defined as the current per unit area. Because the current $I = nqv_dA$, the current density is

$$J \equiv \frac{I}{A} = nqv_d \tag{27.5}$$

where J has SI units of A/m^2. This expression is valid only if the current density is uniform and only if the surface of cross-sectional area A is perpendicular to the direction of the current. In general, the current density is a vector quantity:

Current density

$$\mathbf{J} = nq\mathbf{v}_d \tag{27.6}$$

From this equation, we see that current density, like current, is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers.

A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor. If the potential difference is constant, then the current also is constant. In some materials, the current density is proportional to the electric field:

Ohm's law

$$\mathbf{J} = \sigma\mathbf{E} \tag{27.7}$$

where the constant of proportionality σ is called the **conductivity** of the conductor.[1] Materials that obey Equation 27.7 are said to follow **Ohm's law,** named after Georg Simon Ohm (1787–1854). More specifically, Ohm's law states that

> for many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.

Materials that obey Ohm's law and hence demonstrate this simple relationship between **E** and **J** are said to be *ohmic.* Experimentally, it is found that not all materials have this property, however, and materials that do not obey Ohm's law are said to

[1] Do not confuse conductivity σ with surface charge density, for which the same symbol is used.

be *nonohmic.* Ohm's law is not a fundamental law of nature but rather an empirical relationship valid only for certain materials.

Quick Quiz 27.2

Suppose that a current-carrying ohmic metal wire has a cross-sectional area that gradually becomes smaller from one end of the wire to the other. How do drift velocity, current density, and electric field vary along the wire? Note that the current must have the same value everywhere in the wire so that charge does not accumulate at any one point.

We can obtain a form of Ohm's law useful in practical applications by considering a segment of straight wire of uniform cross-sectional area A and length ℓ, as shown in Figure 27.5. A potential difference $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current. If the field is assumed to be uniform, the potential difference is related to the field through the relationship[2]

$$\Delta V = E\ell$$

Therefore, we can express the magnitude of the current density in the wire as

$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

Because $J = I/A$, we can write the potential difference as

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I$$

The quantity $\ell/\sigma A$ is called the **resistance** R of the conductor. We can define the resistance as the ratio of the potential difference across a conductor to the current through the conductor:

$$R \equiv \frac{\ell}{\sigma A} \equiv \frac{\Delta V}{I} \tag{27.8}$$

Resistance of a conductor

From this result we see that resistance has SI units of volts per ampere. One volt per ampere is defined to be 1 **ohm** (Ω):

$$1\ \Omega \equiv \frac{1\ \text{V}}{1\ \text{A}} \tag{27.9}$$

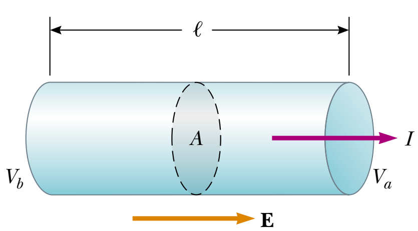


Figure 27.5 A uniform conductor of length ℓ and cross-sectional area A. A potential difference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field $\mathbf{E}$, and this field produces a current I that is proportional to the potential difference.

[2] This result follows from the definition of potential difference:

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s} = E\int_0^\ell dx = E\ell$$

An assortment of resistors used in electric circuits.

This expression shows that if a potential difference of 1 V across a conductor causes a current of 1 A, the resistance of the conductor is 1 Ω. For example, if an electrical appliance connected to a 120-V source of potential difference carries a current of 6 A, its resistance is 20 Ω.

Equation 27.8 solved for potential difference ($\Delta V = I\ell/\sigma A$) explains part of the chapter-opening puzzler: How can a bird perch on a high-voltage power line without being electrocuted? Even though the potential difference between the ground and the wire might be hundreds of thousands of volts, that between the bird's feet (which is what determines how much current flows through the bird) is very small.

The inverse of conductivity is **resistivity**[3] ρ:

Resistivity

$$\rho \equiv \frac{1}{\sigma} \tag{27.10}$$

where ρ has the units ohm-meters ($\Omega \cdot \mathrm{m}$). We can use this definition and Equation 27.8 to express the resistance of a uniform block of material as

Resistance of a uniform conductor

$$R = \rho \frac{\ell}{A} \tag{27.11}$$

Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature. Additionally, as you can see from Equation 27.11, the resistance of a sample depends on geometry as well as on resistivity. Table 27.1 gives the resistivities of a variety of materials at 20°C. Note the enormous range, from very low values for good conductors such as copper and silver, to very high values for good insulators such as glass and rubber. An ideal conductor would have zero resistivity, and an ideal insulator would have infinite resistivity.

Equation 27.11 shows that the resistance of a given cylindrical conductor is proportional to its length and inversely proportional to its cross-sectional area. If the length of a wire is doubled, then its resistance doubles. If its cross-sectional area is doubled, then its resistance decreases by one half. The situation is analogous to the flow of a liquid through a pipe. As the pipe's length is increased, the

[3] Do not confuse resistivity with mass density or charge density, for which the same symbol is used.

TABLE 27.1 Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[b]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\approx 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C.

[b] A nickel–chromium alloy commonly used in heating elements.

resistance to flow increases. As the pipe's cross-sectional area is increased, more liquid crosses a given cross-section of the pipe per unit time. Thus, more liquid flows for the same pressure differential applied to the pipe, and the resistance to flow decreases.

Most electric circuits use devices called **resistors** to control the current level in the various parts of the circuit. Two common types of resistors are the *composition resistor,* which contains carbon, and the *wire-wound resistor,* which consists of a coil of wire. Resistors' values in ohms are normally indicated by color-coding, as shown in Figure 27.6 and Table 27.2.

Ohmic materials have a linear current–potential difference relationship over a broad range of applied potential differences (Fig. 27.7a). The slope of the I-versus-ΔV curve in the linear region yields a value for $1/R$. Nonohmic materials

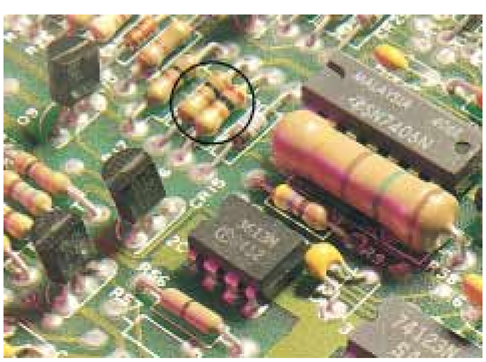


Figure 27.6 The colored bands on a resistor represent a code for determining resistance. The first two colors give the first two digits in the resistance value. The third color represents the power of ten for the multiplier of the resistance value. The last color is the tolerance of the resistance value. As an example, the four colors on the circled resistors are red (= 2), black (= 0), orange (= 10^3), and gold (= 5%), and so the resistance value is $20 \times 10^3\ \Omega = 20\ k\Omega$ with a tolerance value of $5\% = 1\ k\Omega$. (The values for the colors are from Table 27.2.)

TABLE 27.2 Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

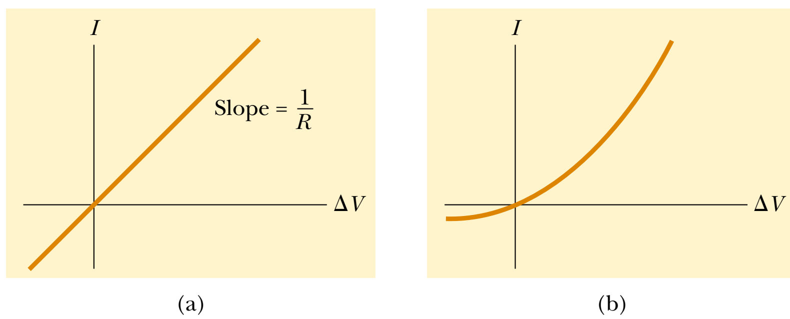


Figure 27.7 (a) The current–potential difference curve for an ohmic material. The curve is linear, and the slope is equal to the inverse of the resistance of the conductor. (b) A nonlinear current–potential difference curve for a semiconducting diode. This device does not obey Ohm's law.

have a nonlinear current–potential difference relationship. One common semiconducting device that has nonlinear I-versus-ΔV characteristics is the *junction diode* (Fig. 27.7b). The resistance of this device is low for currents in one direction (positive ΔV) and high for currents in the reverse direction (negative ΔV). In fact, most modern electronic devices, such as transistors, have nonlinear current–potential difference relationships; their proper operation depends on the particular way in which they violate Ohm's law.

Quick Quiz 27.3

What does the slope of the curved line in Figure 27.7b represent?

Quick Quiz 27.4

Your boss asks you to design an automobile battery jumper cable that has a low resistance. In view of Equation 27.11, what factors would you consider in your design?

EXAMPLE 27.2 The Resistance of a Conductor

Calculate the resistance of an aluminum cylinder that is 10.0 cm long and has a cross-sectional area of $2.00 \times 10^{-4}\ \text{m}^2$. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10}\ \Omega \cdot \text{m}$.

Solution From Equation 27.11 and Table 27.1, we can calculate the resistance of the aluminum cylinder as follows:

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega \cdot \text{m})\left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2}\right)$$

$$= \boxed{1.41 \times 10^{-5}\ \Omega}$$

Similarly, for glass we find that

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega \cdot \text{m})\left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2}\right)$$

$$= \boxed{1.5 \times 10^{13}\ \Omega}$$

As you might guess from the large difference in resistivities, the resistance of identically shaped cylinders of aluminum and glass differ widely. The resistance of the glass cylinder is 18 orders of magnitude greater than that of the aluminum cylinder.

Electrical insulators on telephone poles are often made of glass because of its low electrical conductivity.

EXAMPLE 27.3 The Resistance of Nichrome Wire

(a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

Solution The cross-sectional area of this wire is

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega \cdot \text{m}$ (see Table 27.1). Thus, we can use Equation 27.11 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega \cdot \text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = \boxed{4.6\ \Omega/\text{m}}$$

(b) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Solution Because a 1.0-m length of this wire has a resistance of 4.6 Ω, Equation 27.8 gives

$$I = \frac{\Delta V}{R} = \frac{10\ \text{V}}{4.6\ \Omega} = \boxed{2.2\ \text{A}}$$

Note from Table 27.1 that the resistivity of Nichrome wire is about 100 times that of copper. A copper wire of the same radius would have a resistance per unit length of only 0.052 Ω/m. A 1.0-m length of copper wire of the same radius would carry the same current (2.2 A) with an applied potential difference of only 0.11 V.

Because of its high resistivity and its resistance to oxidation, Nichrome is often used for heating elements in toasters, irons, and electric heaters.

Exercise What is the resistance of a 6.0-m length of 22-gauge Nichrome wire? How much current does the wire carry when connected to a 120-V source of potential difference?

Answer 28 Ω; 4.3 A.

Exercise Calculate the current density and electric field in the wire when it carries a current of 2.2 A.

Answer $6.8 \times 10^6\ \text{A/m}^2$; 10 N/C.

EXAMPLE 27.4 The Radial Resistance of a Coaxial Cable

Coaxial cables are used extensively for cable television and other electronic applications. A coaxial cable consists of two cylindrical conductors. The gap between the conductors is completely filled with silicon, as shown in Figure 27.8a, and current leakage through the silicon is unwanted. (The cable is designed to conduct current along its length.) The radius of the inner conductor is $a = 0.500$ cm, the radius of the outer one is $b = 1.75$ cm, and the length of the cable is $L = 15.0$ cm. Calculate the resistance of the silicon between the two conductors.

Solution In this type of problem, we must divide the object whose resistance we are calculating into concentric elements of infinitesimal thickness dr (Fig. 27.8b). We start by using the differential form of Equation 27.11, replacing ℓ with r for the distance variable: $dR = \rho\, dr/A$, where dR is the resistance of an element of silicon of thickness dr and surface area A. In this example, we take as our representative concentric element a hollow silicon cylinder of radius r, thickness dr, and length L, as shown in Figure 27.8. Any current that passes from the inner conductor to the outer one must pass radially through this concentric element, and the area through which this current passes is $A = 2\pi rL$. (This is the curved surface area—circumference multiplied by length—of our hollow silicon cylinder of thickness dr.) Hence, we can write the resistance of our hollow cylinder of silicon as

$$dR = \frac{\rho}{2\pi rL}\, dr$$

Because we wish to know the total resistance across the entire thickness of the silicon, we must integrate this expression from $r = a$ to $r = b$:

$$R = \int_a^b dR = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$

Substituting in the values given, and using $\rho = 640\ \Omega\cdot\text{m}$ for silicon, we obtain

$$R = \frac{640\ \Omega\cdot\text{m}}{2\pi(0.150\ \text{m})}\ln\left(\frac{1.75\ \text{cm}}{0.500\ \text{cm}}\right) = \boxed{851\ \Omega}$$

Exercise If a potential difference of 12.0 V is applied between the inner and outer conductors, what is the value of the total current that passes between them?

Answer 14.1 mA.

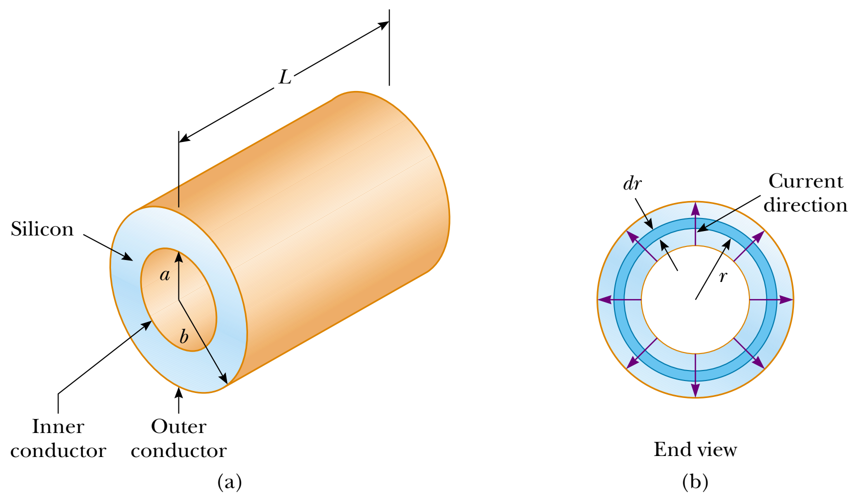


Figure 27.8 A coaxial cable. (a) Silicon fills the gap between the two conductors. (b) End view, showing current leakage.

27.3 A MODEL FOR ELECTRICAL CONDUCTION

In this section we describe a classical model of electrical conduction in metals that was first proposed by Paul Drude in 1900. This model leads to Ohm's law and shows that resistivity can be related to the motion of electrons in metals. Although the Drude model described here does have limitations, it nevertheless introduces concepts that are still applied in more elaborate treatments.

Consider a conductor as a regular array of atoms plus a collection of free electrons, which are sometimes called *conduction* electrons. The conduction electrons, although bound to their respective atoms when the atoms are not part of a solid, gain mobility when the free atoms condense into a solid. In the absence of an electric field, the conduction electrons move in random directions through the conductor with average speeds of the order of 10^6 m/s. The situation is similar to the motion of gas molecules confined in a vessel. In fact, some scientists refer to conduction electrons in a metal as an *electron gas*. There is no current through the conductor in the absence of an electric field because the drift velocity of the free electrons is zero. That is, on the average, just as many electrons move in one direction as in the opposite direction, and so there is no net flow of charge.

This situation changes when an electric field is applied. Now, in addition to undergoing the random motion just described, the free electrons drift slowly in a direction opposite that of the electric field, with an average drift speed v_d that is much smaller (typically 10^{-4} m/s) than their average speed between collisions (typically 10^6 m/s).

Figure 27.9 provides a crude description of the motion of free electrons in a conductor. In the absence of an electric field, there is no net displacement after many collisions (Fig. 27.9a). An electric field $\mathbf{E}$ modifies the random motion and causes the electrons to drift in a direction opposite that of $\mathbf{E}$ (Fig. 27.9b). The slight curvature in the paths shown in Figure 27.9b results from the acceleration of the electrons between collisions, which is caused by the applied field.

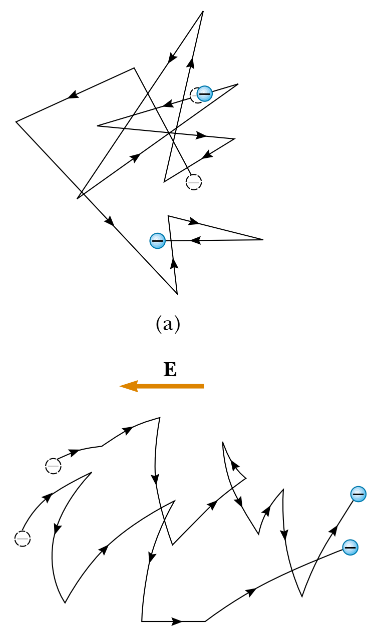


Figure 27.9 (a) A schematic diagram of the random motion of two charge carriers in a conductor in the absence of an electric field. The drift velocity is zero. (b) The motion of the charge carriers in a conductor in the presence of an electric field. Note that the random motion is modified by the field, and the charge carriers have a drift velocity.

In our model, we assume that the motion of an electron after a collision is independent of its motion before the collision. We also assume that the excess energy acquired by the electrons in the electric field is lost to the atoms of the conductor when the electrons and atoms collide. The energy given up to the atoms increases their vibrational energy, and this causes the temperature of the conductor to increase. The temperature increase of a conductor due to resistance is utilized in electric toasters and other familiar appliances.

We are now in a position to derive an expression for the drift velocity. When a free electron of mass m_e and charge $q\,(=-e)$ is subjected to an electric field $\mathbf{E}$, it experiences a force $\mathbf{F} = q\mathbf{E}$. Because $\Sigma\mathbf{F} = m_e\mathbf{a}$, we conclude that the acceleration of the electron is

$$\mathbf{a} = \frac{q\mathbf{E}}{m_e} \qquad \textbf{(27.12)}$$

This acceleration, which occurs for only a short time between collisions, enables the electron to acquire a small drift velocity. If t is the time since the last collision and $\mathbf{v}_i$ is the electron's initial velocity the instant after that collision, then the velocity of the electron after a time t is

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \mathbf{v}_i + \frac{q\mathbf{E}}{m_e}\,t \qquad \textbf{(27.13)}$$

We now take the average value of $\mathbf{v}_f$ over all possible times t and all possible values of $\mathbf{v}_i$. If we assume that the initial velocities are randomly distributed over all possible values, we see that the average value of $\mathbf{v}_i$ is zero. The term $(q\mathbf{E}/m_e)t$ is the velocity added by the field during one trip between atoms. If the electron starts with zero velocity, then the average value of the second term of Equation 27.13 is $(q\mathbf{E}/m_e)\tau$, where τ is the *average time interval between successive collisions*. Because the average value of $\mathbf{v}_f$ is equal to the drift velocity,[4] we have

Drift velocity

$$\overline{\mathbf{v}}_f = \mathbf{v}_d = \frac{q\mathbf{E}}{m_e}\,\tau \qquad \textbf{(27.14)}$$

[4] Because the collision process is random, each collision event is *independent* of what happened earlier. This is analogous to the random process of throwing a die. The probability of rolling a particular number on one throw is independent of the result of the previous throw. On average, the particular number comes up every sixth throw, starting at any arbitrary time.

We can relate this expression for drift velocity to the current in the conductor. Substituting Equation 27.14 into Equation 27.6, we find that the magnitude of the current density is

Current density

$$J = nqv_d = \frac{nq^2E}{m_e}\tau \tag{27.15}$$

where n is the number of charge carriers per unit volume. Comparing this expression with Ohm's law, $J = \sigma E$, we obtain the following relationships for conductivity and resistivity:

Conductivity

$$\sigma = \frac{nq^2\tau}{m_e} \tag{27.16}$$

Resistivity

$$\rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau} \tag{27.17}$$

According to this classical model, conductivity and resistivity do not depend on the strength of the electric field. This feature is characteristic of a conductor obeying Ohm's law.

The average time between collisions τ is related to the average distance between collisions ℓ (that is, the *mean free path;* see Section 21.7) and the average speed $\bar{v}$ through the expression

$$\tau = \frac{\ell}{\bar{v}} \tag{27.18}$$

EXAMPLE 27.5 Electron Collisions in a Wire

(a) Using the data and results from Example 27.1 and the classical model of electron conduction, estimate the average time between collisions for electrons in household copper wiring.

Solution From Equation 27.17, we see that

$$\tau = \frac{m_e}{nq^2\rho}$$

where $\rho = 1.7 \times 10^{-8}\ \Omega\cdot\text{m}$ for copper and the carrier density is $n = 8.49 \times 10^{28}$ electrons/m^3 for the wire described in Example 27.1. Substitution of these values into the expression above gives

$$\tau = \frac{(9.11 \times 10^{-31}\ \text{kg})}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})^2(1.7 \times 10^{-8}\ \Omega\cdot\text{m})}$$

$$= \boxed{2.5 \times 10^{-14}\ \text{s}}$$

(b) Assuming that the average speed for free electrons in copper is 1.6×10^6 m/s and using the result from part (a), calculate the mean free path for electrons in copper.

Solution

$$\ell = \bar{v}\tau = (1.6 \times 10^6\ \text{m/s})(2.5 \times 10^{-14}\ \text{s})$$

$$= \boxed{4.0 \times 10^{-8}\ \text{m}}$$

which is equivalent to 40 nm (compared with atomic spacings of about 0.2 nm). Thus, although the time between collisions is very short, an electron in the wire travels about 200 atomic spacings between collisions.

Although this classical model of conduction is consistent with Ohm's law, it is not satisfactory for explaining some important phenomena. For example, classical values for $\bar{v}$ calculated on the basis of an ideal-gas model (see Section 21.6) are smaller than the true values by about a factor of ten. Furthermore, if we substitute $\ell/\bar{v}$ for τ in Equation 27.17 and rearrange terms so that $\bar{v}$ appears in the numerator, we find that the resistivity ρ is proportional to $\bar{v}$. According to the ideal-gas model, $\bar{v}$ is proportional to $\sqrt{T}$; hence, it should also be true that $\rho \propto \sqrt{T}$. This is in disagreement with the fact that, for pure metals, resistivity depends linearly on temperature. We are able to account for the linear dependence only by using a quantum mechanical model, which we now describe briefly.

According to quantum mechanics, electrons have wave-like properties. If the array of atoms in a conductor is regularly spaced (that is, it is periodic), then the wave-like character of the electrons enables them to move freely through the conductor, and a collision with an atom is unlikely. For an idealized conductor, no collisions would occur, the mean free path would be infinite, and the resistivity would be zero. Electron waves are scattered only if the atomic arrangement is irregular (not periodic) as a result of, for example, structural defects or impurities. At low temperatures, the resistivity of metals is dominated by scattering caused by collisions between electrons and defects or impurities. At high temperatures, the resistivity is dominated by scattering caused by collisions between electrons and atoms of the conductor, which are continuously displaced from the regularly spaced array as a result of thermal agitation. The thermal motion of the atoms causes the structure to be irregular (compared with an atomic array at rest), thereby reducing the electron's mean free path.

27.4 RESISTANCE AND TEMPERATURE

Over a limited temperature range, the resistivity of a metal varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)] \tag{27.19}$$

Variation of ρ with temperature

where ρ is the resistivity at some temperature T (in degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually taken to be 20°C), and α is the **temperature coefficient of resistivity.** From Equation 27.19, we see that the temperature coefficient of resistivity can be expressed as

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T} \tag{27.20}$$

Temperature coefficient of resistivity

where $\Delta\rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

The temperature coefficients of resistivity for various materials are given in Table 27.1. Note that the unit for α is degrees Celsius^{-1} $[(°\text{C})^{-1}]$. Because resistance is proportional to resistivity (Eq. 27.11), we can write the variation of resistance as

$$R = R_0[1 + \alpha(T - T_0)] \tag{27.21}$$

Use of this property enables us to make precise temperature measurements, as shown in the following example.

EXAMPLE 27.6 A Platinum Resistance Thermometer

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Calculate the melting point of the indium.

Solution Solving Equation 27.21 for ΔT and using the α value for platinum given in Table 27.1, we obtain

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3}\ (°\text{C})^{-1}](50.0\ \Omega)} = 137°\text{C}$$

Because $T_0 = 20.0$°C, we find that T, the temperature of the melting indium sample, is 157°C.

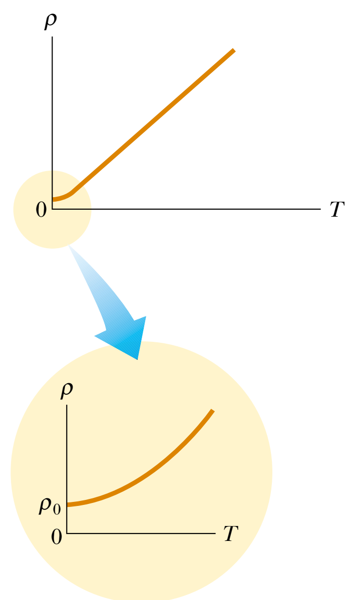


Figure 27.10 Resistivity versus temperature for a metal such as copper. The curve is linear over a wide range of temperatures, and ρ increases with increasing temperature. As T approaches absolute zero (inset), the resistivity approaches a finite value ρ_0.

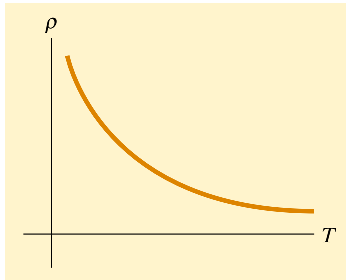


Figure 27.11 Resistivity versus temperature for a pure semiconductor, such as silicon or germanium.

For metals like copper, resistivity is nearly proportional to temperature, as shown in Figure 27.10. However, a nonlinear region always exists at very low temperatures, and the resistivity usually approaches some finite value as the temperature nears absolute zero. This residual resistivity near absolute zero is caused primarily by the collision of electrons with impurities and imperfections in the metal. In contrast, high-temperature resistivity (the linear region) is predominantly characterized by collisions between electrons and metal atoms.

Notice that three of the α values in Table 27.1 are negative; this indicates that the resistivity of these materials decreases with increasing temperature (Fig. 27.11). This behavior is due to an increase in the density of charge carriers at higher temperatures.

Because the charge carriers in a semiconductor are often associated with impurity atoms, the resistivity of these materials is very sensitive to the type and concentration of such impurities. We shall return to the study of semiconductors in Chapter 43 of the extended version of this text.

Quick Quiz 27.5

When does a lightbulb carry more current—just after it is turned on and the glow of the metal filament is increasing, or after it has been on for a few milliseconds and the glow is steady?

Optional Section

27.5 SUPERCONDUCTORS

There is a class of metals and compounds whose resistance decreases to zero when they are below a certain temperature T_c, known as the *critical temperature*. These materials are known as **superconductors.** The resistance–temperature graph for a superconductor follows that of a normal metal at temperatures above T_c (Fig. 27.12). When the temperature is at or below T_c, the resistivity drops suddenly to zero. This phenomenon was discovered in 1911 by the Dutch physicist Heike Kamerlingh-Onnes (1853–1926) as he worked with mercury, which is a superconductor below 4.2 K. Recent measurements have shown that the resistivities of superconductors below their T_c values are less than $4 \times 10^{-25}\ \Omega \cdot \text{m}$—around 10^{17} times smaller than the resistivity of copper and in practice considered to be zero.

Today thousands of superconductors are known, and as Figure 27.13 illustrates, the critical temperatures of recently discovered superconductors are substantially higher than initially thought possible. Two kinds of superconductors are recognized. The more recently identified ones, such as $YBa_2Cu_3O_7$, are essentially ceramics with high critical temperatures, whereas superconducting materials such

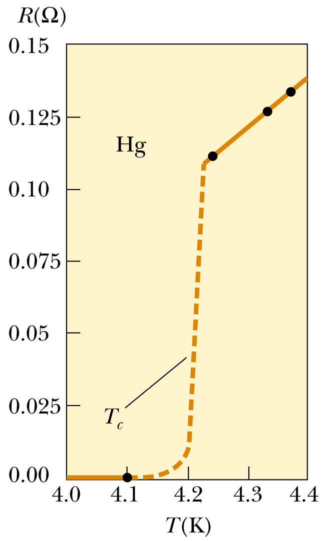


Figure 27.12 Resistance versus temperature for a sample of mercury (Hg). The graph follows that of a normal metal above the critical temperature T_c. The resistance drops to zero at T_c, which is 4.2 K for mercury.

A small permanent magnet levitated above a disk of the superconductor $YBa_2Cu_3O_7$, which is at 77 K.

as those observed by Kamerlingh-Onnes are metals. If a room-temperature superconductor is ever identified, its impact on technology could be tremendous.

The value of T_c is sensitive to chemical composition, pressure, and molecular structure. It is interesting to note that copper, silver, and gold, which are excellent conductors, do not exhibit superconductivity.

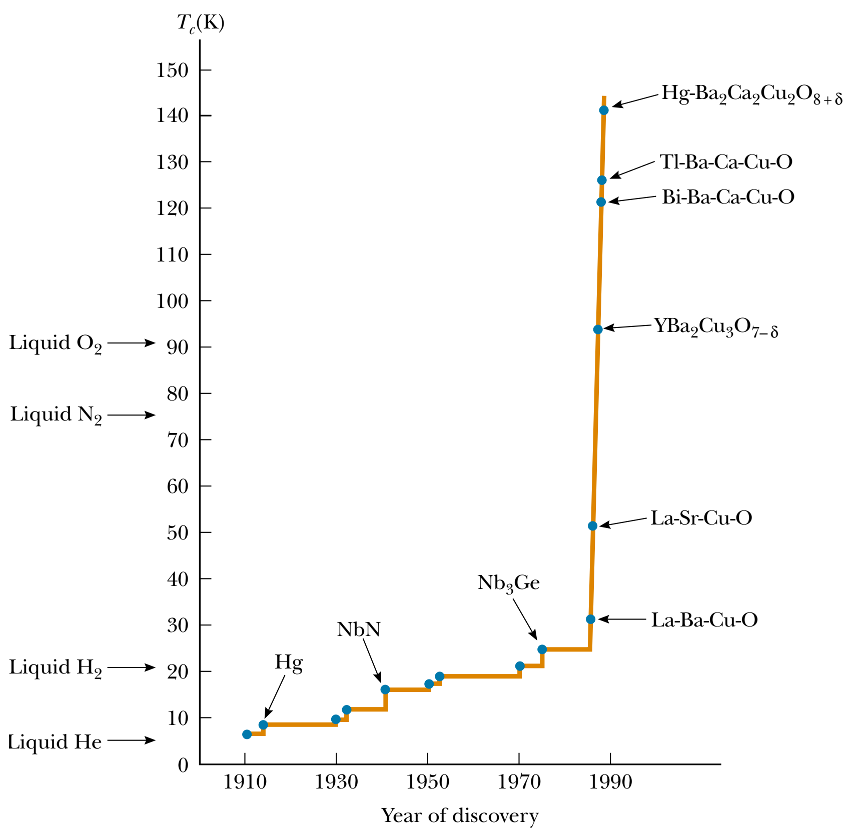


Figure 27.13 Evolution of the superconducting critical temperature since the discovery of the phenomenon.

One of the truly remarkable features of superconductors is that once a current is set up in them, it persists *without any applied potential difference* (because $R = 0$). Steady currents have been observed to persist in superconducting loops for several years with no apparent decay!

An important and useful application of superconductivity is in the development of superconducting magnets, in which the magnitudes of the magnetic field are about ten times greater than those produced by the best normal electromagnets. Such superconducting magnets are being considered as a means of storing energy. Superconducting magnets are currently used in medical magnetic resonance imaging (MRI) units, which produce high-quality images of internal organs without the need for excessive exposure of patients to x-rays or other harmful radiation.

For further information on superconductivity, see Section 43.8.

27.6 ELECTRICAL ENERGY AND POWER

13.3

If a battery is used to establish an electric current in a conductor, the chemical energy stored in the battery is continuously transformed into kinetic energy of the charge carriers. In the conductor, this kinetic energy is quickly lost as a result of collisions between the charge carriers and the atoms making up the conductor, and this leads to an increase in the temperature of the conductor. In other words, the chemical energy stored in the battery is continuously transformed to internal energy associated with the temperature of the conductor.

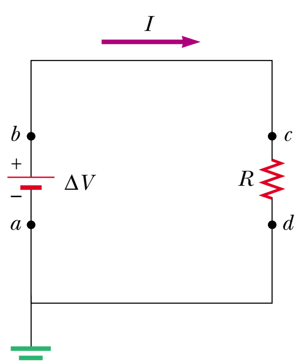


Figure 27.14 A circuit consisting of a resistor of resistance R and a battery having a potential difference ΔV across its terminals. Positive charge flows in the clockwise direction. Points a and d are grounded.

Consider a simple circuit consisting of a battery whose terminals are connected to a resistor, as shown in Figure 27.14. (Resistors are designated by the symbol .) Now imagine following a positive quantity of charge ΔQ that is moving clockwise around the circuit from point a through the battery and resistor back to point a. Points a and d are *grounded* (ground is designated by the symbol); that is, we take the electric potential at these two points to be zero. As the charge moves from a to b through the battery, its electric potential energy U *increases* by an amount $\Delta V \Delta Q$ (where ΔV is the potential difference between b and a), while the chemical potential energy in the battery *decreases* by the same amount. (Recall from Eq. 25.9 that $\Delta U = q \Delta V$.) However, as the charge moves from c to d through the resistor, it *loses* this electric potential energy as it collides with atoms in the resistor, thereby producing internal energy. If we neglect the resistance of the connecting wires, no loss in energy occurs for paths bc and da. When the charge arrives at point a, it must have the same electric potential energy (zero) that it had at the start.[5] Note that because charge cannot build up at any point, the current is the same everywhere in the circuit.

The rate at which the charge ΔQ loses potential energy in going through the resistor is

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t} \Delta V = I \Delta V$$

where I is the current in the circuit. In contrast, the charge regains this energy when it passes through the battery. Because the rate at which the charge loses energy equals the power $\mathcal{P}$ delivered to the resistor (which appears as internal energy), we have

Power

$$\mathcal{P} = I \Delta V \qquad \textbf{(27.22)}$$

[5] Note that once the current reaches its steady-state value, there is *no* change in the kinetic energy of the charge carriers creating the current.

In this case, the power is supplied to a resistor by a battery. However, we can use Equation 27.22 to determine the power transferred to *any* device carrying a current I and having a potential difference ΔV between its terminals.

Using Equation 27.22 and the fact that $\Delta V = IR$ for a resistor, we can express the power delivered to the resistor in the alternative forms

$$\mathcal{P} = I^2R = \frac{(\Delta V)^2}{R} \quad \textbf{(27.23)}$$

Power delivered to a resistor

When I is expressed in amperes, ΔV in volts, and R in ohms, the SI unit of power is the watt, as it was in Chapter 7 in our discussion of mechanical power. The power lost as internal energy in a conductor of resistance R is called *joule heating*[6]; this transformation is also often referred to as an I^2R loss.

A battery, a device that supplies electrical energy, is called either a *source of electromotive force* or, more commonly, an *emf source.* The concept of emf is discussed in greater detail in Chapter 28. (The phrase *electromotive force* is an unfortunate choice because it describes not a force but rather a potential difference in volts.) **When the internal resistance of the battery is neglected, the potential difference between points a and b in Figure 27.14 is equal to the emf $\mathcal{E}$ of the battery**—that is, $\Delta V = V_b - V_a = \mathcal{E}$. This being true, we can state that the current in the circuit is $I = \Delta V/R = \mathcal{E}/R$. Because $\Delta V = \mathcal{E}$, the power supplied by the emf source can be expressed as $\mathcal{P} = I\mathcal{E}$, which equals the power delivered to the resistor, I^2R.

When transporting electrical energy through power lines, such as those shown in Figure 27.15, utility companies seek to minimize the power transformed to internal energy in the lines and maximize the energy delivered to the consumer. Because $\mathcal{P} = I\Delta V$, the same amount of power can be transported either at high currents and low potential differences or at low currents and high potential differences. Utility companies choose to transport electrical energy at low currents and high potential differences primarily for economic reasons. Copper wire is very expensive, and so it is cheaper to use high-resistance wire (that is, wire having a small cross-sectional area; see Eq. 27.11). Thus, in the expression for the power delivered to a resistor, $\mathcal{P} = I^2R$, the resistance of the wire is fixed at a relatively high value for economic considerations. The I^2R loss can be reduced by keeping the current I as low as possible. In some instances, power is transported at potential differences as great as 765 kV. Once the electricity reaches your city, the potential difference is usually reduced to 4 kV by a device called a *transformer.* Another transformer drops the potential difference to 240 V before the electricity finally reaches your home. Of course, each time the potential difference decreases, the current increases by the same factor, and the power remains the same. We shall discuss transformers in greater detail in Chapter 33.

Figure 27.15 Power companies transfer electrical energy at high potential differences.

Quick Quiz 27.6

The same potential difference is applied to the two lightbulbs shown in Figure 27.16. Which one of the following statements is true?

(a) The 30-W bulb carries the greater current and has the higher resistance.
(b) The 30-W bulb carries the greater current, but the 60-W bulb has the higher resistance.

QuickLab

If you have access to an ohmmeter, verify your answer to Quick Quiz 27.6 by testing the resistance of a few lightbulbs.

[6] It is called *joule heating* even though the process of heat does not occur. This is another example of incorrect usage of the word *heat* that has become entrenched in our language.

Figure 27.16 These lightbulbs operate at their rated power only when they are connected to a 120-V source.

(c) The 30-W bulb has the higher resistance, but the 60-W bulb carries the greater current.
(d) The 60-W bulb carries the greater current and has the higher resistance.

QuickLab

From the labels on household appliances such as hair dryers, televisions, and stereos, estimate the annual cost of operating them.

Quick Quiz 27.7

For the two lightbulbs shown in Figure 27.17, rank the current values at points *a* through *f*, from greatest to least.

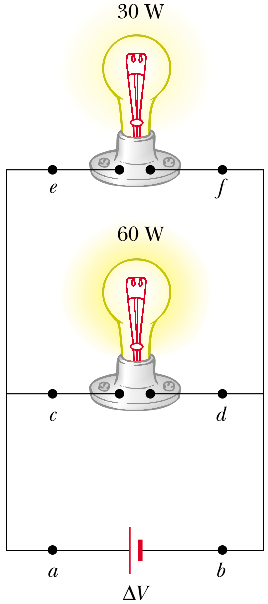


Figure 27.17 Two lightbulbs connected across the same potential difference. The bulbs operate at their rated power only if they are connected to a 120-V battery.

EXAMPLE 27.7 Power in an Electric Heater

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

Solution Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120\text{ V}}{8.00\ \Omega} = \boxed{15.0\text{ A}}$$

We can find the power rating using the expression $\mathcal{P} = I^2R$:

$$\mathcal{P} = I^2R = (15.0\text{ A})^2(8.00\ \Omega) = \boxed{1.80\text{ kW}}$$

If we doubled the applied potential difference, the current would double but the power would quadruple because $\mathcal{P} = (\Delta V)^2/R$.

EXAMPLE 27.8 The Cost of Making Dinner

Estimate the cost of cooking a turkey for 4 h in an oven that operates continuously at 20.0 A and 240 V.

Solution The power used by the oven is

$$\mathcal{P} = I\Delta V = (20.0\text{ A})(240\text{ V}) = 4\,800\text{ W} = 4.80\text{ kW}$$

Because the energy consumed equals power × time, the amount of energy for which you must pay is

$$\text{Energy} = \mathcal{P}t = (4.80\text{ kW})(4\text{ h}) = 19.2\text{ kWh}$$

If the energy is purchased at an estimated price of 8.00¢ per kilowatt hour, the cost is

$$\text{Cost} = (19.2\text{ kWh})(\$0.080/\text{kWh}) = \boxed{\$1.54}$$

Demands on our dwindling energy supplies have made it necessary for us to be aware of the energy requirements of our electrical devices. Every electrical appliance carries a label that contains the information you need to calculate the appliance's power requirements. In many cases, the power consumption in watts is stated directly, as it is on a lightbulb. In other cases, the amount of current used by the device and the potential difference at which it operates are given. This information and Equation 27.22 are sufficient for calculating the operating cost of any electrical device.

Exercise What does it cost to operate a 100-W lightbulb for 24 h if the power company charges $0.08/kWh?

Answer $0.19.

EXAMPLE 27.9 Current in an Electron Beam

In a certain particle accelerator, electrons emerge with an energy of 40.0 MeV ($1\text{ MeV} = 1.60 \times 10^{-13}\text{ J}$). The electrons emerge not in a steady stream but rather in pulses at the rate of 250 pulses/s. This corresponds to a time between pulses of 4.00 ms (Fig. 27.18). Each pulse has a duration of 200 ns, and the electrons in the pulse constitute a current of 250 mA. The current is zero between pulses. (a) How many electrons are delivered by the accelerator per pulse?

Solution We use Equation 27.2 in the form $dQ = I\,dt$ and integrate to find the charge per pulse. While the pulse is on, the current is constant; thus,

$$Q_{\text{pulse}} = I\int dt = I\Delta t = (250 \times 10^{-3}\text{ A})(200 \times 10^{-9}\text{ s})$$
$$= 5.00 \times 10^{-8}\text{ C}$$

Dividing this quantity of charge per pulse by the electronic charge gives the number of electrons per pulse:

$$\text{Electrons per pulse} = \frac{5.00 \times 10^{-8}\text{ C/pulse}}{1.60 \times 10^{-19}\text{ C/electron}}$$
$$= \boxed{3.13 \times 10^{11}\text{ electrons/pulse}}$$

(b) What is the average current per pulse delivered by the accelerator?

Solution Average current is given by Equation 27.1, $I_{\text{av}} = \Delta Q/\Delta t$. Because the time interval between pulses is 4.00 ms, and because we know the charge per pulse from part (a), we obtain

$$I_{\text{av}} = \frac{Q_{\text{pulse}}}{\Delta t} = \frac{5.00 \times 10^{-8}\text{ C}}{4.00 \times 10^{-3}\text{ s}} = \boxed{12.5\ \mu\text{A}}$$

This represents only 0.005% of the peak current, which is 250 mA.

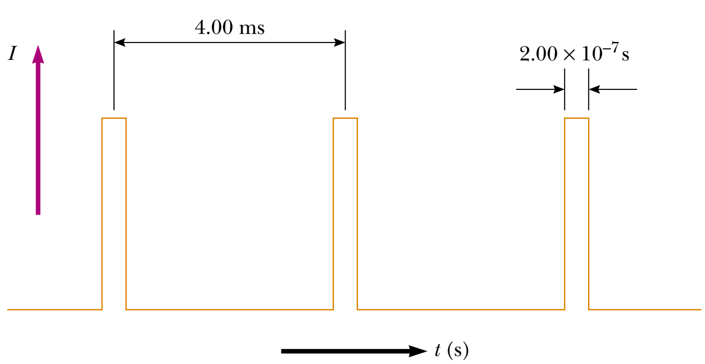


Figure 27.18 Current versus time for a pulsed beam of electrons.

(c) What is the maximum power delivered by the electron beam?

Solution By definition, power is energy delivered per unit time. Thus, the maximum power is equal to the energy delivered by a pulse divided by the pulse duration:

$$\mathcal{P} = \frac{E}{\Delta t}$$

$$= \frac{(3.13 \times 10^{11} \text{ electrons/pulse})(40.0 \text{ MeV/electron})}{2.00 \times 10^{-7} \text{ s/pulse}}$$

$$= (6.26 \times 10^{19} \text{ MeV/s})(1.60 \times 10^{-13} \text{ J/MeV})$$

$$= 1.00 \times 10^{7} \text{ W} = \boxed{10.0 \text{ MW}}$$

We could also compute this power directly. We assume that each electron had zero energy before being accelerated. Thus, by definition, each electron must have gone through a potential difference of 40.0 MV to acquire a final energy of 40.0 MeV. Hence, we have

$$\mathcal{P} = I\,\Delta V = (250 \times 10^{-3} \text{ A})(40.0 \times 10^{6} \text{ V}) = \boxed{10.0 \text{ MW}}$$

SUMMARY

The **electric current** I in a conductor is defined as

$$I \equiv \frac{dQ}{dt} \tag{27.2}$$

where dQ is the charge that passes through a cross-section of the conductor in a time dt. The SI unit of current is the **ampere** (A), where 1 A = 1 C/s.

The average current in a conductor is related to the motion of the charge carriers through the relationship

$$I_{\text{av}} = nqv_d A \tag{27.4}$$

where n is the density of charge carriers, q is the charge on each carrier, v_d is the drift speed, and A is the cross-sectional area of the conductor.

The magnitude of the **current density** J in a conductor is the current per unit area:

$$J \equiv \frac{I}{A} = nqv_d \tag{27.5}$$

The current density in a conductor is proportional to the electric field according to the expression

$$\mathbf{J} = \sigma \mathbf{E} \tag{27.7}$$

The proportionality constant σ is called the **conductivity** of the material of which the conductor is made. The inverse of σ is known as **resistivity** ρ ($\rho = 1/\sigma$). Equation 27.7 is known as **Ohm's law,** and a material is said to obey this law if the ratio of its current density $\mathbf{J}$ to its applied electric field $\mathbf{E}$ is a constant that is independent of the applied field.

The **resistance** R of a conductor is defined either in terms of the length of the conductor or in terms of the potential difference across it:

$$R \equiv \frac{\ell}{\sigma A} \equiv \frac{\Delta V}{I} \tag{27.8}$$

where ℓ is the length of the conductor, σ is the conductivity of the material of which it is made, A is its cross-sectional area, ΔV is the potential difference across it, and I is the current it carries.

The SI unit of resistance is volts per ampere, which is defined to be 1 **ohm** (Ω); that is, $1\ \Omega = 1\ \text{V/A}$. If the resistance is independent of the applied potential difference, the conductor obeys Ohm's law.

In a classical model of electrical conduction in metals, the electrons are treated as molecules of a gas. In the absence of an electric field, the average velocity of the electrons is zero. When an electric field is applied, the electrons move (on the average) with a **drift velocity** $\mathbf{v}_d$ that is opposite the electric field and given by the expression

$$\mathbf{v}_d = \frac{q\mathbf{E}}{m_e}\tau \qquad \textbf{(27.14)}$$

where τ is the average time between electron–atom collisions, m_e is the mass of the electron, and q is its charge. According to this model, the resistivity of the metal is

$$\rho = \frac{m_e}{nq^2\tau} \qquad \textbf{(27.17)}$$

where n is the number of free electrons per unit volume.

The resistivity of a conductor varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)] \qquad \textbf{(27.19)}$$

where α is the **temperature coefficient of resistivity** and ρ_0 is the resistivity at some reference temperature T_0.

If a potential difference ΔV is maintained across a resistor, the **power,** or rate at which energy is supplied to the resistor, is

$$\mathcal{P} = I\,\Delta V \qquad \textbf{(27.22)}$$

Because the potential difference across a resistor is given by $\Delta V = IR$, we can express the power delivered to a resistor in the form

$$\mathcal{P} = I^2R = \frac{(\Delta V)^2}{R} \qquad \textbf{(27.23)}$$

The electrical energy supplied to a resistor appears in the form of internal energy in the resistor.

Questions

1. Newspaper articles often contain statements such as "10 000 volts of electricity surged through the victim's body." What is wrong with this statement?
2. What is the difference between resistance and resistivity?
3. Two wires A and B of circular cross-section are made of the same metal and have equal lengths, but the resistance of wire A is three times greater than that of wire B. What is the ratio of their cross-sectional areas? How do their radii compare?
4. What is required in order to maintain a steady current in a conductor?
5. Do all conductors obey Ohm's law? Give examples to justify your answer.
6. When the voltage across a certain conductor is doubled, the current is observed to increase by a factor of three. What can you conclude about the conductor?
7. In the water analogy of an electric circuit, what corresponds to the power supply, resistor, charge, and potential difference?
8. Why might a "good" electrical conductor also be a "good" thermal conductor?
9. On the basis of the atomic theory of matter, explain why the resistance of a material should increase as its temperature increases.
10. How does the resistance for copper and silicon change with temperature? Why are the behaviors of these two materials different?
11. Explain how a current can persist in a superconductor in the absence of any applied voltage.
12. What single experimental requirement makes superconducting devices expensive to operate? In principle, can this limitation be overcome?

13. What would happen to the drift velocity of the electrons in a wire and to the current in the wire if the electrons could move freely without resistance through the wire?
14. If charges flow very slowly through a metal, why does it not require several hours for a light to turn on when you throw a switch?
15. In a conductor, the electric field that drives the electrons through the conductor propagates with a speed that is almost the same as the speed of light, even though the drift velocity of the electrons is very small. Explain how these can both be true. Does a given electron move from one end of the conductor to the other?
16. Two conductors of the same length and radius are connected across the same potential difference. One conductor has twice the resistance of the other. To which conductor is more power delivered?
17. Car batteries are often rated in ampere-hours. Does this designate the amount of current, power, energy, or charge that can be drawn from the battery?
18. If you were to design an electric heater using Nichrome wire as the heating element, what parameters of the wire could you vary to meet a specific power output, such as 1 000 W?
19. Consider the following typical monthly utility rate structure: \$2.00 for the first 16 kWh, 8.00¢/kWh for the next 34 kWh, 6.50¢/kWh for the next 50 kWh, 5.00¢/kWh for the next 100 kWh, 4.00¢/kWh for the next 200 kWh, and 3.50¢/kWh for all kilowatt-hours in excess of 400 kWh. On the basis of these rates, determine the amount charged for 327 kWh.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
☐ = paired numerical/symbolic problems

Section 27.1 Electric Current

1. In a particular cathode ray tube, the measured beam current is 30.0 μA. How many electrons strike the tube screen every 40.0 s?
2. A teapot with a surface area of 700 cm^2 is to be silver plated. It is attached to the negative electrode of an electrolytic cell containing silver nitrate ($Ag^+NO_3^-$). If the cell is powered by a 12.0-V battery and has a resistance of 1.80 Ω, how long does it take for a 0.133-mm layer of silver to build up on the teapot? (The density of silver is 10.5×10^3 kg/m^3.)

WEB 3. Suppose that the current through a conductor decreases exponentially with time according to the expression $I(t) = I_0 e^{-t/\tau}$, where I_0 is the initial current (at $t = 0$) and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor. (a) How much charge passes this point between $t = 0$ and $t = \tau$? (b) How much charge passes this point between $t = 0$ and $t = 10\tau$? (c) How much charge passes this point between $t = 0$ and $t = \infty$?
4. In the Bohr model of the hydrogen atom, an electron in the lowest energy state follows a circular path at a distance of 5.29×10^{-11} m from the proton. (a) Show that the speed of the electron is 2.19×10^6 m/s. (b) What is the effective current associated with this orbiting electron?
5. A small sphere that carries a charge of 8.00 nC is whirled in a circle at the end of an insulating string. The angular frequency of rotation is 100π rad/s. What average current does this rotating charge represent?
6. A small sphere that carries a charge q is whirled in a circle at the end of an insulating string. The angular frequency of rotation is ω. What average current does this rotating charge represent?
7. The quantity of charge q (in coulombs) passing through a surface of area 2.00 cm^2 varies with time according to the equation $q = 4.00t^3 + 5.00t + 6.00$, where t is in seconds. (a) What is the instantaneous current through the surface at $t = 1.00$ s? (b) What is the value of the current density?
8. An electric current is given by the expression $I(t) = 100 \sin(120\pi t)$, where I is in amperes and t is in seconds. What is the total charge carried by the current from $t = 0$ to $t = 1/240$ s?
9. Figure P27.9 represents a section of a circular conductor of nonuniform diameter carrying a current of 5.00 A. The radius of cross-section A_1 is 0.400 cm. (a) What is the magnitude of the current density across A_1? (b) If the current density across A_2 is one-fourth the value across A_1, what is the radius of the conductor at A_2?

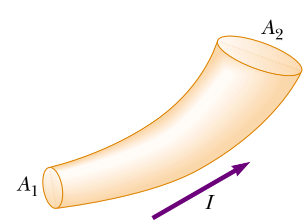


Figure P27.9

10. A Van de Graaff generator produces a beam of 2.00-MeV *deuterons,* which are heavy hydrogen nuclei containing a proton and a neutron. (a) If the beam current is 10.0 μA, how far apart are the deuterons? (b) Is their electrostatic repulsion a factor in beam stability? Explain.

11. The electron beam emerging from a certain high-energy electron accelerator has a circular cross-section of radius 1.00 mm. (a) If the beam current is 8.00 μA, what is the current density in the beam, assuming that it is uniform throughout? (b) The speed of the electrons is so close to the speed of light that their speed can be taken as $c = 3.00 \times 10^8$ m/s with negligible error. Find the electron density in the beam. (c) How long does it take for Avogadro's number of electrons to emerge from the accelerator?

12. An aluminum wire having a cross-sectional area of 4.00×10^{-6} m^2 carries a current of 5.00 A. Find the drift speed of the electrons in the wire. The density of aluminum is 2.70 g/cm^3. (Assume that one electron is supplied by each atom.)

Section 27.2 Resistance and Ohm's Law

13. A lightbulb has a resistance of 240 Ω when operating at a voltage of 120 V. What is the current through the lightbulb?

14. A resistor is constructed of a carbon rod that has a uniform cross-sectional area of 5.00 mm^2. When a potential difference of 15.0 V is applied across the ends of the rod, there is a current of 4.00×10^{-3} A in the rod. Find (a) the resistance of the rod and (b) the rod's length.

WEB 15. A 0.900-V potential difference is maintained across a 1.50-m length of tungsten wire that has a cross-sectional area of 0.600 mm^2. What is the current in the wire?

16. A conductor of uniform radius 1.20 cm carries a current of 3.00 A produced by an electric field of 120 V/m. What is the resistivity of the material?

17. Suppose that you wish to fabricate a uniform wire out of 1.00 g of copper. If the wire is to have a resistance of $R = 0.500\ \Omega$, and if all of the copper is to be used, what will be (a) the length and (b) the diameter of this wire?

18. (a) Make an order-of-magnitude estimate of the resistance between the ends of a rubber band. (b) Make an order-of-magnitude estimate of the resistance between the 'heads' and 'tails' sides of a penny. In each case, state what quantities you take as data and the values you measure or estimate for them. (c) What would be the order of magnitude of the current that each carries if it were connected across a 120-V power supply? (WARNING! Do not try this at home!)

19. A solid cube of silver (density = 10.5 g/cm^3) has a mass of 90.0 g. (a) What is the resistance between opposite faces of the cube? (b) If there is one conduction electron for each silver atom, what is the average drift speed of electrons when a potential difference of 1.00×10^{-5} V is applied to opposite faces? (The atomic number of silver is 47, and its molar mass is 107.87 g/mol.)

20. A metal wire of resistance R is cut into three equal pieces that are then connected side by side to form a new wire whose length is equal to one-third the original length. What is the resistance of this new wire?

21. A wire with a resistance R is lengthened to 1.25 times its original length by being pulled through a small hole. Find the resistance of the wire after it has been stretched.

22. Aluminum and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?

23. A current density of 6.00×10^{-13} A/m^2 exists in the atmosphere where the electric field (due to charged thunderclouds in the vicinity) is 100 V/m. Calculate the electrical conductivity of the Earth's atmosphere in this region.

24. The rod in Figure P27.24 (not drawn to scale) is made of two materials. Both have a square cross section of 3.00 mm on a side. The first material has a resistivity of $4.00 \times 10^{-3}\ \Omega \cdot$m and is 25.0 cm long, while the second material has a resistivity of $6.00 \times 10^{-3}\ \Omega \cdot$m and is 40.0 cm long. What is the resistance between the ends of the rod?

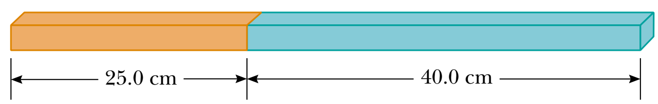


Figure P27.24

Section 27.3 A Model for Electrical Conduction

WEB 25. If the drift velocity of free electrons in a copper wire is 7.84×10^{-4} m/s, what is the electric field in the conductor?

26. If the current carried by a conductor is doubled, what happens to the (a) charge carrier density? (b) current density? (c) electron drift velocity? (d) average time between collisions?

27. Use data from Example 27.1 to calculate the collision mean free path of electrons in copper, assuming that the average thermal speed of conduction electrons is 8.60×10^5 m/s.

Section 27.4 Resistance and Temperature

28. While taking photographs in Death Valley on a day when the temperature is 58.0°C, Bill Hiker finds that a certain voltage applied to a copper wire produces a current of 1.000 A. Bill then travels to Antarctica and applies the same voltage to the same wire. What current does he register there if the temperature is −88.0°C? Assume that no change occurs in the wire's shape and size.

29. A certain lightbulb has a tungsten filament with a resistance of 19.0 Ω when cold and of 140 Ω when hot. Assuming that Equation 27.21 can be used over the large

temperature range involved here, find the temperature of the filament when hot. (Assume an initial temperature of 20.0°C.)

30. A carbon wire and a Nichrome wire are connected in series. If the combination has a resistance of 10.0 kΩ at 0°C, what is the resistance of each wire at 0°C such that the resistance of the combination does not change with temperature? (Note that the equivalent resistance of two resistors in series is the sum of their resistances.)

31. An aluminum wire with a diameter of 0.100 mm has a uniform electric field with a magnitude of 0.200 V/m imposed along its entire length. The temperature of the wire is 50.0°C. Assume one free electron per atom. (a) Using the information given in Table 27.1, determine the resistivity. (b) What is the current density in the wire? (c) What is the total current in the wire? (d) What is the drift speed of the conduction electrons? (e) What potential difference must exist between the ends of a 2.00-m length of the wire if the stated electric field is to be produced?

32. **Review Problem.** An aluminum rod has a resistance of 1.234 Ω at 20.0°C. Calculate the resistance of the rod at 120°C by accounting for the changes in both the resistivity and the dimensions of the rod.

33. What is the fractional change in the resistance of an iron filament when its temperature changes from 25.0°C to 50.0°C?

34. The resistance of a platinum wire is to be calibrated for low-temperature measurements. A platinum wire with a resistance of 1.00 Ω at 20.0°C is immersed in liquid nitrogen at 77 K (−196°C). If the temperature response of the platinum wire is linear, what is the expected resistance of the platinum wire at −196°C? ($\alpha_{\text{platinum}} = 3.92 \times 10^{-3}/°\text{C}$)

35. The temperature of a tungsten sample is raised while a copper sample is maintained at 20°C. At what temperature will the resistivity of the tungsten sample be four times that of the copper sample?

36. A segment of Nichrome wire is initially at 20.0°C. Using the data from Table 27.1, calculate the temperature to which the wire must be heated if its resistance is to be doubled.

Section 27.6 Electrical Energy and Power

37. A toaster is rated at 600 W when connected to a 120-V source. What current does the toaster carry, and what is its resistance?

38. In a hydroelectric installation, a turbine delivers 1 500 hp to a generator, which in turn converts 80.0% of the mechanical energy into electrical energy. Under these conditions, what current does the generator deliver at a terminal potential difference of 2 000 V?

WEB 39. **Review Problem.** What is the required resistance of an immersion heater that increases the temperature of 1.50 kg of water from 10.0°C to 50.0°C in 10.0 min while operating at 110 V?

40. **Review Problem.** What is the required resistance of an immersion heater that increases the temperature of a mass m of liquid water from T_1 to T_2 in a time t while operating at a voltage ΔV?

41. Suppose that a voltage surge produces 140 V for a moment. By what percentage does the power output of a 120-V, 100-W lightbulb increase? (Assume that its resistance does not change.)

42. A 500-W heating coil designed to operate from 110 V is made of Nichrome wire 0.500 mm in diameter. (a) Assuming that the resistivity of the Nichrome remains constant at its 20.0°C value, find the length of wire used. (b) Now consider the variation of resistivity with temperature. What power does the coil of part (a) actually deliver when it is heated to 1 200°C?

43. A coil of Nichrome wire is 25.0 m long. The wire has a diameter of 0.400 mm and is at 20.0°C. If it carries a current of 0.500 A, what are (a) the magnitude of the electric field in the wire and (b) the power delivered to it? (c) If the temperature is increased to 340°C and the potential difference across the wire remains constant, what is the power delivered?

44. Batteries are rated in terms of ampere-hours (A·h): For example, a battery that can produce a current of 2.00 A for 3.00 h is rated at 6.00 A·h. (a) What is the total energy, in kilowatt-hours, stored in a 12.0-V battery rated at 55.0 A·h? (b) At a rate of $0.060 0 per kilowatt-hour, what is the value of the electricity produced by this battery?

45. A 10.0-V battery is connected to a 120-Ω resistor. Neglecting the internal resistance of the battery, calculate the power delivered to the resistor.

46. It is estimated that each person in the United States (population = 270 million) has one electric clock, and that each clock uses energy at a rate of 2.50 W. To supply this energy, about how many metric tons of coal are burned per hour in coal-fired electricity generating plants that are, on average, 25.0% efficient? (The heat of combustion for coal is 33.0 MJ/kg.)

47. Compute the cost per day of operating a lamp that draws 1.70 A from a 110-V line if the cost of electrical energy is $0.060 0/kWh.

48. **Review Problem.** The heating element of a coffeemaker operates at 120 V and carries a current of 2.00 A. Assuming that all of the energy transferred from the heating element is absorbed by the water, calculate how long it takes to heat 0.500 kg of water from room temperature (23.0°C) to the boiling point.

49. A certain toaster has a heating element made of Nichrome resistance wire. When the toaster is first connected to a 120-V source of potential difference (and the wire is at a temperature of 20.0°C), the initial current is 1.80 A. However, the current begins to decrease as the resistive element warms up. When the toaster has reached its final operating temperature, the current has dropped to 1.53 A. (a) Find the power the toaster con-

sumes when it is at its operating temperature. (b) What is the final temperature of the heating element?

50. To heat a room having ceilings 8.0 ft high, about 10.0 W of electric power are required per square foot. At a cost of \$0.080 0/kWh, how much does it cost per day to use electricity to heat a room measuring 10.0 ft × 15.0 ft?

51. Estimate the cost of one person's routine use of a hair dryer for 1 yr. If you do not use a blow dryer yourself, observe or interview someone who does. State the quantities you estimate and their values.

ADDITIONAL PROBLEMS

52. One lightbulb is marked "25 W 120 V," and another "100 W 120 V"; this means that each bulb converts its respective power when plugged into a constant 120-V potential difference. (a) Find the resistance of each bulb. (b) How long does it take for 1.00 C to pass through the dim bulb? How is this charge different at the time of its exit compared with the time of its entry? (c) How long does it take for 1.00 J to pass through the dim bulb? How is this energy different at the time of its exit compared with the time of its entry? (d) Find the cost of running the dim bulb continuously for 30.0 days if the electric company sells its product at \$0.070 0 per kWh. What product *does* the electric company sell? What is its price for one SI unit of this quantity?

53. A high-voltage transmission line with a diameter of 2.00 cm and a length of 200 km carries a steady current of 1 000 A. If the conductor is copper wire with a free charge density of 8.00×10^{28} electrons/m^3, how long does it take one electron to travel the full length of the cable?

54. A high-voltage transmission line carries 1 000 A starting at 700 kV for a distance of 100 mi. If the resistance in the wire is 0.500 Ω/mi, what is the power loss due to resistive losses?

55. A more general definition of the temperature coefficient of resistivity is

$$\alpha = \frac{1}{\rho}\frac{d\rho}{dT}$$

where ρ is the resistivity at temperature T. (a) Assuming that α is constant, show that

$$\rho = \rho_0 e^{\alpha(T-T_0)}$$

where ρ_0 is the resistivity at temperature T_0. (b) Using the series expansion ($e^x \cong 1 + x$ for $x \ll 1$), show that the resistivity is given approximately by the expression $\rho = \rho_0[1 + \alpha(T - T_0)]$ for $\alpha(T - T_0) \ll 1$.

56. A copper cable is to be designed to carry a current of 300 A with a power loss of only 2.00 W/m. What is the required radius of the copper cable?

WEB **57.** An experiment is conducted to measure the electrical resistivity of Nichrome in the form of wires with different lengths and cross-sectional areas. For one set of measurements, a student uses 30-gauge wire, which has a cross-sectional area of 7.30×10^{-8} m^2. The student measures the potential difference across the wire and the current in the wire with a voltmeter and ammeter, respectively. For each of the measurements given in the table taken on wires of three different lengths, calculate the resistance of the wires and the corresponding values of the resistivity. What is the average value of the resistivity, and how does this value compare with the value given in Table 27.1?

L (m)	ΔV (V)	I (A)	R (Ω)	ρ (Ω·m)
0.540	5.22	0.500		
1.028	5.82	0.276		
1.543	5.94	0.187		

58. An electric utility company supplies a customer's house from the main power lines (120 V) with two copper wires, each of which is 50.0 m long and has a resistance of 0.108 Ω per 300 m. (a) Find the voltage at the customer's house for a load current of 110 A. For this load current, find (b) the power that the customer is receiving and (c) the power lost in the copper wires.

59. A straight cylindrical wire lying along the x axis has a length of 0.500 m and a diameter of 0.200 mm. It is made of a material described by Ohm's law with a resistivity of $\rho = 4.00 \times 10^{-8}$ Ω·m. Assume that a potential of 4.00 V is maintained at $x = 0$, and that $V = 0$ at $x = 0.500$ m. Find (a) the electric field $\mathbf{E}$ in the wire, (b) the resistance of the wire, (c) the electric current in the wire, and (d) the current density $\mathbf{J}$ in the wire. Express vectors in vector notation. (e) Show that $\mathbf{E} = \rho\mathbf{J}$.

60. A straight cylindrical wire lying along the x axis has a length L and a diameter d. It is made of a material described by Ohm's law with a resistivity ρ. Assume that a potential V is maintained at $x = 0$, and that $V = 0$ at $x = L$. In terms of L, d, V, ρ, and physical constants, derive expressions for (a) the electric field in the wire, (b) the resistance of the wire, (c) the electric current in the wire, and (d) the current density in the wire. Express vectors in vector notation. (e) Show that $\mathbf{E} = \rho\mathbf{J}$.

61. The potential difference across the filament of a lamp is maintained at a constant level while equilibrium temperature is being reached. It is observed that the steady-state current in the lamp is only one tenth of the current drawn by the lamp when it is first turned on. If the temperature coefficient of resistivity for the lamp at 20.0°C is 0.004 50 (°C)$^{-1}$, and if the resistance increases linearly with increasing temperature, what is the final operating temperature of the filament?

62. The current in a resistor decreases by 3.00 A when the potential difference applied across the resistor decreases from 12.0 V to 6.00 V. Find the resistance of the resistor.

63. An electric car is designed to run off a bank of 12.0-V batteries with a total energy storage of 2.00×10^7 J. (a) If the electric motor draws 8.00 kW, what is the current delivered to the motor? (b) If the electric motor draws 8.00 kW as the car moves at a steady speed of 20.0 m/s, how far will the car travel before it is "out of juice"?

64. **Review Problem.** When a straight wire is heated, its resistance is given by the expression $R = R_0[1 + \alpha(T - T_0)]$ according to Equation 27.21, where α is the temperature coefficient of resistivity. (a) Show that a more precise result, one that accounts for the fact that the length and area of the wire change when heated, is

$$R = \frac{R_0[1 + \alpha(T - T_0)][1 + \alpha'(T - T_0)]}{[1 + 2\alpha'(T - T_0)]}$$

where α' is the coefficient of linear expansion (see Chapter 19). (b) Compare these two results for a 2.00-m-long copper wire of radius 0.100 mm, first at 20.0°C and then heated to 100.0°C.

65. The temperature coefficients of resistivity in Table 27.1 were determined at a temperature of 20°C. What would they be at 0°C? (*Hint:* The temperature coefficient of resistivity at 20°C satisfies the expression $\rho = \rho_0[1 + \alpha(T - T_0)]$, where ρ_0 is the resistivity of the material at $T_0 = 20$°C. The temperature coefficient of resistivity α' at 0°C must satisfy the expression $\rho = \rho_0'[1 + \alpha' T]$, where ρ_0' is the resistivity of the material at 0°C.)

66. A resistor is constructed by shaping a material of resistivity ρ into a hollow cylinder of length L and with inner and outer radii r_a and r_b, respectively (Fig. P27.66). In use, the application of a potential difference between the ends of the cylinder produces a current parallel to the axis. (a) Find a general expression for the resistance of such a device in terms of L, ρ, r_a, and r_b. (b) Obtain a numerical value for R when $L = 4.00$ cm, $r_a = 0.500$ cm, $r_b = 1.20$ cm, and $\rho = 3.50 \times 10^5\ \Omega\cdot$m. (c) Now suppose that the potential difference is applied between the inner and outer surfaces so that the resulting current flows radially outward. Find a general expression for the resistance of the device in terms of L, ρ, r_a, and r_b. (d) Calculate the value of R, using the parameter values given in part (b).

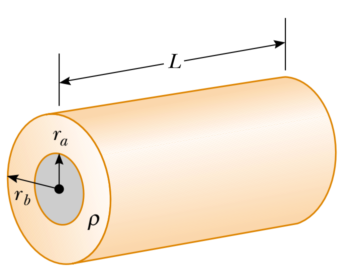


Figure P27.66

67. In a certain stereo system, each speaker has a resistance of 4.00 Ω. The system is rated at 60.0 W in each channel, and each speaker circuit includes a fuse rated at 4.00 A. Is this system adequately protected against overload? Explain your reasoning.

68. A close analogy exists between the flow of energy due to a temperature difference (see Section 20.7) and the flow of electric charge due to a potential difference. The energy dQ and the electric charge dq are both transported by free electrons in the conducting material. Consequently, a good electrical conductor is usually a good thermal conductor as well. Consider a thin conducting slab of thickness dx, area A, and electrical conductivity σ, with a potential difference dV between opposite faces. Show that the current $I = dq/dt$ is given by the equation on the left:

Charge conduction	Analogous thermal conduction (Eq. 20.14)
$\dfrac{dq}{dt} = \sigma A \left\lvert \dfrac{dV}{dx} \right\rvert$	$\dfrac{dQ}{dt} = kA \left\lvert \dfrac{dT}{dx} \right\rvert$

In the analogous thermal conduction equation on the right, the rate of energy flow dQ/dt (in SI units of joules per second) is due to a temperature gradient dT/dx in a material of thermal conductivity k. State analogous rules relating the direction of the electric current to the change in potential and relating the direction of energy flow to the change in temperature.

69. Material with uniform resistivity ρ is formed into a wedge, as shown in Figure P27.69. Show that the resistance between face A and face B of this wedge is

$$R = \rho \frac{L}{w(y_2 - y_1)} \ln\left(\frac{y_2}{y_1}\right)$$

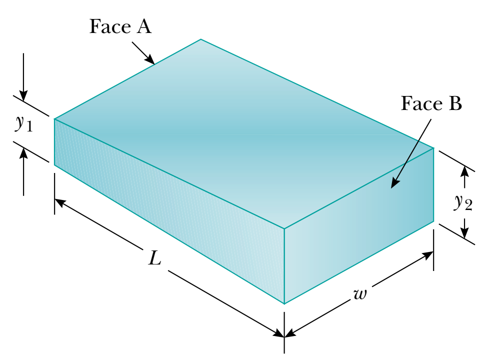


Figure P27.69

70. A material of resistivity ρ is formed into the shape of a truncated cone of altitude h, as shown in Figure P27.70.

The bottom end has a radius b, and the top end has a radius a. Assuming that the current is distributed uniformly over any particular cross-section of the cone so that the current density is not a function of radial position (although it does vary with position along the axis of the cone), show that the resistance between the two ends is given by the expression

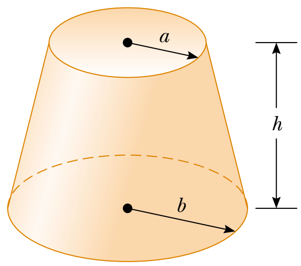


Figure P27.70

$$R = \frac{\rho}{\pi}\left(\frac{h}{ab}\right)$$

71. The current–voltage characteristic curve for a semiconductor diode as a function of temperature T is given by the equation

$$I = I_0(e^{e\Delta V/k_BT} - 1)$$

Here, the first symbol e represents the base of the natural logarithm. The second e is the charge on the electron. The k_B is Boltzmann's constant, and T is the absolute temperature. Set up a spreadsheet to calculate I and $R = (\Delta V)/I$ for $\Delta V = 0.400$ V to 0.600 V in increments of 0.005 V. Assume that $I_0 = 1.00$ nA. Plot R versus ΔV for $T = 280$ K, 300 K, and 320 K.

Answers to Quick Quizzes

27.1 d, b = c, a. The current in part (d) is equivalent to two positive charges moving to the left. Parts (b) and (c) each represent four positive charges moving in the same direction because negative charges moving to the left are equivalent to positive charges moving to the right. The current in part (a) is equivalent to five positive charges moving to the right.

27.2 Every portion of the wire carries the same current even though the wire constricts. As the cross-sectional area decreases, the drift velocity must increase in order for the constant current to be maintained, in accordance with Equation 27.4. Equations 27.5 and 27.6 indicate that the current density also increases. An increasing electric field must be causing the increasing current density, as indicated by Equation 27.7. If you were to draw this situation, you would show the electric field lines being compressed into the smaller area, indicating increasing magnitude of the electric field.

27.3 The curvature of the line indicates that the device is nonohmic (that is, its resistance varies with potential difference). Being the definition of resistance, Equation 27.8 still applies, giving different values for R at different points on the curve. The slope of the tangent to the graph line at a point is the reciprocal of the "dynamic resistance" at that point. Note that the resistance of the device (as measured by an ohmmeter) is the reciprocal of the slope of a secant line joining the origin to a particular point on the curve.

27.4 The cable should be as short as possible but still able to reach from one vehicle to another (small ℓ), it should be quite thick (large A), and it should be made of a material with a low resistivity ρ. Referring to Table 27.1, you should probably choose copper or aluminum because the only two materials in the table that have lower ρ values—silver and gold—are prohibitively expensive for your purposes.

27.5 Just after it is turned on. When the filament is at room temperature, its resistance is low, and hence the current is relatively large ($I = \Delta V/R$). As the filament warms up, its resistance increases, and the current decreases. Older lightbulbs often fail just as they are turned on because this large initial current "spike" produces rapid temperature increase and stress on the filament.

27.6 (c). Because the potential difference ΔV is the same across the two bulbs and because the power delivered to a conductor is $\mathcal{P} = I\Delta V$, the 60-W bulb, with its higher power rating, must carry the greater current. The 30-W bulb has the higher resistance because it draws less current at the same potential difference.

27.7 $I_a = I_b > I_c = I_d > I_e = I_f$. The current I_a leaves the positive terminal of the battery and then splits to flow through the two bulbs; thus, $I_a = I_c + I_e$. From Quick Quiz 27.6, we know that the current in the 60-W bulb is greater than that in the 30-W bulb. (Note that all the current does not follow the "path of least resistance," which in this case is through the 60-W bulb.) Because charge does not build up in the bulbs, we know that all the charge flowing into a bulb from the left must flow out on the right; consequently, $I_c = I_d$ and $I_e = I_f$. The two currents leaving the bulbs recombine to form the current back into the battery, $I_f + I_d = I_b$.

PUZZLER

If all these appliances were operating at one time, a circuit breaker would probably be tripped, preventing a potentially dangerous situation. What causes a circuit breaker to trip when too many electrical devices are plugged into one circuit? *(George Semple)*

chapter

28 Direct Current Circuits

Chapter Outline

This chapter is concerned with the analysis of some simple electric circuits that contain batteries, resistors, and capacitors in various combinations. The analysis of these circuits is simplified by the use of two rules known as *Kirchhoff's rules,* which follow from the laws of conservation of energy and conservation of electric charge. Most of the circuits analyzed are assumed to be in *steady state,* which means that the currents are constant in magnitude and direction. In Section 28.4 we discuss circuits in which the current varies with time. Finally, we describe a variety of common electrical devices and techniques for measuring current, potential difference, resistance, and emf.

28.1 ELECTROMOTIVE FORCE

In Section 27.6 we found that a constant current can be maintained in a closed circuit through the use of a source of *emf,* which is a device (such as a battery or generator) that produces an electric field and thus may cause charges to move around a circuit. One can think of a source of emf as a "charge pump." When an electric potential difference exists between two points, the source moves charges "uphill" from the lower potential to the higher. The emf $\mathcal{E}$ describes the work done per unit charge, and hence the SI unit of emf is the volt.

Consider the circuit shown in Figure 28.1, consisting of a battery connected to a resistor. We assume that the connecting wires have no resistance. The positive terminal of the battery is at a higher potential than the negative terminal. If we neglect the internal resistance of the battery, the potential difference across it (called the *terminal voltage*) equals its emf. However, because a real battery always has some internal resistance r, the terminal voltage is not equal to the emf for a battery in a circuit in which there is a current. To understand why this is so, consider the circuit diagram in Figure 28.2a, where the battery of Figure 28.1 is represented by the dashed rectangle containing an emf $\mathcal{E}$ in series with an internal resistance r. Now imagine moving through the battery clockwise from a to b and measuring the electric potential at various locations. As we pass from the negative terminal to the positive terminal, the potential *increases* by an amount $\mathcal{E}$. However, as we move through the resistance r, the potential *decreases* by an amount Ir, where I is the current in the circuit. Thus, the terminal voltage of the battery $\Delta V = V_b - V_a$ is[1]

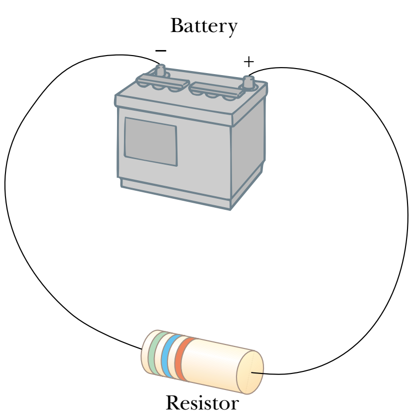
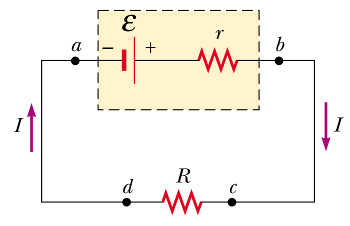
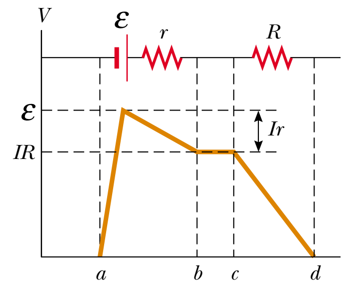


Figure 28.1 A circuit consisting of a resistor connected to the terminals of a battery.

[1] The terminal voltage in this case is less than the emf by an amount Ir. In some situations, the terminal voltage may *exceed* the emf by an amount Ir. This happens when the direction of the current is *opposite* that of the emf, as in the case of charging a battery with another source of emf.



(a)



(b)

Figure 28.2 (a) Circuit diagram of a source of emf $\mathcal{E}$ (in this case, a battery), of internal resistance r, connected to an external resistor of resistance R. (b) Graphical representation showing how the electric potential changes as the circuit in part (a) is traversed clockwise.

$$\Delta V = \mathcal{E} - Ir \tag{28.1}$$

From this expression, note that $\mathcal{E}$ is equivalent to the **open-circuit voltage**—that is, the *terminal voltage when the current is zero.* The emf is the voltage labeled on a battery—for example, the emf of a D cell is 1.5 V. The actual potential difference between the terminals of the battery depends on the current through the battery, as described by Equation 28.1.

Figure 28.2b is a graphical representation of the changes in electric potential as the circuit is traversed in the clockwise direction. By inspecting Figure 28.2a, we see that the terminal voltage ΔV must equal the potential difference across the external resistance R, often called the **load resistance.** The load resistor might be a simple resistive circuit element, as in Figure 28.1, or it could be the resistance of some electrical device (such as a toaster, an electric heater, or a lightbulb) connected to the battery (or, in the case of household devices, to the wall outlet). The resistor represents a *load* on the battery because the battery must supply energy to operate the device. The potential difference across the load resistance is $\Delta V = IR$. Combining this expression with Equation 28.1, we see that

$$\mathcal{E} = IR + Ir \tag{28.2}$$

Solving for the current gives

$$I = \frac{\mathcal{E}}{R + r} \tag{28.3}$$

This equation shows that the current in this simple circuit depends on both the load resistance R external to the battery and the internal resistance r. If R is much greater than r, as it is in many real-world circuits, we can neglect r.

If we multiply Equation 28.2 by the current I, we obtain

$$I\mathcal{E} = I^2R + I^2r \tag{28.4}$$

This equation indicates that, because power $\mathcal{P} = I\Delta V$ (see Eq. 27.22), the total power output $I\mathcal{E}$ of the battery is delivered to the external load resistance in the amount I^2R and to the internal resistance in the amount I^2r. Again, if $r \ll R$, then most of the power delivered by the battery is transferred to the load resistance.

EXAMPLE 28.1 Terminal Voltage of a Battery

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω. Its terminals are connected to a load resistance of 3.00 Ω. (a) Find the current in the circuit and the terminal voltage of the battery.

Solution Using first Equation 28.3 and then Equation 28.1, we obtain

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0\ \text{V}}{3.05\ \Omega} = \boxed{3.93\ \text{A}}$$

$$\Delta V = \mathcal{E} - Ir = 12.0\ \text{V} - (3.93\ \text{A})(0.05\ \Omega) = \boxed{11.8\ \text{V}}$$

To check this result, we can calculate the voltage across the load resistance R:

$$\Delta V = IR = (3.93\ \text{A})(3.00\ \Omega) = 11.8\ \text{V}$$

(b) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

Solution The power delivered to the load resistor is

$$\mathcal{P}_R = I^2R = (3.93\ \text{A})^2\ (3.00\ \Omega) = \boxed{46.3\ \text{W}}$$

The power delivered to the internal resistance is

$$\mathcal{P}_r = I^2r = (3.93\ \text{A})^2\ (0.05\ \Omega) = \boxed{0.772\ \text{W}}$$

Hence, the power delivered by the battery is the sum of these quantities, or 47.1 W. You should check this result, using the expression $\mathcal{P} = I\mathcal{E}$.

EXAMPLE 28.2 Matching the Load

Show that the maximum power delivered to the load resistance R in Figure 28.2a occurs when the load resistance matches the internal resistance—that is, when $R = r$.

Solution The power delivered to the load resistance is equal to I^2R, where I is given by Equation 28.3:

$$\mathcal{P} = I^2R = \frac{\mathcal{E}^2R}{(R + r)^2}$$

When $\mathcal{P}$ is plotted versus R as in Figure 28.3, we find that $\mathcal{P}$ reaches a maximum value of $\mathcal{E}^2/4r$ at $R = r$. We can also prove this by differentiating $\mathcal{P}$ with respect to R, setting the result equal to zero, and solving for R. The details are left as a problem for you to solve (Problem 57).

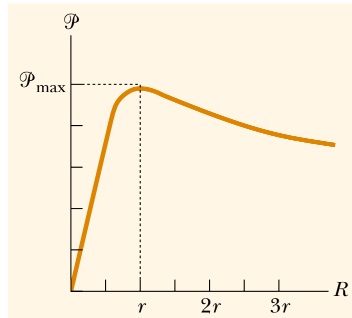


Figure 28.3 Graph of the power $\mathcal{P}$ delivered by a battery to a load resistor of resistance R as a function of R. The power delivered to the resistor is a maximum when the load resistance equals the internal resistance of the battery.

28.2 RESISTORS IN SERIES AND IN PARALLEL

Suppose that you and your friends are at a crowded basketball game in a sports arena and decide to leave early. You have two choices: (1) your whole group can exit through a single door and walk down a long hallway containing several concession stands, each surrounded by a large crowd of people waiting to buy food or souvenirs; or (b) each member of your group can exit through a separate door in the main hall of the arena, where each will have to push his or her way through a single group of people standing by the door. In which scenario will less time be required for your group to leave the arena?

It should be clear that your group will be able to leave faster through the separate doors than down the hallway where each of you has to push through several groups of people. We could describe the groups of people in the hallway as acting in *series,* because each of you must push your way through all of the groups. The groups of people around the doors in the arena can be described as acting in *parallel.* Each member of your group must push through only one group of people, and each member pushes through a *different* group of people. This simple analogy will help us understand the behavior of currents in electric circuits containing more than one resistor.

When two or more resistors are connected together as are the lightbulbs in Figure 28.4a, they are said to be in *series.* Figure 28.4b is the circuit diagram for the lightbulbs, which are shown as resistors, and the battery. In a series connection, all the charges moving through one resistor must also pass through the second resistor. (This is analogous to all members of your group pushing through the crowds in the single hallway of the sports arena.) Otherwise, charge would accumulate between the resistors. Thus,

> for a series combination of resistors, the currents in the two resistors are the same because any charge that passes through R_1 must also pass through R_2.

The potential difference applied across the series combination of resistors will divide between the resistors. In Figure 28.4b, because the voltage drop[2] from a to b

[2] The term *voltage drop* is synonymous with a decrease in electric potential across a resistor and is used often by individuals working with electric circuits.

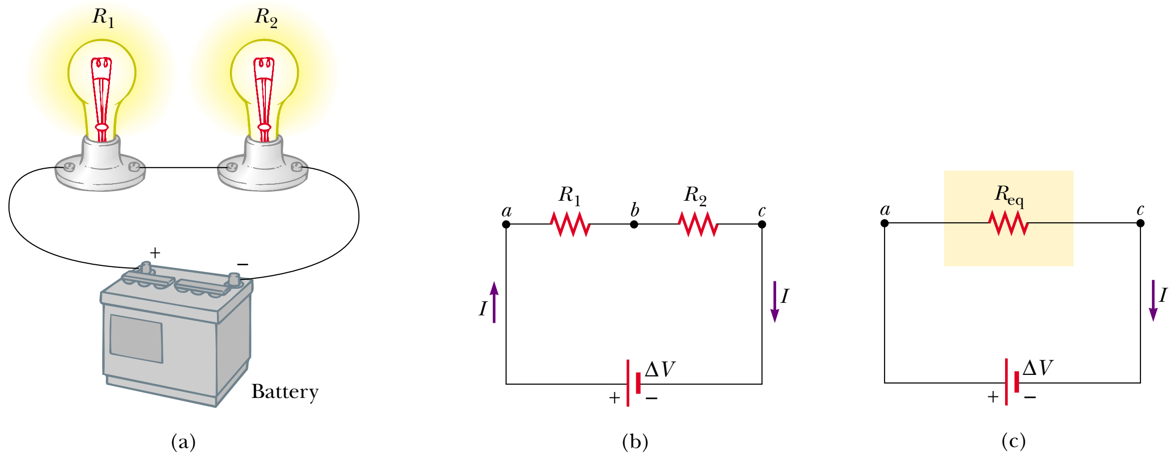


Figure 28.4 (a) A series connection of two resistors R_1 and R_2. The current in R_1 is the same as that in R_2. (b) Circuit diagram for the two-resistor circuit. (c) The resistors replaced with a single resistor having an equivalent resistance $R_{eq} = R_1 + R_2$.

equals IR_1 and the voltage drop from b to c equals IR_2, the voltage drop from a to c is

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

Therefore, we can replace the two resistors in series with a single resistor having an *equivalent resistance* R_{eq}, where

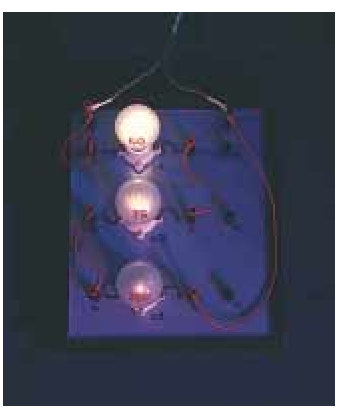
$$R_{eq} = R_1 + R_2 \qquad \textbf{(28.5)}$$

The resistance R_{eq} is equivalent to the series combination $R_1 + R_2$ in the sense that the circuit current is unchanged when R_{eq} replaces $R_1 + R_2$.

The equivalent resistance of three or more resistors connected in series is

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \cdots} \qquad \textbf{(28.6)}$$

This relationship indicates that **the equivalent resistance of a series connection of resistors is always greater than any individual resistance.**



A series connection of three lightbulbs, all rated at 120 V but having power ratings of 60 W, 75 W, and 200 W. Why are the intensities of the bulbs different? Which bulb has the greatest resistance? How would their relative intensities differ if they were connected in parallel?

Quick Quiz 28.1

If a piece of wire is used to connect points b and c in Figure 28.4b, does the brightness of bulb R_1 increase, decrease, or stay the same? What happens to the brightness of bulb R_2?

Now consider two resistors connected in *parallel,* as shown in Figure 28.5. When the current I reaches point a in Figure 28.5b, called a *junction,* it splits into two parts, with I_1 going through R_1 and I_2 going through R_2. A **junction** is any point in a circuit where a current can split (just as your group might split up and leave the arena through several doors, as described earlier.) This split results in less current in each individual resistor than the current leaving the battery. Because charge must be conserved, the current I that enters point a must equal the total current leaving that point:

$$I = I_1 + I_2$$

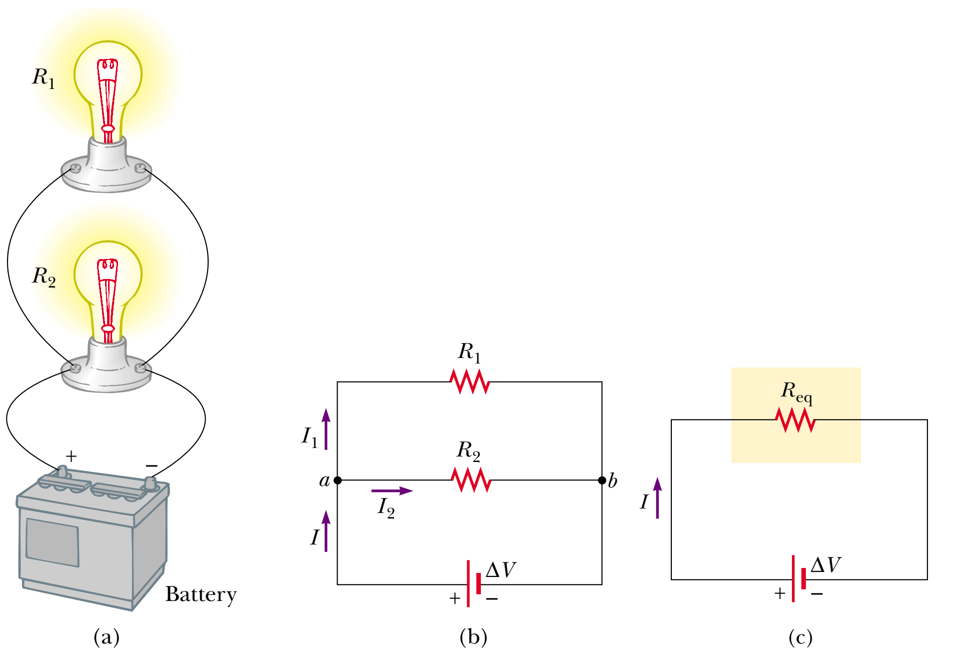
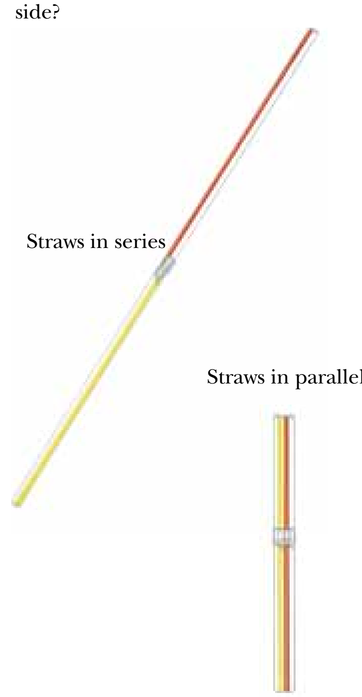


Figure 28.5 (a) A parallel connection of two resistors R_1 and R_2. The potential difference across R_1 is the same as that across R_2. (b) Circuit diagram for the two-resistor circuit. (c) The resistors replaced with a single resistor having an equivalent resistance $R_{eq} = (R_1^{-1} + R_2^{-1})^{-1}$.

QuickLab

Tape one pair of drinking straws end to end, and tape a second pair side by side. Which pair is easier to blow through? What would happen if you were comparing three straws taped end to end with three taped side by side?



As can be seen from Figure 28.5, both resistors are connected directly across the terminals of the battery. Thus,

when resistors are connected in parallel, the potential differences across them are the same.

Because the potential differences across the resistors are the same, the expression $\Delta V = IR$ gives

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$

From this result, we see that the equivalent resistance of two resistors in parallel is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \textbf{(28.7)}$$

or

$$R_{eq} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$$

An extension of this analysis to three or more resistors in parallel gives

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad \textbf{(28.8)}$$

The equivalent resistance of several resistors in parallel

Three lightbulbs having power ratings of 25 W, 75 W, and 150 W, connected in parallel to a voltage source of about 100 V. All bulbs are rated at the same voltage. Why do the intensities differ? Which bulb draws the most current? Which has the least resistance?

We can see from this expression that **the equivalent resistance of two or more resistors connected in parallel is always less than the least resistance in the group.**

Household circuits are always wired such that the appliances are connected in parallel. Each device operates independently of the others so that if one is switched off, the others remain on. In addition, the devices operate on the same voltage.

Quick Quiz 28.2

Assume that the battery of Figure 28.1 has zero internal resistance. If we add a second resistor in series with the first, does the current in the battery increase, decrease, or stay the same? How about the potential difference across the battery terminals? Would your answers change if the second resistor were connected in parallel to the first one?

Quick Quiz 28.3

Are automobile headlights wired in series or in parallel? How can you tell?

EXAMPLE 28.3 Find the Equivalent Resistance

Four resistors are connected as shown in Figure 28.6a. (a) Find the equivalent resistance between points *a* and *c*.

Solution The combination of resistors can be reduced in steps, as shown in Figure 28.6. The 8.0-Ω and 4.0-Ω resistors are in series; thus, the equivalent resistance between *a* and *b* is 12 Ω (see Eq. 28.5). The 6.0-Ω and 3.0-Ω resistors are in parallel, so from Equation 28.7 we find that the equivalent resistance from *b* to *c* is 2.0 Ω. Hence, the equivalent resistance from *a* to *c* is 14 Ω.

(b) What is the current in each resistor if a potential difference of 42 V is maintained between *a* and *c*?

Solution The currents in the 8.0-Ω and 4.0-Ω resistors are the same because they are in series. In addition, this is the same as the current that would exist in the 14-Ω equivalent resistor subject to the 42-V potential difference. Therefore, using Equation 27.8 ($R = \Delta V/I$) and the results from part (a), we obtain

$$I = \frac{\Delta V_{ac}}{R_{\text{eq}}} = \frac{42\text{ V}}{14\ \Omega} = 3.0\text{ A}$$

This is the current in the 8.0-Ω and 4.0-Ω resistors. When this 3.0-A current enters the junction at *b*, however, it splits, with part passing through the 6.0-Ω resistor (I_1) and part through the 3.0-Ω resistor (I_2). Because the potential difference is ΔV_{bc} across each of these resistors (since they are in parallel), we see that $(6.0\ \Omega)I_1 = (3.0\ \Omega)I_2$, or $I_2 = 2I_1$. Using this result and the fact that $I_1 + I_2 = 3.0$ A, we find that $I_1 = 1.0$ A and $I_2 = 2.0$ A. We could have guessed this at the start by noting that the current through the 3.0-Ω resistor has to be twice that through the 6.0-Ω resistor, in view of their relative resistances and the fact that the same voltage is applied to each of them.

As a final check of our results, note that $\Delta V_{bc} = (6.0\ \Omega)I_1 = (3.0\ \Omega)I_2 = 6.0\text{ V}$ and $\Delta V_{ab} = (12\ \Omega)I = 36\text{ V}$; therefore, $\Delta V_{ac} = \Delta V_{ab} + \Delta V_{bc} = 42$ V, as it must.

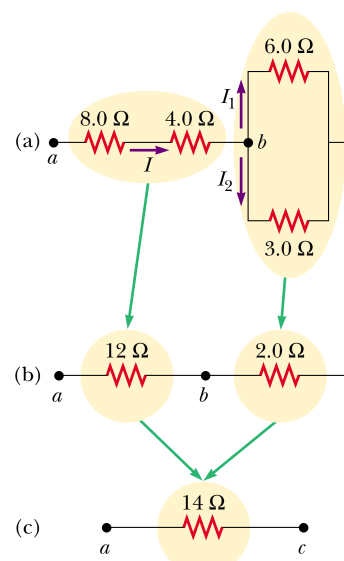


Figure 28.6

EXAMPLE 28.4 Three Resistors in Parallel

Three resistors are connected in parallel as shown in Figure 28.7. A potential difference of 18 V is maintained between points a and b. (a) Find the current in each resistor.

Solution The resistors are in parallel, and so the potential difference across each must be 18 V. Applying the relationship $\Delta V = IR$ to each resistor gives

$$I_1 = \frac{\Delta V}{R_1} = \frac{18\text{ V}}{3.0\ \Omega} = \boxed{6.0\text{ A}}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18\text{ V}}{6.0\ \Omega} = \boxed{3.0\text{ A}}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18\text{ V}}{9.0\ \Omega} = \boxed{2.0\text{ A}}$$

(b) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

Solution We apply the relationship $\mathcal{P} = (\Delta V)^2/R$ to each resistor and obtain

$$\mathcal{P}_1 = \frac{\Delta V^2}{R_1} = \frac{(18\text{ V})^2}{3.0\ \Omega} = \boxed{110\text{ W}}$$

$$\mathcal{P}_2 = \frac{\Delta V^2}{R_2} = \frac{(18\text{ V})^2}{6.0\ \Omega} = \boxed{54\text{ W}}$$

$$\mathcal{P}_3 = \frac{\Delta V^2}{R_3} = \frac{(18\text{ V})^2}{9.0\ \Omega} = \boxed{36\text{ W}}$$

This shows that the smallest resistor receives the most power. Summing the three quantities gives a total power of 200 W.

(c) Calculate the equivalent resistance of the circuit.

Solution We can use Equation 28.8 to find R_{eq}:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{3.0\ \Omega} + \frac{1}{6.0\ \Omega} + \frac{1}{9.0\ \Omega}$$

$$= \frac{6}{18\ \Omega} + \frac{3}{18\ \Omega} + \frac{2}{18\ \Omega} = \frac{11}{18\ \Omega}$$

$$R_{\text{eq}} = \frac{18\ \Omega}{11} = \boxed{1.6\ \Omega}$$

Exercise Use R_{eq} to calculate the total power delivered by the battery.

Answer 200 W.

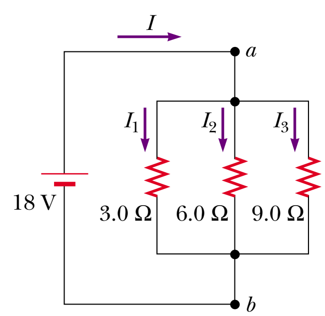


Figure 28.7 Three resistors connected in parallel. The voltage across each resistor is 18 V.

EXAMPLE 28.5 Finding R_{eq} by Symmetry Arguments

Consider five resistors connected as shown in Figure 28.8a. Find the equivalent resistance between points a and b.

Solution In this type of problem, it is convenient to assume a current entering junction a and then apply symmetry

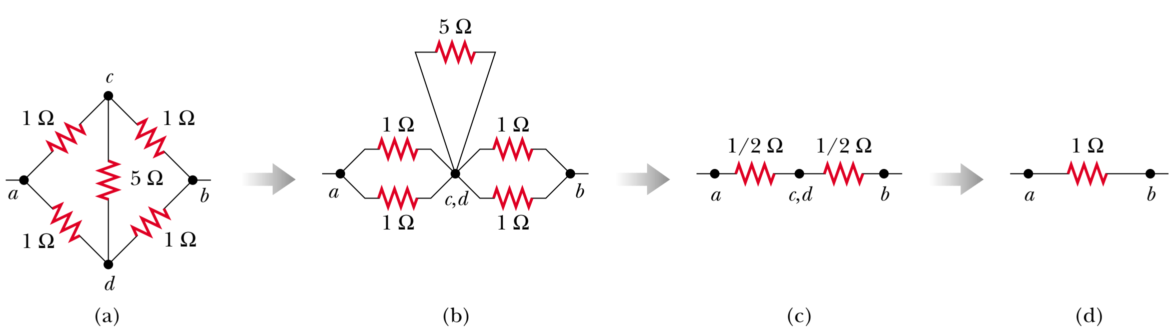


Figure 28.8 Because of the symmetry in this circuit, the 5-Ω resistor does not contribute to the resistance between points a and b and therefore can be disregarded when we calculate the equivalent resistance.

arguments. Because of the symmetry in the circuit (all 1-Ω resistors in the outside loop), the currents in branches *ac* and *ad* must be equal; hence, the electric potentials at points *c* and *d* must be equal. This means that $\Delta V_{cd} = 0$ and, as a result, points *c* and *d* may be connected together without affecting the circuit, as in Figure 28.8b. Thus, the 5-Ω resistor may be removed from the circuit and the remaining circuit then reduced as in Figures 28.8c and d. From this reduction we see that the equivalent resistance of the combination is 1 Ω. Note that the result is 1 Ω regardless of the value of the resistor connected between *c* and *d*.

CONCEPTUAL EXAMPLE 28.6 Operation of a Three-Way Lightbulb

Figure 28.9 illustrates how a three-way lightbulb is constructed to provide three levels of light intensity. The socket of the lamp is equipped with a three-way switch for selecting different light intensities. The bulb contains two filaments. When the lamp is connected to a 120-V source, one filament receives 100 W of power, and the other receives 75 W. Explain how the two filaments are used to provide three different light intensities.

Solution The three light intensities are made possible by applying the 120 V to one filament alone, to the other filament alone, or to the two filaments in parallel. When switch S_1 is closed and switch S_2 is opened, current passes only through the 75-W filament. When switch S_1 is open and switch S_2 is closed, current passes only through the 100-W filament. When both switches are closed, current passes through both filaments, and the total power is 175 W.

If the filaments were connected in series and one of them were to break, no current could pass through the bulb, and the bulb would give no illumination, regardless of the switch position. However, with the filaments connected in parallel, if one of them (for example, the 75-W filament) breaks, the bulb will still operate in two of the switch positions as current passes through the other (100-W) filament.

Exercise Determine the resistances of the two filaments and their parallel equivalent resistance.

Answer 144 Ω, 192 Ω, 82.3 Ω.

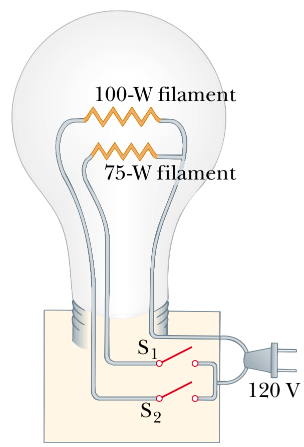


Figure 28.9 A three-way lightbulb.

APPLICATION Strings of Lights

Strings of lights are used for many ornamental purposes, such as decorating Christmas trees. Over the years, both parallel and series connections have been used for multilight strings powered by 120 V.[3] Series-wired bulbs are safer than parallel-wired bulbs for indoor Christmas-tree use because series-wired bulbs operate with less light per bulb and at a lower temperature. However, if the filament of a single bulb fails (or if the bulb is removed from its socket), all the lights on the string are extinguished. The popularity of series-wired light strings diminished because troubleshooting a failed bulb was a tedious, time-consuming chore that involved trial-and-error substitution of a good bulb in each socket along the string until the defective bulb was found.

In a parallel-wired string, each bulb operates at 120 V. By design, the bulbs are brighter and hotter than those on a series-wired string. As a result, these bulbs are inherently more dangerous (more likely to start a fire, for instance), but if one bulb in a parallel-wired string fails or is removed, the rest of the bulbs continue to glow. (A 25-bulb string of 4-W bulbs results in a power of 100 W; the total power becomes substantial when several strings are used.)

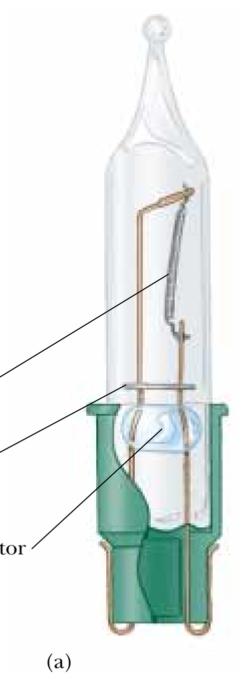
A new design was developed for so-called "miniature" lights wired in series, to prevent the failure of one bulb from extinguishing the entire string. The solution is to create a connection (called a jumper) across the filament after it fails. (If an alternate connection existed across the filament before

[3] These and other household devices, such as the three-way lightbulb in Conceptual Example 28.6 and the kitchen appliances shown in this chapter's Puzzler, actually operate on alternating current (ac), to be introduced in Chapter 33.

it failed, each bulb would represent a parallel circuit; in this circuit, the current would flow through the alternate connection, forming a short circuit, and the bulb would not glow.) When the filament breaks in one of these miniature lightbulbs, 120 V appears across the bulb because no current is present in the bulb and therefore no drop in potential occurs across the other bulbs. Inside the lightbulb, a small loop covered by an insulating material is wrapped around the filament leads. An arc burns the insulation and connects the filament leads when 120 V appears across the bulb—that is, when the filament fails. This "short" now completes the circuit through the bulb even though the filament is no longer active (Fig. 28.10).

Suppose that all the bulbs in a 50-bulb miniature-light string are operating. A 2.4-V potential drop occurs across each bulb because the bulbs are in series. The power input to this style of bulb is 0.34 W, so the total power supplied to the string is only 17 W. We calculate the filament resistance at the operating temperature to be $(2.4\text{ V})^2/(0.34\text{ W}) = 17\ \Omega$. When the bulb fails, the resistance across its terminals is reduced to zero because of the alternate jumper connection mentioned in the preceding paragraph. All the other bulbs not only stay on but glow more brightly because the total resistance of the string is reduced and consequently the current in each bulb increases.

Let us assume that the operating resistance of a bulb remains at 17 Ω even though its temperature rises as a result of the increased current. If one bulb fails, the potential drop across each of the remaining bulbs increases to 2.45 V, the current increases from 0.142 A to 0.145 A, and the power increases to 0.354 W. As more lights fail, the current keeps rising, the filament of each bulb operates at a higher temperature, and the lifetime of the bulb is reduced. It is therefore a good idea to check for failed (nonglowing) bulbs in such a series-wired string and replace them as soon as possible, in order to maximize the lifetimes of all the bulbs.



(a)


(b)

Figure 28.10 (a) Schematic diagram of a modern "miniature" holiday lightbulb, with a jumper connection to provide a current path if the filament breaks. (b) A Christmas-tree lightbulb.

28.3 KIRCHHOFF'S RULES

13.4 As we saw in the preceding section, we can analyze simple circuits using the expression $\Delta V = IR$ and the rules for series and parallel combinations of resistors. Very often, however, it is not possible to reduce a circuit to a single loop. The procedure for analyzing more complex circuits is greatly simplified if we use two principles called **Kirchhoff's rules:**

1. The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}} \qquad (28.9)$$

Gustav Kirchhoff (1824–1887) Kirchhoff, a professor at Heidelberg, Germany, and Robert Bunsen invented the spectroscope and founded the science of spectroscopy, which we shall study in Chapter 40. They discovered the elements cesium and rubidium and invented astronomical spectroscopy. Kirchhoff formulated another Kirchhoff's rule, namely, "a cool substance will absorb light of the same wavelengths that it emits when hot." *(AIP ESVA/W. F. Meggers Collection)*

2. The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0 \qquad \textbf{(28.10)}$$

Kirchhoff's first rule is a statement of conservation of electric charge. All current that enters a given point in a circuit must leave that point because charge cannot build up at a point. If we apply this rule to the junction shown in Figure 28.11a, we obtain

$$I_1 = I_2 + I_3$$

Figure 28.11b represents a mechanical analog of this situation, in which water flows through a branched pipe having no leaks. The flow rate into the pipe equals the total flow rate out of the two branches on the right.

Kirchhoff's second rule follows from the law of conservation of energy. Let us imagine moving a charge around the loop. When the charge returns to the starting point, the charge–circuit system must have the same energy as when the charge started from it. The sum of the increases in energy in some circuit elements must equal the sum of the decreases in energy in other elements. The potential energy decreases whenever the charge moves through a potential drop $-IR$ across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal. Kirchhoff's second rule applies only for circuits in which an electric potential is defined at each point; this criterion may not be satisfied if changing electromagnetic fields are present, as we shall see in Chapter 31.

In justifying our claim that Kirchhoff's second rule is a statement of conservation of energy, we imagined carrying a charge around a loop. When applying this rule, we imagine *traveling* around the loop and consider changes in *electric potential,* rather than the changes in *potential energy* described in the previous paragraph. You should note the following sign conventions when using the second rule:

- Because charges move from the high-potential end of a resistor to the low-potential end, if a resistor is traversed in the direction of the current, the change in potential ΔV across the resistor is $-IR$ (Fig. 28.12a).
- If a resistor is traversed in the direction *opposite* the current, the change in potential ΔV across the resistor is $+IR$ (Fig. 28.12b).
- If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from $-$ to $+$), the change in potential ΔV is $+\mathcal{E}$ (Fig. 28.12c). The emf of the battery increases the electric potential as we move through it in this direction.
- If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from $+$ to $-$), the change in potential ΔV is $-\mathcal{E}$ (Fig. 28.12d). In this case the emf of the battery reduces the electric potential as we move through it.

Limitations exist on the numbers of times you can usefully apply Kirchhoff's rules in analyzing a given circuit. You can use the junction rule as often as you need, so long as each time you write an equation you include in it a current that has not been used in a preceding junction-rule equation. In general, the number of times you can use the junction rule is one fewer than the number of junction

QuickLab

Draw an arbitrarily shaped closed loop that does not cross over itself. Label five points on the loop *a*, *b*, *c*, *d*, and *e*, and assign a random number to each point. Now start at *a* and work your way around the loop, calculating the difference between each pair of adjacent numbers. Some of these differences will be positive, and some will be negative. Add the differences together, making sure you accurately keep track of the algebraic signs. What is the sum of the differences all the way around the loop?

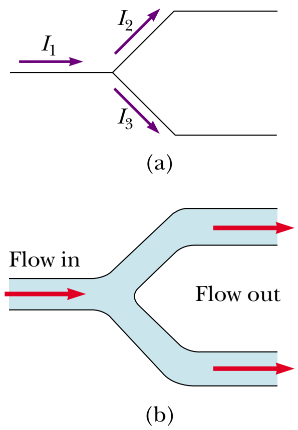


Figure 28.11 (a) Kirchhoff's junction rule. Conservation of charge requires that all current entering a junction must leave that junction. Therefore, $I_1 = I_2 + I_3$. (b) A mechanical analog of the junction rule: the amount of water flowing out of the branches on the right must equal the amount flowing into the single branch on the left.

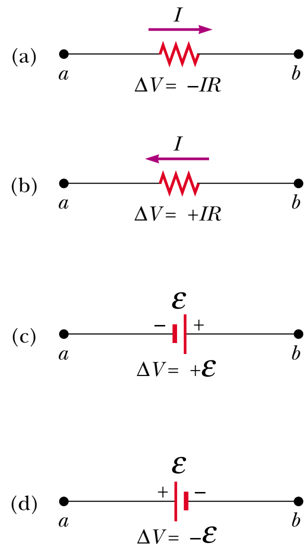


Figure 28.12 Rules for determining the potential changes across a resistor and a battery. (The battery is assumed to have no internal resistance.) Each circuit element is traversed from left to right.

points in the circuit. You can apply the loop rule as often as needed, so long as a new circuit element (resistor or battery) or a new current appears in each new equation. In general, **in order to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents.**

Complex networks containing many loops and junctions generate great numbers of independent linear equations and a correspondingly great number of unknowns. Such situations can be handled formally through the use of matrix algebra. Computer programs can also be written to solve for the unknowns.

The following examples illustrate how to use Kirchhoff's rules. In all cases, it is assumed that the circuits have reached steady-state conditions—that is, the currents in the various branches are constant. Any capacitor **acts as an open circuit;** that is, the current in the branch containing the capacitor is zero under steady-state conditions.

Problem-Solving Hints

Kirchhoff's Rules

- Draw a circuit diagram, and label all the known and unknown quantities. You must assign a *direction* to the current in each branch of the circuit. Do not be alarmed if you guess the direction of a current incorrectly; your result will be negative, but *its magnitude will be correct.* Although the assignment of current directions is arbitrary, you must adhere rigorously to the assigned directions when applying Kirchhoff's rules.
- Apply the junction rule to any junctions in the circuit that provide new relationships among the various currents.

- Apply the loop rule to as many loops in the circuit as are needed to solve for the unknowns. To apply this rule, you must correctly identify the change in potential as you imagine crossing each element in traversing the closed loop (either clockwise or counterclockwise). Watch out for errors in sign!
- Solve the equations simultaneously for the unknown quantities.

Example 28.7 A Single-Loop Circuit

A single-loop circuit contains two resistors and two batteries, as shown in Figure 28.13. (Neglect the internal resistances of the batteries.) (a) Find the current in the circuit.

Solution We do not need Kirchhoff's rules to analyze this simple circuit, but let us use them anyway just to see how they are applied. There are no junctions in this single-loop circuit; thus, the current is the same in all elements. Let us assume that the current is clockwise, as shown in Figure 28.13. Traversing the circuit in the clockwise direction, starting at *a*, we see that $a \rightarrow b$ represents a potential change of $+\mathcal{E}_1$, $b \rightarrow c$ represents a potential change of $-IR_1$, $c \rightarrow d$ represents a potential change of $-\mathcal{E}_2$, and $d \rightarrow a$ represents a potential change of $-IR_2$. Applying Kirchhoff's loop rule gives

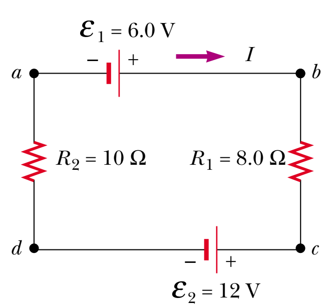


Figure 28.13 A series circuit containing two batteries and two resistors, where the polarities of the batteries are in opposition.

$$\sum \Delta V = 0$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

Solving for I and using the values given in Figure 28.13, we obtain

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\text{ V} - 12\text{ V}}{8.0\ \Omega + 10\ \Omega} = \boxed{-0.33\text{ A}}$$

The negative sign for I indicates that the direction of the current is opposite the assumed direction.

(b) What power is delivered to each resistor? What power is delivered by the 12-V battery?

Solution

$$\mathcal{P}_1 = I^2R_1 = (0.33\text{ A})^2(8.0\ \Omega) = \boxed{0.87\text{ W}}$$

$$\mathcal{P}_2 = I^2R_2 = (0.33\text{ A})^2(10\ \Omega) = \boxed{1.1\text{ W}}$$

Hence, the total power delivered to the resistors is $\mathcal{P}_1 + \mathcal{P}_2 = 2.0$ W.

The 12-V battery delivers power $I\mathcal{E}_2 = 4.0$ W. Half of this power is delivered to the two resistors, as we just calculated. The other half is delivered to the 6-V battery, which is being charged by the 12-V battery. If we had included the internal resistances of the batteries in our analysis, some of the power would appear as internal energy in the batteries; as a result, we would have found that less power was being delivered to the 6-V battery.

Example 28.8 Applying Kirchhoff's Rules

Find the currents I_1, I_2, and I_3 in the circuit shown in Figure 28.14.

Solution Notice that we cannot reduce this circuit to a simpler form by means of the rules of adding resistances in series and in parallel. We must use Kirchhoff's rules to analyze this circuit. We arbitrarily choose the directions of the currents as labeled in Figure 28.14. Applying Kirchhoff's junction rule to junction *c* gives

$$(1) \qquad I_1 + I_2 = I_3$$

We now have one equation with three unknowns—I_1, I_2, and I_3. There are three loops in the circuit—*abcda, befcb,* and *aefda.* We therefore need only two loop equations to determine the unknown currents. (The third loop equation would give no new information.) Applying Kirchhoff's loop rule to loops *abcda* and *befcb* and traversing these loops clockwise, we obtain the expressions

$$(2) \quad abcda \quad 10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)I_3 = 0$$

$$(3) \quad befcb \quad -14\text{ V} + (6\ \Omega)I_1 - 10\text{ V} - (4\ \Omega)I_2 = 0$$

Note that in loop *befcb* we obtain a positive value when traversing the 6-Ω resistor because our direction of travel is opposite the assumed direction of I_1.

Expressions (1), (2), and (3) represent three independent equations with three unknowns. Substituting Equation (1) into Equation (2) gives

$$10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)\ (I_1 + I_2) = 0$$

$$(4) \qquad 10\text{ V} = (8\ \Omega)I_1 + (2\ \Omega)I_2$$

Dividing each term in Equation (3) by 2 and rearranging gives

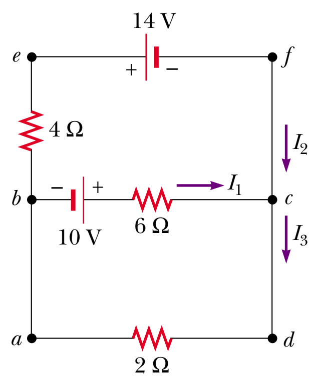


Figure 28.14 A circuit containing three loops.

$$(5) \qquad -12\text{ V} = -(3\ \Omega)I_1 + (2\ \Omega)I_2$$

Subtracting Equation (5) from Equation (4) eliminates I_2, giving

$$22\text{ V} = (11\ \Omega)I_1$$

$$I_1 = \boxed{2\text{ A}}$$

Using this value of I_1 in Equation (5) gives a value for I_2:

$$(2\ \Omega)I_2 = (3\ \Omega)I_1 - 12\text{ V} = (3\ \Omega)\ (2\text{ A}) - 12\text{ V} = -6\text{ V}$$

$$I_2 = \boxed{-3\text{ A}}$$

Finally,

$$I_3 = I_1 + I_2 = \boxed{-1\text{ A}}$$

The fact that I_2 and I_3 are both negative indicates only that the currents are opposite the direction we chose for them. However, the numerical values are correct. What would have happened had we left the current directions as labeled in Figure 28.14 but traversed the loops in the opposite direction?

Exercise Find the potential difference between points *b* and *c*.

Answer 2 V.

EXAMPLE 28.9 A Multiloop Circuit

(a) Under steady-state conditions, find the unknown currents I_1, I_2, and I_3 in the multiloop circuit shown in Figure 28.15.

Solution First note that because the capacitor represents an open circuit, there is no current between *g* and *b* along path *ghab* under steady-state conditions. Therefore, when the charges associated with I_1 reach point *g*, they all go through the 8.00-V battery to point *b*; hence, $I_{gb} = I_1$. Labeling the currents as shown in Figure 28.15 and applying Equation 28.9 to junction *c*, we obtain

$$(1) \quad I_1 + I_2 = I_3$$

Equation 28.10 applied to loops *defcd* and *cfgbc*, traversed clockwise, gives

$$(2)\ \ defcd \qquad 4.00\text{ V} - (3.00\ \Omega)I_2 - (5.00\ \Omega)I_3 = 0$$

$$(3)\ \ cfgbc \qquad (3.00\ \Omega)I_2 - (5.00\ \Omega)I_1 + 8.00\text{ V} = 0$$

From Equation (1) we see that $I_1 = I_3 - I_2$, which, when substituted into Equation (3), gives

$$(4) \quad (8.00\ \Omega)I_2 - (5.00\ \Omega)I_3 + 8.00\text{ V} = 0$$

Subtracting Equation (4) from Equation (2), we eliminate I_3 and find that

$$I_2 = -\frac{4.00\text{ V}}{11.0\ \Omega} = \boxed{-0.364\text{ A}}$$

Because our value for I_2 is negative, we conclude that the direction of I_2 is from *c* to *f* through the 3.00-Ω resistor. Despite

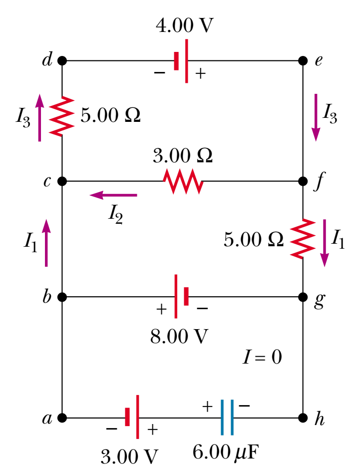


Figure 28.15 A multiloop circuit. Kirchhoff's loop rule can be applied to *any* closed loop, including the one containing the capacitor.

this interpretation of the direction, however, we must continue to use this negative value for I_2 in subsequent calculations because our equations were established with our original choice of direction.

Using $I_2 = -0.364$ A in Equations (3) and (1) gives

$$I_1 = \boxed{1.38 \text{ A}} \qquad I_3 = \boxed{1.02 \text{ A}}$$

(b) What is the charge on the capacitor?

Solution We can apply Kirchhoff's loop rule to loop *bghab* (or any other loop that contains the capacitor) to find the potential difference ΔV_{cap} across the capacitor. We enter this potential difference in the equation without reference to a sign convention because the charge on the capacitor depends only on the magnitude of the potential difference. Moving clockwise around this loop, we obtain

$$-8.00 \text{ V} + \Delta V_{\text{cap}} - 3.00 \text{ V} = 0$$

$$\Delta V_{\text{cap}} = 11.0 \text{ V}$$

Because $Q = C\Delta V_{\text{cap}}$ (see Eq. 26.1), the charge on the capacitor is

$$Q = (6.00\ \mu\text{F})(11.0 \text{ V}) = \boxed{66.0\ \mu\text{C}}$$

Why is the left side of the capacitor positively charged?

Exercise Find the voltage across the capacitor by traversing any other loop.

Answer 11.0 V.

Exercise Reverse the direction of the 3.00-V battery and answer parts (a) and (b) again.

Answer (a) $I_1 = 1.38$ A, $I_2 = -0.364$ A, $I_3 = 1.02$ A; (b) 30 μC.

28.4 *RC* CIRCUITS

So far we have been analyzing steady-state circuits, in which the current is constant. In circuits containing capacitors, the current may vary in time. A circuit containing a series combination of a resistor and a capacitor is called an ***RC* circuit.**

Charging a Capacitor

Let us assume that the capacitor in Figure 28.16 is initially uncharged. There is no current while switch S is open (Fig. 28.16b). If the switch is closed at $t = 0$, however, charge begins to flow, setting up a current in the circuit, and the capacitor begins to charge.[4] Note that during charging, charges do not jump across the capacitor plates because the gap between the plates represents an open circuit. Instead, charge is transferred between each plate and its connecting wire due to the electric field established in the wires by the battery, until the capacitor is fully charged. As the plates become charged, the potential difference across the capacitor increases. The value of the maximum charge depends on the voltage of the battery. Once the maximum charge is reached, the current in the circuit is zero because the potential difference across the capacitor matches that supplied by the battery.

To analyze this circuit quantitatively, let us apply Kirchhoff's loop rule to the circuit after the switch is closed. Traversing the loop clockwise gives

$$\mathcal{E} - \frac{q}{C} - IR = 0 \tag{28.11}$$

where q/C is the potential difference across the capacitor and IR is the potential

[4] In previous discussions of capacitors, we assumed a steady-state situation, in which no current was present in any branch of the circuit containing a capacitor. Now we are considering the case *before* the steady-state condition is realized; in this situation, charges are moving and a current exists in the wires connected to the capacitor.

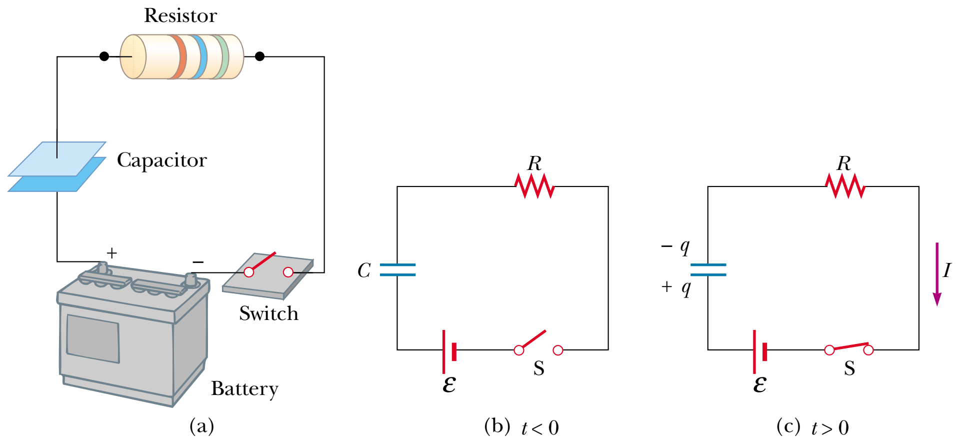


Figure 28.16 (a) A capacitor in series with a resistor, switch, and battery. (b) Circuit diagram representing this system at time $t < 0$, before the switch is closed. (c) Circuit diagram at time $t > 0$, after the switch has been closed.

difference across the resistor. We have used the sign conventions discussed earlier for the signs on $\mathcal{E}$ and IR. For the capacitor, notice that we are traveling in the direction from the positive plate to the negative plate; this represents a decrease in potential. Thus, we use a negative sign for this voltage in Equation 28.11. Note that q and I are *instantaneous* values that depend on time (as opposed to steady-state values) as the capacitor is being charged.

We can use Equation 28.11 to find the initial current in the circuit and the maximum charge on the capacitor. At the instant the switch is closed ($t = 0$), the charge on the capacitor is zero, and from Equation 28.11 we find that the initial current in the circuit I_0 is a maximum and is equal to

$$I_0 = \frac{\mathcal{E}}{R} \qquad \text{(current at } t = 0\text{)} \tag{28.12}$$

Maximum current

At this time, the potential difference from the battery terminals appears entirely across the resistor. Later, when the capacitor is charged to its maximum value Q, charges cease to flow, the current in the circuit is zero, and the potential difference from the battery terminals appears entirely across the capacitor. Substituting $I = 0$ into Equation 28.11 gives the charge on the capacitor at this time:

$$Q = C\mathcal{E} \qquad \text{(maximum charge)} \tag{28.13}$$

Maximum charge on the capacitor

To determine analytical expressions for the time dependence of the charge and current, we must solve Equation 28.11—a single equation containing two variables, q and I. The current in all parts of the series circuit must be the same. Thus, the current in the resistance R must be the same as the current flowing out of and into the capacitor plates. This current is equal to the time rate of change of the charge on the capacitor plates. Thus, we substitute $I = dq/dt$ into Equation 28.11 and rearrange the equation:

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

To find an expression for q, we first combine the terms on the right-hand side:

$$\frac{dq}{dt} = \frac{C\mathcal{E}}{RC} - \frac{q}{RC} = -\frac{q - C\mathcal{E}}{RC}$$

Now we multiply by dt and divide by $q - C\mathcal{E}$ to obtain

$$\frac{dq}{q - C\mathcal{E}} = -\frac{1}{RC}\,dt$$

Integrating this expression, using the fact that $q = 0$ at $t = 0$, we obtain

$$\int_0^q \frac{dq}{q - C\mathcal{E}} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

From the definition of the natural logarithm, we can write this expression as

Charge versus time for a capacitor being charged

$$q(t) = C\mathcal{E}\,(1 - e^{-t/RC}) = Q(1 - e^{-t/RC}) \qquad \textbf{(28.14)}$$

where e is the base of the natural logarithm and we have made the substitution $C\mathcal{E} = Q$ from Equation 28.13.

We can find an expression for the charging current by differentiating Equation 28.14 with respect to time. Using $I = dq/dt$, we find that

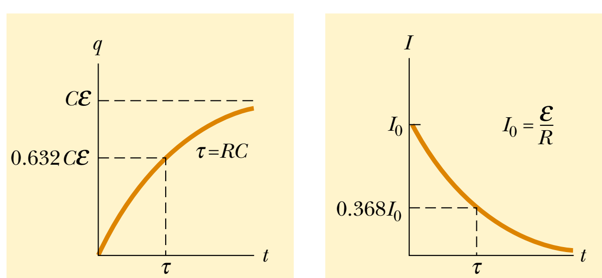
Current versus time for a charging capacitor

$$I(t) = \frac{\mathcal{E}}{R}\,e^{-t/RC} \qquad \textbf{(28.15)}$$

Plots of capacitor charge and circuit current versus time are shown in Figure 28.17. Note that the charge is zero at $t = 0$ and approaches the maximum value $C\mathcal{E}$ as $t \rightarrow \infty$. The current has its maximum value $I_0 = \mathcal{E}/R$ at $t = 0$ and decays exponentially to zero as $t \rightarrow \infty$. The quantity RC, which appears in the exponents of Equations 28.14 and 28.15, is called the **time constant** τ of the circuit. It represents the time it takes the current to decrease to $1/e$ of its initial value; that is, in a time τ, $I = e^{-1}I_0 = 0.368I_0$. In a time 2τ, $I = e^{-2}I_0 = 0.135I_0$, and so forth. Likewise, in a time τ, the charge increases from zero to $C\mathcal{E}\,(1 - e^{-1}) = 0.632C\mathcal{E}$.

The following dimensional analysis shows that τ has the units of time:

$$[\tau] = [RC] = \left[\frac{\Delta V}{I} \times \frac{Q}{\Delta V}\right] = \left[\frac{Q}{Q/\Delta t}\right] = [\Delta t] = \mathrm{T}$$



(a) (b)

Figure 28.17 (a) Plot of capacitor charge versus time for the circuit shown in Figure 28.16. After a time interval equal to one time constant τ has passed, the charge is 63.2% of the maximum value $C\mathcal{E}$. The charge approaches its maximum value as t approaches infinity. (b) Plot of current versus time for the circuit shown in Figure 28.16. The current has its maximum value $I_0 = \mathcal{E}/R$ at $t = 0$ and decays to zero exponentially as t approaches infinity. After a time interval equal to one time constant τ has passed, the current is 36.8% of its initial value.

Because $\tau = RC$ has units of time, the combination t/RC is dimensionless, as it must be in order to be an exponent of e in Equations 28.14 and 28.15.

The energy output of the battery as the capacitor is fully charged is $Q\mathcal{E} = C\mathcal{E}^2$. After the capacitor is fully charged, the energy stored in the capacitor is $\frac{1}{2}Q\mathcal{E} = \frac{1}{2}C\mathcal{E}^2$, which is just half the energy output of the battery. It is left as a problem (Problem 60) to show that the remaining half of the energy supplied by the battery appears as internal energy in the resistor.

Discharging a Capacitor

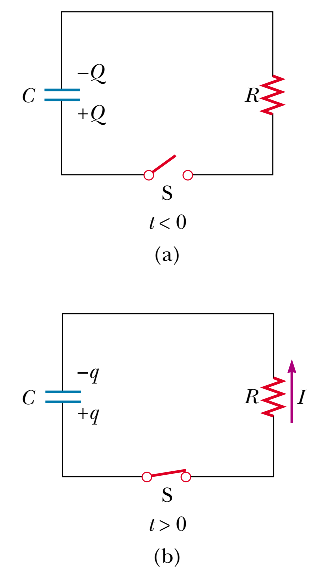


Figure 28.18 (a) A charged capacitor connected to a resistor and a switch, which is open at $t < 0$. (b) After the switch is closed, a current that decreases in magnitude with time is set up in the direction shown, and the charge on the capacitor decreases exponentially with time.

Now let us consider the circuit shown in Figure 28.18, which consists of a capacitor carrying an initial charge Q, a resistor, and a switch. The *initial* charge Q is not the same as the *maximum* charge Q in the previous discussion, unless the discharge occurs after the capacitor is fully charged (as described earlier). When the switch is open, a potential difference Q/C exists across the capacitor and there is zero potential difference across the resistor because $I = 0$. If the switch is closed at $t = 0$, the capacitor begins to discharge through the resistor. At some time t during the discharge, the current in the circuit is I and the charge on the capacitor is q (Fig. 28.18b). The circuit in Figure 28.18 is the same as the circuit in Figure 28.16 except for the absence of the battery. Thus, we eliminate the emf $\mathcal{E}$ from Equation 28.11 to obtain the appropriate loop equation for the circuit in Figure 28.18:

$$-\frac{q}{C} - IR = 0 \qquad \textbf{(28.16)}$$

When we substitute $I = dq/dt$ into this expression, it becomes

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{1}{RC}\,dt$$

Integrating this expression, using the fact that $q = Q$ at $t = 0$, gives

$$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

Charge versus time for a discharging capacitor

$$q(t) = Qe^{-t/RC} \qquad \textbf{(28.17)}$$

Differentiating this expression with respect to time gives the instantaneous current as a function of time:

Current versus time for a discharging capacitor

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}(Qe^{-t/RC}) = -\frac{Q}{RC}e^{-t/RC} \qquad \textbf{(28.18)}$$

where $Q/RC = I_0$ is the initial current. The negative sign indicates that the current direction now that the capacitor is discharging is opposite the current direction when the capacitor was being charged. (Compare the current directions in Figs. 28.16c and 28.18b.) We see that both the charge on the capacitor and the current decay exponentially at a rate characterized by the time constant $\tau = RC$.

CONCEPTUAL EXAMPLE 28.10 Intermittent Windshield Wipers

Many automobiles are equipped with windshield wipers that can operate intermittently during a light rainfall. How does the operation of such wipers depend on the charging and discharging of a capacitor?

Solution The wipers are part of an *RC* circuit whose time constant can be varied by selecting different values of *R* through a multiposition switch. As it increases with time, the voltage across the capacitor reaches a point at which it triggers the wipers and discharges, ready to begin another charging cycle. The time interval between the individual sweeps of the wipers is determined by the value of the time constant.

EXAMPLE 28.11 Charging a Capacitor in an *RC* Circuit

An uncharged capacitor and a resistor are connected in series to a battery, as shown in Figure 28.19. If $\mathcal{E} = 12.0$ V, $C = 5.00\ \mu$F, and $R = 8.00 \times 10^5\ \Omega$, find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as functions of time.

Solution The time constant of the circuit is $\tau = RC = (8.00 \times 10^5\ \Omega)(5.00 \times 10^{-6}\ \text{F}) = 4.00$ s. The maximum charge on the capacitor is $Q = C\mathcal{E} = (5.00\ \mu\text{F})(12.0\ \text{V}) = 60.0\ \mu\text{C}$. The maximum current in the circuit is $I_0 = \mathcal{E}/R = (12.0\ \text{V})/(8.00 \times 10^5\ \Omega) = 15.0\ \mu\text{A}$. Using these values and Equations 28.14 and 28.15, we find that

$$q(t) = (60.0\ \mu\text{C})(1 - e^{-t/4.00\ \text{s}})$$

$$I(t) = (15.0\ \mu\text{A})\ e^{-t/4.00\ \text{s}}$$

Graphs of these functions are provided in Figure 28.20.

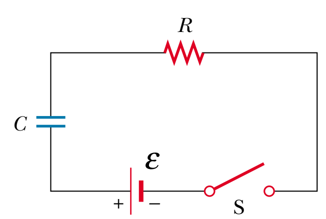


Figure 28.19 The switch of this series *RC* circuit, open for times $t < 0$, is closed at $t = 0$.

Exercise Calculate the charge on the capacitor and the current in the circuit after one time constant has elapsed.

Answer 37.9 μC, 5.52 μA.

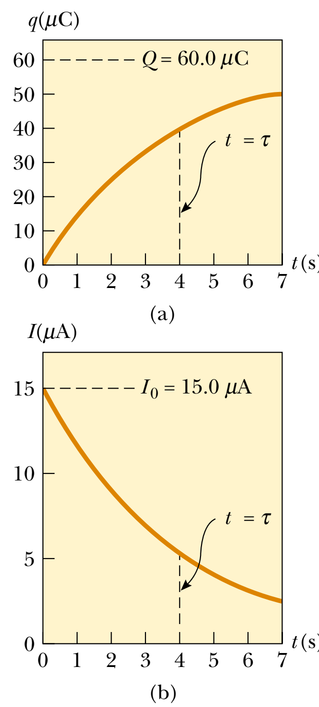


Figure 28.20 Plots of (a) charge versus time and (b) current versus time for the *RC* circuit shown in Figure 28.19, with $\mathcal{E} = 12.0$ V, $R = 8.00 \times 10^5\ \Omega$, and $C = 5.00\ \mu$F.

EXAMPLE 28.12 Discharging a Capacitor in an *RC* Circuit

Consider a capacitor of capacitance *C* that is being discharged through a resistor of resistance *R*, as shown in Figure 28.18. (a) After how many time constants is the charge on the capacitor one-fourth its initial value?

Solution The charge on the capacitor varies with time according to Equation 28.17, $q(t) = Qe^{-t/RC}$. To find the time it takes q to drop to one-fourth its initial value, we substitute $q(t) = Q/4$ into this expression and solve for t:

$$\frac{Q}{4} = Qe^{-t/RC}$$

$$\tfrac{1}{4} = e^{-t/RC}$$

Taking logarithms of both sides, we find

$$-\ln 4 = -\frac{t}{RC}$$

$$t = RC(\ln 4) = 1.39RC = \boxed{1.39\tau}$$

(b) The energy stored in the capacitor decreases with time as the capacitor discharges. After how many time constants is this stored energy one-fourth its initial value?

Solution Using Equations 26.11 ($U = Q^2/2C$) and 28.17, we can express the energy stored in the capacitor at any time t as

$$U = \frac{q^2}{2C} = \frac{(Qe^{-t/RC})^2}{2C} = \frac{Q^2}{2C}e^{-2t/RC} = U_0 e^{-2t/RC}$$

where $U_0 = Q^2/2C$ is the initial energy stored in the capacitor. As in part (a), we now set $U = U_0/4$ and solve for t:

$$\frac{U_0}{4} = U_0 e^{-2t/RC}$$

$$\tfrac{1}{4} = e^{-2t/RC}$$

Again, taking logarithms of both sides and solving for t gives

$$t = \tfrac{1}{2}RC(\ln 4) = 0.693RC = \boxed{0.693\tau}$$

Exercise After how many time constants is the current in the circuit one-half its initial value?

Answer $0.693RC = 0.693\tau$.

EXAMPLE 28.13 Energy Delivered to a Resistor

A 5.00-μF capacitor is charged to a potential difference of 800 V and then discharged through a 25.0-kΩ resistor. How much energy is delivered to the resistor in the time it takes to fully discharge the capacitor?

Solution We shall solve this problem in two ways. The first way is to note that the initial energy in the circuit equals the energy stored in the capacitor, $C\mathcal{E}^2/2$ (see Eq. 26.11). Once the capacitor is fully discharged, the energy stored in it is zero. Because energy is conserved, the initial energy stored in the capacitor is transformed into internal energy in the resistor. Using the given values of C and $\mathcal{E}$, we find

$$\text{Energy} = \tfrac{1}{2}C\mathcal{E}^2 = \tfrac{1}{2}(5.00 \times 10^{-6}\text{ F})(800\text{ V})^2 = \boxed{1.60\text{ J}}$$

The second way, which is more difficult but perhaps more instructive, is to note that as the capacitor discharges through the resistor, the rate at which energy is delivered to the resistor is given by I^2R, where I is the instantaneous current given by Equation 28.18. Because power is defined as the time rate of change of energy, we conclude that the energy delivered to the resistor must equal the time integral of $I^2R\,dt$:

$$\text{Energy} = \int_0^\infty I^2 R\,dt = \int_0^\infty (I_0 e^{-t/RC})^2 R\,dt$$

To evaluate this integral, we note that the initial current I_0 is equal to $\mathcal{E}/R$ and that all parameters except t are constant. Thus, we find

$$(1) \qquad \text{Energy} = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-2t/RC}\,dt$$

This integral has a value of $RC/2$; hence, we find

$$\text{Energy} = \tfrac{1}{2}C\mathcal{E}^2$$

which agrees with the result we obtained using the simpler approach, as it must. Note that we can use this second approach to find the total energy delivered to the resistor at *any* time after the switch is closed by simply replacing the upper limit in the integral with that specific value of t.

Exercise Show that the integral in Equation (1) has the value $RC/2$.

Optional Section

28.5 ELECTRICAL INSTRUMENTS

The Ammeter

A device that measures current is called an **ammeter.** The current to be measured must pass directly through the ammeter, so the ammeter must be connected in se-

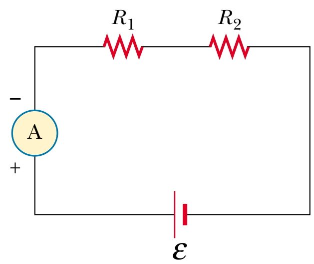


Figure 28.21 Current can be measured with an ammeter connected in series with the resistor and battery of a circuit. An ideal ammeter has zero resistance.

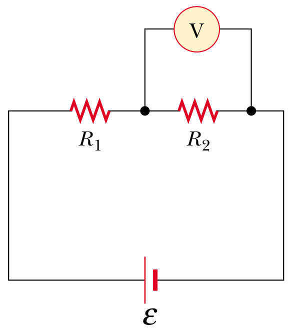


Figure 28.22 The potential difference across a resistor can be measured with a voltmeter connected in parallel with the resistor. An ideal voltmeter has infinite resistance.

ries with other elements in the circuit, as shown in Figure 28.21. When using an ammeter to measure direct currents, you must be sure to connect it so that current enters the instrument at the positive terminal and exits at the negative terminal.

Ideally, an ammeter should have zero resistance so that the current being measured is not altered. In the circuit shown in Figure 28.21, this condition requires that the resistance of the ammeter be much less than $R_1 + R_2$. Because any ammeter always has some internal resistance, the presence of the ammeter in the circuit slightly reduces the current from the value it would have in the meter's absence.

The Voltmeter

A device that measures potential difference is called a **voltmeter.** The potential difference between any two points in a circuit can be measured by attaching the terminals of the voltmeter between these points without breaking the circuit, as shown in Figure 28.22. The potential difference across resistor R_2 is measured by connecting the voltmeter in parallel with R_2. Again, it is necessary to observe the polarity of the instrument. The positive terminal of the voltmeter must be connected to the end of the resistor that is at the higher potential, and the negative terminal to the end of the resistor at the lower potential.

An ideal voltmeter has infinite resistance so that no current passes through it. In Figure 28.22, this condition requires that the voltmeter have a resistance much greater than R_2. In practice, if this condition is not met, corrections should be made for the known resistance of the voltmeter.

The Galvanometer

The **galvanometer** is the main component in analog ammeters and voltmeters. Figure 28.23a illustrates the essential features of a common type called the *D'Arsonval galvanometer.* It consists of a coil of wire mounted so that it is free to rotate on a pivot in a magnetic field provided by a permanent magnet. The basic op-

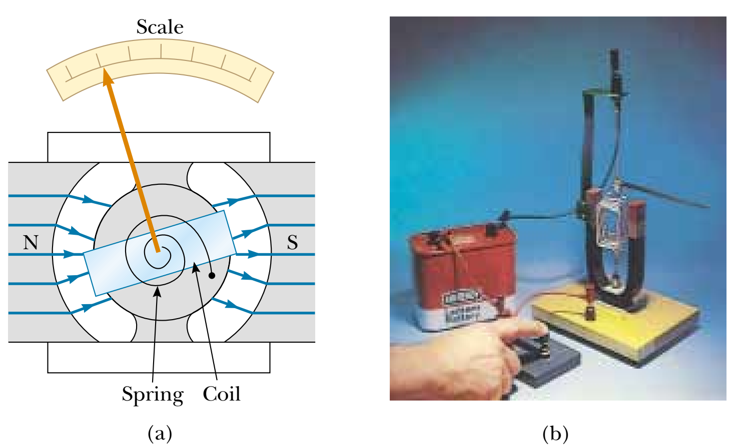


Figure 28.23 (a) The principal components of a D'Arsonval galvanometer. When the coil situated in a magnetic field carries a current, the magnetic torque causes the coil to twist. The angle through which the coil rotates is proportional to the current in the coil because of the counteracting torque of the spring. (b) A large-scale model of a galvanometer movement. Why does the coil rotate about the vertical axis after the switch is closed?

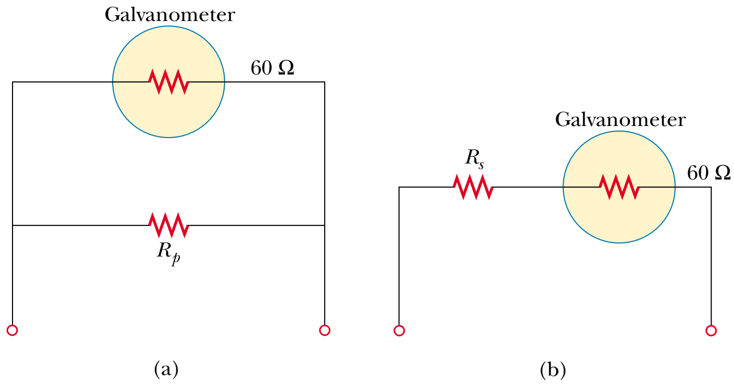


Figure 28.24 (a) When a galvanometer is to be used as an ammeter, a shunt resistor R_p is connected in parallel with the galvanometer. (b) When the galvanometer is used as a voltmeter, a resistor R_s is connected in series with the galvanometer.

eration of the galvanometer makes use of the fact that a torque acts on a current loop in the presence of a magnetic field (Chapter 29). The torque experienced by the coil is proportional to the current through it: the larger the current, the greater the torque and the more the coil rotates before the spring tightens enough to stop the rotation. Hence, the deflection of a needle attached to the coil is proportional to the current. Once the instrument is properly calibrated, it can be used in conjunction with other circuit elements to measure either currents or potential differences.

A typical off-the-shelf galvanometer is often not suitable for use as an ammeter, primarily because it has a resistance of about 60 Ω. An ammeter resistance this great considerably alters the current in a circuit. You can understand this by considering the following example: The current in a simple series circuit containing a 3-V battery and a 3-Ω resistor is 1 A. If you insert a 60-Ω galvanometer in this circuit to measure the current, the total resistance becomes 63 Ω and the current is reduced to 0.048 A!

A second factor that limits the use of a galvanometer as an ammeter is the fact that a typical galvanometer gives a full-scale deflection for currents of the order of 1 mA or less. Consequently, such a galvanometer cannot be used directly to measure currents greater than this value. However, it can be converted to a useful ammeter by placing a shunt resistor R_p in parallel with the galvanometer, as shown in Figure 28.24a. The value of R_p must be much less than the galvanometer resistance so that most of the current to be measured passes through the shunt resistor.

A galvanometer can also be used as a voltmeter by adding an external resistor R_s in series with it, as shown in Figure 28.24b. In this case, the external resistor must have a value much greater than the resistance of the galvanometer to ensure that the galvanometer does not significantly alter the voltage being measured.

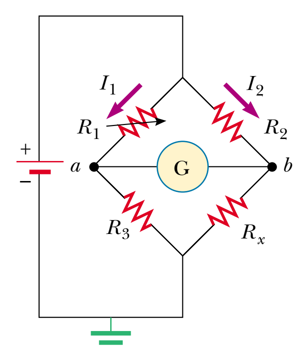
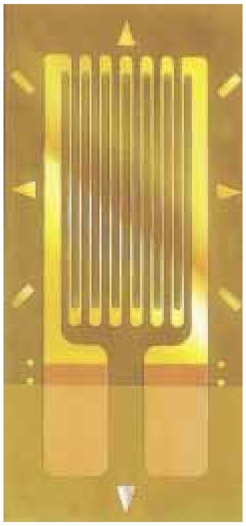


Figure 28.25 Circuit diagram for a Wheatstone bridge, an instrument used to measure an unknown resistance R_x in terms of known resistances R_1, R_2, and R_3. When the bridge is balanced, no current is present in the galvanometer. The arrow superimposed on the circuit symbol for resistor R_1 indicates that the value of this resistor can be varied by the person operating the bridge.

The Wheatstone Bridge

An unknown resistance value can be accurately measured using a circuit known as a **Wheatstone bridge** (Fig. 28.25). This circuit consists of the unknown resistance R_x, three known resistances R_1, R_2, and R_3 (where R_1 is a calibrated variable resistor), a galvanometer, and a battery. The known resistor R_1 is varied until the galvanometer reading is zero—that is, until there is no current from a to b. Under this condition the bridge is said to be balanced. Because the electric potential at



The strain gauge, a device used for experimental stress analysis, consists of a thin coiled wire bonded to a flexible plastic backing. The gauge measures stresses by detecting changes in the resistance of the coil as the strip bends. Resistance measurements are made with this device as one element of a Wheatstone bridge. Strain gauges are commonly used in modern electronic balances to measure the masses of objects.

point a must equal the potential at point b when the bridge is balanced, the potential difference across R_1 must equal the potential difference across R_2. Likewise, the potential difference across R_3 must equal the potential difference across R_x. From these considerations we see that

$$(1) \qquad I_1R_1 = I_2R_2$$

$$(2) \qquad I_1R_3 = I_2R_x$$

Dividing Equation (1) by Equation (2) eliminates the currents, and solving for R_x, we find that

$$R_x = \frac{R_2R_3}{R_1} \tag{28.19}$$

A number of similar devices also operate on the principle of null measurement (that is, adjustment of one circuit element to make the galvanometer read zero). One example is the capacitance bridge used to measure unknown capacitances. These devices do not require calibrated meters and can be used with any voltage source.

Wheatstone bridges are not useful for resistances above $10^5\ \Omega$, but modern electronic instruments can measure resistances as high as $10^{12}\ \Omega$. Such instruments have an extremely high resistance between their input terminals. For example, input resistances of $10^{10}\ \Omega$ are common in most digital multimeters, which are devices that are used to measure voltage, current, and resistance (Fig. 28.26).

Figure 28.26 Voltages, currents, and resistances are frequently measured with digital multimeters like this one.

The Potentiometer

A **potentiometer** is a circuit that is used to measure an unknown emf $\mathcal{E}_x$ by comparison with a known emf. In Figure 28.27, point d represents a sliding contact that is used to vary the resistance (and hence the potential difference) between points a and d. The other required components are a galvanometer, a battery of known emf $\mathcal{E}_0$, and a battery of unknown emf $\mathcal{E}_x$.

With the currents in the directions shown in Figure 28.27, we see from Kirchhoff's junction rule that the current in the resistor R_x is $I - I_x$, where I is the current in the left branch (through the battery of emf $\mathcal{E}_0$) and I_x is the current in the right branch. Kirchhoff's loop rule applied to loop *abcda* traversed clockwise gives

$$-\mathcal{E}_x + (I - I_x)R_x = 0$$

Because current I_x passes through it, the galvanometer displays a nonzero reading. The sliding contact at d is now adjusted until the galvanometer reads zero (indicating a balanced circuit and that the potentiometer is another null-measurement device). Under this condition, the current in the galvanometer is zero, and the potential difference between a and d must equal the unknown emf $\mathcal{E}_x$:

$$\mathcal{E}_x = IR_x$$

Next, the battery of unknown emf is replaced by a standard battery of known emf $\mathcal{E}_s$, and the procedure is repeated. If R_s is the resistance between a and d when balance is achieved this time, then

$$\mathcal{E}_s = IR_s$$

where it is assumed that I remains the same. Combining this expression with the preceding one, we see that

$$\mathcal{E}_x = \frac{R_x}{R_s}\mathcal{E}_s \tag{28.20}$$

If the resistor is a wire of resistivity ρ, its resistance can be varied by using the sliding contact to vary the length L, indicating how much of the wire is part of the circuit. With the substitutions $R_s = \rho L_s/A$ and $R_x = \rho L_x/A$, Equation 28.20 becomes

$$\mathcal{E}_x = \frac{L_x}{L_s}\mathcal{E}_s \quad (28.21)$$

where L_x is the resistor length when the battery of unknown emf $\mathcal{E}_x$ is in the circuit and L_s is the resistor length when the standard battery is in the circuit.

The sliding-wire circuit of Figure 28.27 without the unknown emf and the galvanometer is sometimes called a *voltage divider.* This circuit makes it possible to tap into any desired smaller portion of the emf $\mathcal{E}_0$ by adjusting the length of the resistor.

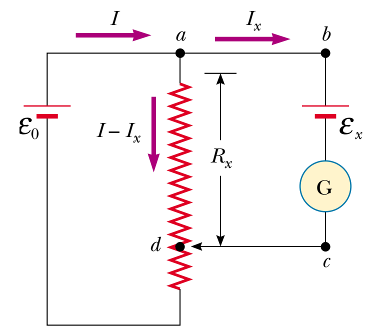


Figure 28.27 Circuit diagram for a potentiometer. The circuit is used to measure an unknown emf $\mathcal{E}_x$.

Optional Section

28.6 HOUSEHOLD WIRING AND ELECTRICAL SAFETY

Household circuits represent a practical application of some of the ideas presented in this chapter. In our world of electrical appliances, it is useful to understand the power requirements and limitations of conventional electrical systems and the safety measures that prevent accidents.

In a conventional installation, the utility company distributes electric power to individual homes by means of a pair of wires, with each home connected in parallel to these wires. One wire is called the *live wire,*[5] as illustrated in Figure 28.28, and the other is called the *neutral wire.* The potential difference between these two wires is about 120 V. This voltage alternates in time, with the neutral wire connected to ground and the potential of the live wire oscillating relative to ground. Much of what we have learned so far for the constant-emf situation (direct current) can also be applied to the alternating current that power companies supply to businesses and households. (Alternating voltage and current are discussed in Chapter 33.)

A meter is connected in series with the live wire entering the house to record the household's usage of electricity. After the meter, the wire splits so that there are several separate circuits in parallel distributed throughout the house. Each circuit contains a circuit breaker (or, in older installations, a fuse). The wire and circuit breaker for each circuit are carefully selected to meet the current demands for that circuit. If a circuit is to carry currents as large as 30 A, a heavy wire and an appropriate circuit breaker must be selected to handle this current. A circuit used to power only lamps and small appliances often requires only 15 A. Each circuit has its own circuit breaker to accommodate various load conditions.

As an example, consider a circuit in which a toaster oven, a microwave oven, and a coffee maker are connected (corresponding to R_1, R_2, and R_3 in Figure 28.28 and as shown in the chapter-opening photograph). We can calculate the current drawn by each appliance by using the expression $\mathcal{P} = I\Delta V$. The toaster oven, rated at 1 000 W, draws a current of 1 000 W/120 V = 8.33 A. The microwave oven, rated at 1 300 W, draws 10.8 A, and the coffee maker, rated at 800 W, draws 6.67 A. If the three appliances are operated simultaneously, they draw a total cur-

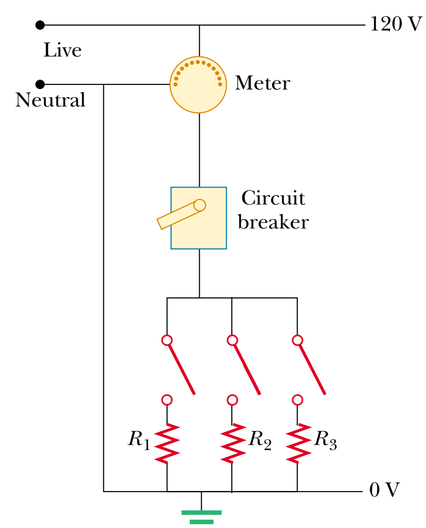


Figure 28.28 Wiring diagram for a household circuit. The resistances represent appliances or other electrical devices that operate with an applied voltage of 120 V.

[5] *Live wire* is a common expression for a conductor whose electric potential is above or below ground potential.

Figure 28.29 A power connection for a 240-V appliance.

rent of 25.8 A. Therefore, the circuit should be wired to handle at least this much current. If the rating of the circuit breaker protecting the circuit is too small—say, 20 A—the breaker will be tripped when the third appliance is turned on, preventing all three appliances from operating. To avoid this situation, the toaster oven and coffee maker can be operated on one 20-A circuit and the microwave oven on a separate 20-A circuit.

Many heavy-duty appliances, such as electric ranges and clothes dryers, require 240 V for their operation (Fig. 28.29). The power company supplies this voltage by providing a third wire that is 120 V below ground potential. The potential difference between this live wire and the other live wire (which is 120 V above ground potential) is 240 V. An appliance that operates from a 240-V line requires half the current of one operating from a 120-V line; therefore, smaller wires can be used in the higher-voltage circuit without overheating.

Electrical Safety

When the live wire of an electrical outlet is connected directly to ground, the circuit is completed and a short-circuit condition exists. A *short circuit* occurs when almost zero resistance exists between two points at different potentials; this results in a very large current. When this happens accidentally, a properly operating circuit breaker opens the circuit and no damage is done. However, a person in contact with ground can be electrocuted by touching the live wire of a frayed cord or other exposed conductor. An exceptionally good (although very dangerous) ground contact is made when the person either touches a water pipe (normally at ground potential) or stands on the ground with wet feet. The latter situation represents a good ground because normal, nondistilled water is a conductor because it contains a large number of ions associated with impurities. This situation should be avoided at all cost.

Figure 28.30 A three-pronged power cord for a 120-V appliance.

Electric shock can result in fatal burns, or it can cause the muscles of vital organs, such as the heart, to malfunction. The degree of damage to the body depends on the magnitude of the current, the length of time it acts, the part of the body touched by the live wire, and the part of the body through which the current passes. Currents of 5 mA or less cause a sensation of shock but ordinarily do little or no damage. If the current is larger than about 10 mA, the muscles contract and the person may be unable to release the live wire. If a current of about 100 mA passes through the body for only a few seconds, the result can be fatal. Such a large current paralyzes the respiratory muscles and prevents breathing. In some cases, currents of about 1 A through the body can produce serious (and sometimes fatal) burns. In practice, no contact with live wires is regarded as safe whenever the voltage is greater than 24 V.

Many 120-V outlets are designed to accept a three-pronged power cord such as the one shown in Figure 28.30. (This feature is required in all new electrical installations.) One of these prongs is the live wire at a nominal potential of 120 V. The second, called the "neutral," is nominally at 0 V and carries current to ground. The third, round prong is a safety ground wire that normally carries no current but is both grounded and connected directly to the casing of the appliance. If the live wire is accidentally shorted to the casing (which can occur if the wire insulation wears off), most of the current takes the low-resistance path through the appliance to ground. In contrast, if the casing of the appliance is not properly grounded and a short occurs, anyone in contact with the appliance experiences an electric shock because the body provides a low-resistance path to ground.

Special power outlets called *ground-fault interrupters* (GFIs) are now being used in kitchens, bathrooms, basements, exterior outlets, and other hazardous areas of new homes. These devices are designed to protect persons from electric shock by sensing small currents (≈ 5 mA) leaking to ground. (The principle of their operation is described in Chapter 31.) When an excessive leakage current is detected, the current is shut off in less than 1 ms.

Quick Quiz 28.4

Is a circuit breaker wired in series or in parallel with the device it is protecting?

SUMMARY

The **emf** of a battery is equal to the voltage across its terminals when the current is zero. That is, the emf is equivalent to the **open-circuit voltage** of the battery.

The **equivalent resistance** of a set of resistors connected in **series** is

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots \tag{28.6}$$

The **equivalent resistance** of a set of resistors connected in **parallel** is

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \tag{28.8}$$

If it is possible to combine resistors into series or parallel equivalents, the preceding two equations make it easy to determine how the resistors influence the rest of the circuit.

Circuits involving more than one loop are conveniently analyzed with the use of **Kirchhoff's rules:**

1. The sum of the currents entering any junction in an electric circuit must equal the sum of the currents leaving that junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}} \tag{28.9}$$

2. The sum of the potential differences across all elements around any circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0 \tag{28.10}$$

The first rule is a statement of conservation of charge; the second is equivalent to a statement of conservation of energy.

When a resistor is traversed in the direction of the current, the change in potential ΔV across the resistor is $-IR$. When a resistor is traversed in the direction opposite the current, $\Delta V = +IR$. When a source of emf is traversed in the direction of the emf (negative terminal to positive terminal), the change in potential is $+\boldsymbol{\mathcal{E}}$. When a source of emf is traversed opposite the emf (positive to negative), the change in potential is $-\boldsymbol{\mathcal{E}}$. The use of these rules together with Equations 28.9 and 28.10 allows you to analyze electric circuits.

If a capacitor is charged with a battery through a resistor of resistance R, the charge on the capacitor and the current in the circuit vary in time according to the expressions

$$q(t) = Q(1 - e^{-t/RC}) \tag{28.14}$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC} \tag{28.15}$$

where $Q = C\mathcal{E}$ is the maximum charge on the capacitor. The product RC is called the **time constant** τ of the circuit. If a charged capacitor is discharged through a resistor of resistance R, the charge and current decrease exponentially in time according to the expressions

$$q(t) = Qe^{-t/RC} \tag{28.17}$$

$$I(t) = -\frac{Q}{RC} e^{-t/RC} \tag{28.18}$$

where Q is the initial charge on the capacitor and $Q/RC = I_0$ is the initial current in the circuit. Equations 28.14, 28.15, 28.17, and 28.18 permit you to analyze the current and potential differences in an RC circuit and the charge stored in the circuit's capacitor.

QUESTIONS

1. Explain the difference between load resistance in a circuit and internal resistance in a battery.
2. Under what condition does the potential difference across the terminals of a battery equal its emf? Can the terminal voltage ever exceed the emf? Explain.
3. Is the direction of current through a battery always from the negative terminal to the positive one? Explain.
4. How would you connect resistors so that the equivalent resistance is greater than the greatest individual resistance? Give an example involving three resistors.
5. How would you connect resistors so that the equivalent resistance is less than the least individual resistance? Give an example involving three resistors.
6. Given three lightbulbs and a battery, sketch as many different electric circuits as you can.
7. Which of the following are the same for each resistor in a series connection—potential difference, current, power?
8. Which of the following are the same for each resistor in a parallel connection—potential difference, current, power?
9. What advantage might there be in using two identical resistors in parallel connected in series with another identical parallel pair, rather than just using a single resistor?
10. An incandescent lamp connected to a 120-V source with a short extension cord provides more illumination than the same lamp connected to the same source with a very long extension cord. Explain why.
11. When can the potential difference across a resistor be positive?
12. In Figure 28.15, suppose the wire between points g and h is replaced by a 10-Ω resistor. Explain why this change does not affect the currents calculated in Example 28.9.
13. Describe what happens to the lightbulb shown in Figure Q28.13 after the switch is closed. Assume that the capacitor has a large capacitance and is initially uncharged, and assume that the light illuminates when connected directly across the battery terminals.

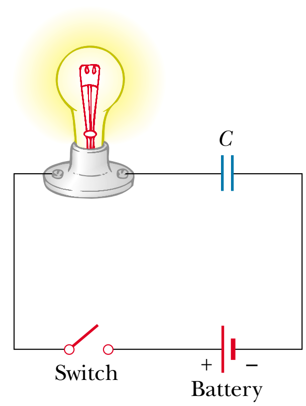


Figure Q28.13

14. What are the internal resistances of an ideal ammeter? of an ideal voltmeter? Do real meters ever attain these ideals?
15. Although the internal resistances of all sources of emf were neglected in the treatment of the potentiometer (Section 28.5), it is really not necessary to make this assumption. Explain why internal resistances play no role in the measurement of $\mathcal{E}_x$.

16. Why is it dangerous to turn on a light when you are in the bathtub?
17. Suppose you fall from a building, and on your way down you grab a high-voltage wire. Assuming that you are hanging from the wire, will you be electrocuted? If the wire then breaks, should you continue to hold onto an end of the wire as you fall?
18. What advantage does 120-V operation offer over 240 V? What are its disadvantages compared with 240 V?
19. When electricians work with potentially live wires, they often use the backs of their hands or fingers to move the wires. Why do you suppose they employ this technique?
20. What procedure would you use to try to save a person who is "frozen" to a live high-voltage wire without endangering your own life?
21. If it is the current through the body that determines the seriousness of a shock, why do we see warnings of high *voltage* rather than high *current* near electrical equipment?
22. Suppose you are flying a kite when it strikes a high-voltage wire. What factors determine how great a shock you receive?
23. A series circuit consists of three identical lamps that are connected to a battery as shown in Figure Q28.23. When switch S is closed, what happens (a) to the intensities of lamps A and B, (b) to the intensity of lamp C, (c) to the current in the circuit, and (d) to the voltage across the three lamps? (e) Does the power delivered to the circuit increase, decrease, or remain the same?

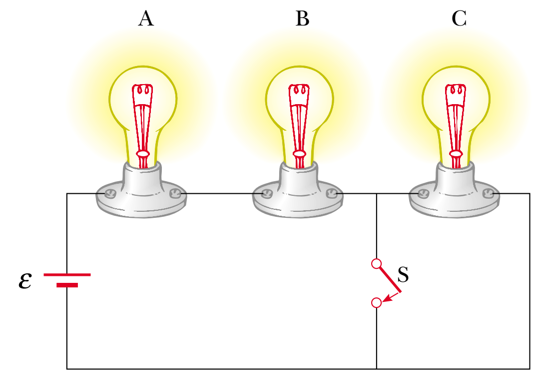


Figure Q28.23

24. If your car's headlights are on when you start the ignition, why do they dim while the car is starting?
25. A ski resort consists of a few chair lifts and several interconnected downhill runs on the side of a mountain, with a lodge at the bottom. The lifts are analogous to batteries, and the runs are analogous to resistors. Describe how two runs can be in series. Describe how three runs can be in parallel. Sketch a junction of one lift and two runs. State Kirchhoff's junction rule for ski resorts. One of the skiers, who happens to be carrying an altimeter, stops to warm up her toes each time she passes the lodge. State Kirchhoff's loop rule for altitude.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
= paired numerical/symbolic problems

Section 28.1 Electromotive Force

WEB 1. A battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor R. (a) What is the value of R? (b) What is the internal resistance of the battery?

2. (a) What is the current in a 5.60-Ω resistor connected to a battery that has a 0.200-Ω internal resistance if the terminal voltage of the battery is 10.0 V? (b) What is the emf of the battery?

3. Two 1.50-V batteries—with their positive terminals in the same direction—are inserted in series into the barrel of a flashlight. One battery has an internal resistance of 0.255 Ω, the other an internal resistance of 0.153 Ω. When the switch is closed, a current of 600 mA occurs in the lamp. (a) What is the lamp's resistance? (b) What percentage of the power from the batteries appears in the batteries themselves, as represented by an increase in temperature?

4. An automobile battery has an emf of 12.6 V and an internal resistance of 0.080 0 Ω. The headlights have a total resistance of 5.00 Ω (assumed constant). What is the potential difference across the headlight bulbs (a) when they are the only load on the battery and (b) when the starter motor, which takes an additional 35.0 A from the battery, is operated?

Section 28.2 Resistors in Series and in Parallel

5. The current in a loop circuit that has a resistance of R_1 is 2.00 A. The current is reduced to 1.60 A when an additional resistor $R_2 = 3.00\ \Omega$ is added in series with R_1. What is the value of R_1?

6. (a) Find the equivalent resistance between points a and b in Figure P28.6. (b) Calculate the current in each resistor if a potential difference of 34.0 V is applied between points a and b.

7. A television repairman needs a 100-Ω resistor to repair a malfunctioning set. He is temporarily out of resistors

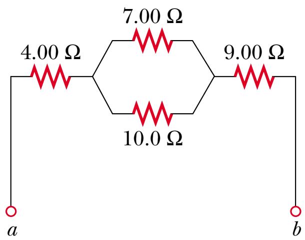


Figure P28.6

of this value. All he has in his toolbox are a 500-Ω resistor and two 250-Ω resistors. How can he obtain the desired resistance using the resistors he has on hand?

8. A lightbulb marked "75 W [at] 120 V" is screwed into a socket at one end of a long extension cord in which each of the two conductors has a resistance of 0.800 Ω. The other end of the extension cord is plugged into a 120-V outlet. Draw a circuit diagram, and find the actual power delivered to the bulb in this circuit.

WEB **9.** Consider the circuit shown in Figure P28.9. Find (a) the current in the 20.0-Ω resistor and (b) the potential difference between points *a* and *b*.

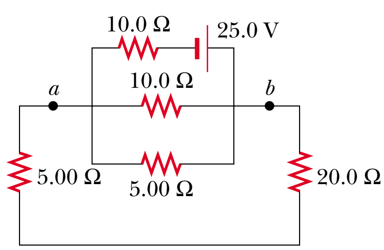


Figure P28.9

10. Four copper wires of equal length are connected in series. Their cross-sectional areas are 1.00 cm^2, 2.00 cm^2, 3.00 cm^2, and 5.00 cm^2. If a voltage of 120 V is applied to the arrangement, what is the voltage across the 2.00-cm^2 wire?

11. Three 100-Ω resistors are connected as shown in Figure P28.11. The maximum power that can safely be delivered to any one resistor is 25.0 W. (a) What is the maximum voltage that can be applied to the terminals *a* and *b*? (b) For the voltage determined in part (a), what is the power delivered to each resistor? What is the total power delivered?

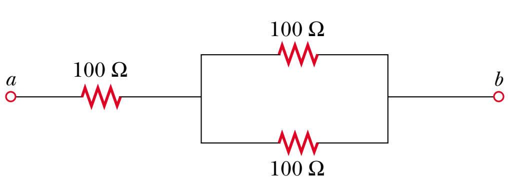


Figure P28.11

12. Using only three resistors—2.00 Ω, 3.00 Ω, and 4.00 Ω—find 17 resistance values that can be obtained with various combinations of one or more resistors. Tabulate the combinations in order of increasing resistance.

13. The current in a circuit is tripled by connecting a 500-Ω resistor in parallel with the resistance of the circuit. Determine the resistance of the circuit in the absence of the 500-Ω resistor.

14. The power delivered to the top part of the circuit shown in Figure P28.14 does not depend on whether the switch is opened or closed. If $R = 1.00\ \Omega$, what is R'? Neglect the internal resistance of the voltage source.

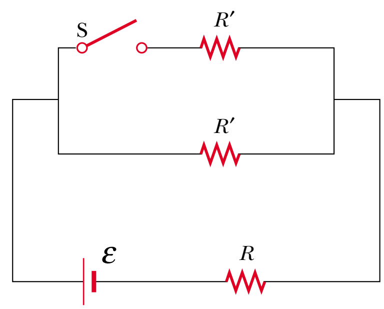


Figure P28.14

15. Calculate the power delivered to each resistor in the circuit shown in Figure P28.15.

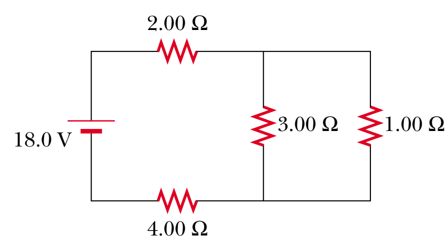


Figure P28.15

16. Two resistors connected in series have an equivalent resistance of 690 Ω. When they are connected in parallel, their equivalent resistance is 150 Ω. Find the resistance of each resistor.

17. In Figures 28.4 and 28.5, let $R_1 = 11.0\ \Omega$, let $R_2 = 22.0\ \Omega$, and let the battery have a terminal voltage of 33.0 V. (a) In the parallel circuit shown in Figure 28.5, which resistor uses more power? (b) Verify that the sum of the power (I^2R) used by each resistor equals the power supplied by the battery ($I\Delta V$). (c) In the series circuit, which resistor uses more power? (d) Verify that the sum of the power (I^2R) used by each resistor equals

the power supplied by the battery ($\mathcal{P} = I\Delta V$).
(e) Which circuit configuration uses more power?

Section 28.3 Kirchhoff's Rules

Note: The currents are not necessarily in the direction shown for some circuits.

18. The ammeter shown in Figure P28.18 reads 2.00 A. Find I_1, I_2, and $\boldsymbol{\mathcal{E}}$.

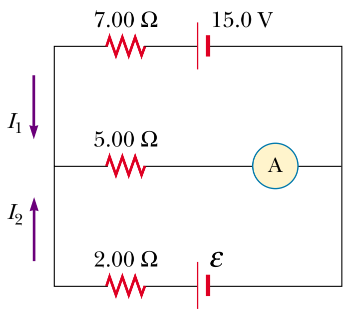


Figure P28.18

WEB **19.** Determine the current in each branch of the circuit shown in Figure P28.19.

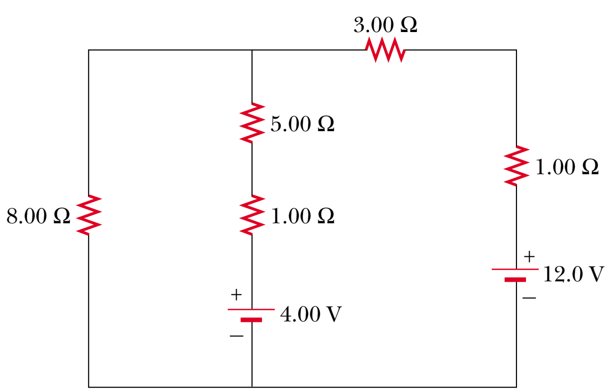


Figure P28.19 Problems 19, 20, and 21.

20. In Figure P28.19, show how to add just enough ammeters to measure every different current that is flowing. Show how to add just enough voltmeters to measure the potential difference across each resistor and across each battery.

21. The circuit considered in Problem 19 and shown in Figure P28.19 is connected for 2.00 min. (a) Find the energy supplied by each battery. (b) Find the energy delivered to each resistor. (c) Find the total amount of energy converted from chemical energy in the battery to internal energy in the circuit resistance.

22. (a) Using Kirchhoff's rules, find the current in each resistor shown in Figure P28.22 and (b) find the potential difference between points *c* and *f*. Which point is at the higher potential?

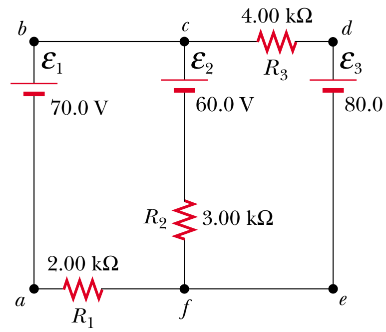


Figure P28.22

23. If $R = 1.00$ kΩ and $\boldsymbol{\mathcal{E}} = 250$ V in Figure P28.23, determine the direction and magnitude of the current in the horizontal wire between *a* and *e*.

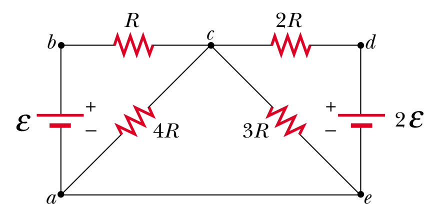


Figure P28.23

24. In the circuit of Figure P28.24, determine the current in each resistor and the voltage across the 200-Ω resistor.

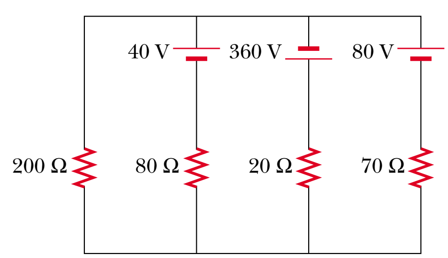


Figure P28.24

25. A dead battery is charged by connecting it to the live battery of another car with jumper cables (Fig. P28.25). Determine the current in the starter and in the dead battery.

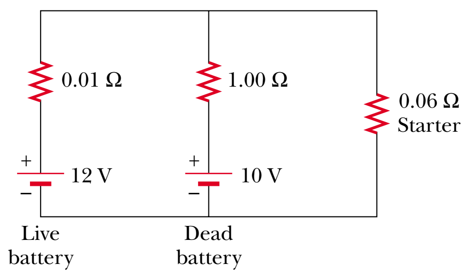


Figure P28.25

26. For the network shown in Figure P28.26, show that the resistance $R_{ab} = \frac{27}{17}\ \Omega$.

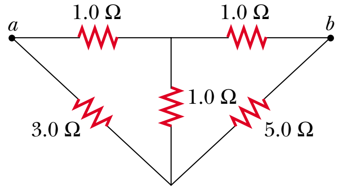


Figure P28.26

27. For the circuit shown in Figure P28.27, calculate (a) the current in the 2.00-Ω resistor and (b) the potential difference between points a and b.

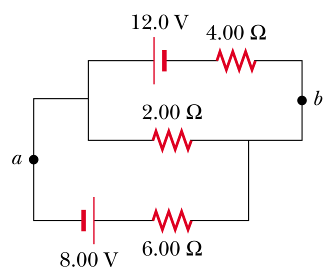


Figure P28.27

28. Calculate the power delivered to each of the resistors shown in Figure P28.28.

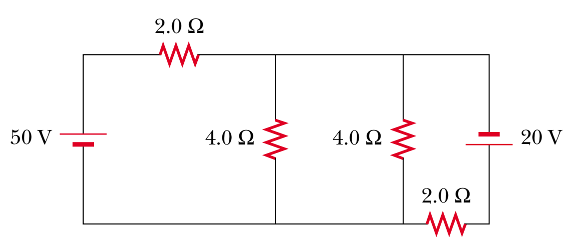


Figure P28.28

Section 28.4 *RC* Circuits

WEB **29.** Consider a series *RC* circuit (see Fig. 28.16) for which $R = 1.00\ \text{M}\Omega$, $C = 5.00\ \mu\text{F}$, and $\mathcal{E} = 30.0$ V. Find (a) the time constant of the circuit and (b) the maximum charge on the capacitor after the switch is closed. (c) If the switch is closed at $t = 0$, find the current in the resistor 10.0 s later.

30. A 2.00-nF capacitor with an initial charge of 5.10 μC is discharged through a 1.30-kΩ resistor. (a) Calculate the current through the resistor 9.00 μs after the resistor is connected across the terminals of the capacitor. (b) What charge remains on the capacitor after 8.00 μs? (c) What is the maximum current in the resistor?

31. A fully charged capacitor stores energy U_0. How much energy remains when its charge has decreased to half its original value?

32. In the circuit of Figure P28.32, switch S has been open for a long time. It is then suddenly closed. Determine the time constant (a) before the switch is closed and (b) after the switch is closed. (c) If the switch is closed at $t = 0$, determine the current through it as a function of time.

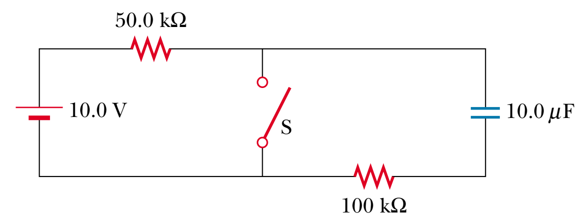


Figure P28.32

33. The circuit shown in Figure P28.33 has been connected for a long time. (a) What is the voltage across the capacitor? (b) If the battery is disconnected, how long does it take the capacitor to discharge to one-tenth its initial voltage?

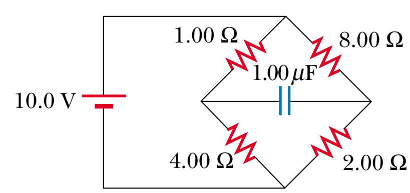


Figure P28.33

34. A 4.00-MΩ resistor and a 3.00-μF capacitor are connected in series with a 12.0-V power supply. (a) What is the time constant for the circuit? (b) Express the current in the circuit and the charge on the capacitor as functions of time.

35. Dielectric materials used in the manufacture of capacitors are characterized by conductivities that are small but not zero. Therefore, a charged capacitor slowly loses its charge by "leaking" across the dielectric. If a certain 3.60-μF capacitor leaks charge such that the potential difference decreases to half its initial value in 4.00 s, what is the equivalent resistance of the dielectric?

36. Dielectric materials used in the manufacture of capacitors are characterized by conductivities that are small but not zero. Therefore, a charged capacitor slowly loses its charge by "leaking" across the dielectric. If a capacitor having capacitance C leaks charge such that the potential difference decreases to half its initial value in a time t, what is the equivalent resistance of the dielectric?

37. A capacitor in an RC circuit is charged to 60.0% of its maximum value in 0.900 s. What is the time constant of the circuit?

(Optional)

Section 28.5 Electrical Instruments

38. A typical galvanometer, which requires a current of 1.50 mA for full-scale deflection and has a resistance of 75.0 Ω, can be used to measure currents of much greater values. A relatively small shunt resistor is wired in parallel with the galvanometer (refer to Fig. 28.24a) so that an operator can measure large currents without causing damage to the galvanometer. Most of the current then flows through the shunt resistor. Calculate the value of the shunt resistor that enables the galvanometer to be used to measure a current of 1.00 A at full-scale deflection. (*Hint:* Use Kirchhoff's rules.)

$\boxed{\textbf{39.}}$ The galvanometer described in the preceding problem can be used to measure voltages. In this case a large resistor is wired in series with the galvanometer in a way similar to that shown in Figure 28.24b. This arrangement, in effect, limits the current that flows through the galvanometer when large voltages are applied. Most of the potential drop occurs across the resistor placed in series. Calculate the value of the resistor that enables the galvanometer to measure an applied voltage of 25.0 V at full-scale deflection.

40. A galvanometer with a full-scale sensitivity of 1.00 mA requires a 900-Ω series resistor to make a voltmeter reading full scale when 1.00 V is measured across the terminals. What series resistor is required to make the same galvanometer into a 50.0-V (full-scale) voltmeter?

$\boxed{\textbf{41.}}$ Assume that a galvanometer has an internal resistance of 60.0 Ω and requires a current of 0.500 mA to produce full-scale deflection. What resistance must be connected in parallel with the galvanometer if the combination is to serve as an ammeter that has a full-scale deflection for a current of 0.100 A?

42. A Wheatstone bridge of the type shown in Figure 28.25 is used to make a precise measurement of the resistance of a wire connector. If $R_3 = 1.00\ \text{k}\Omega$ and the bridge is balanced by adjusting R_1 such that $R_1 = 2.50R_2$, what is R_x?

43. Consider the case in which the Wheatstone bridge shown in Figure 28.25 is unbalanced. Calculate the current through the galvanometer when $R_x = R_3 = 7.00\ \Omega$, $R_2 = 21.0\ \Omega$, and $R_1 = 14.0\ \Omega$. Assume that the voltage across the bridge is 70.0 V, and neglect the galvanometer's resistance.

44. **Review Problem.** A Wheatstone bridge can be used to measure the strain $(\Delta L/L_i)$ of a wire (see Section 12.4), where L_i is the length before stretching, L is the length after stretching, and $\Delta L = L - L_i$. Let $\alpha = \Delta L/L_i$. Show that the resistance is $R = R_i(1 + 2\alpha + \alpha^2)$ for any length, where $R_i = \rho L_i/A_i$. Assume that the resistivity and volume of the wire stay constant.

45. Consider the potentiometer circuit shown in Figure 28.27. If a standard battery with an emf of 1.018 6 V is used in the circuit and the resistance between a and d is 36.0 Ω, the galvanometer reads zero. If the standard battery is replaced by an unknown emf, the galvanometer reads zero when the resistance is adjusted to 48.0 Ω. What is the value of the emf?

46. *Meter loading.* Work this problem to five-digit precision. Refer to Figure P28.46. (a) When a 180.00-Ω resistor is put across a battery with an emf of 6.000 0 V and an internal resistance of 20.000 Ω, what current flows in the resistor? What will be the potential difference across it? (b) Suppose now that an ammeter with a resistance of 0.500 00 Ω and a voltmeter with a resistance of

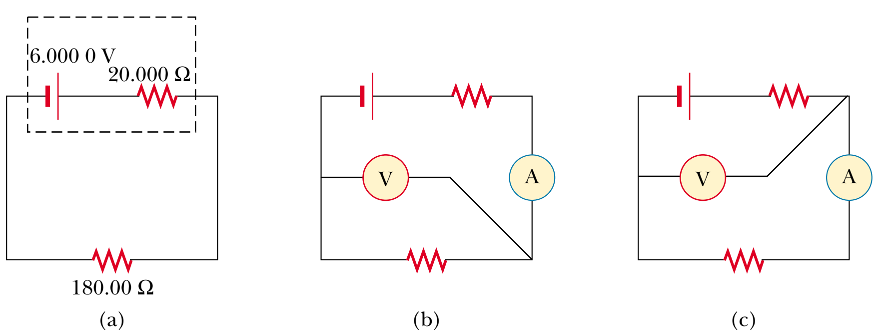


Figure P28.46

20 000 Ω are added to the circuit, as shown in Figure P28.46b. Find the reading of each. (c) One terminal of one wire is moved, as shown in Figure P28.46c. Find the new meter readings.

(Optional)
Section 28.6 Household Wiring and Electrical Safety

WEB 47. An electric heater is rated at 1 500 W, a toaster at 750 W, and an electric grill at 1 000 W. The three appliances are connected to a common 120-V circuit. (a) How much current does each draw? (b) Is a 25.0-A circuit breaker sufficient in this situation? Explain your answer.

48. An 8.00-ft extension cord has two 18-gauge copper wires, each with a diameter of 1.024 mm. What is the I^2R loss in this cord when it carries a current of (a) 1.00 A? (b) 10.0 A?

49. Sometimes aluminum wiring has been used instead of copper for economic reasons. According to the National Electrical Code, the maximum allowable current for 12-gauge copper wire with rubber insulation is 20 A. What should be the maximum allowable current in a 12-gauge aluminum wire if it is to have the same I^2R loss per unit length as the copper wire?

50. Turn on your desk lamp. Pick up the cord with your thumb and index finger spanning its width. (a) Compute an order-of-magnitude estimate for the current that flows through your hand. You may assume that at a typical instant the conductor inside the lamp cord next to your thumb is at potential $\sim 10^2$ V and that the conductor next to your index finger is at ground potential (0 V). The resistance of your hand depends strongly on the thickness and moisture content of the outer layers of your skin. Assume that the resistance of your hand between fingertip and thumb tip is $\sim 10^4$ Ω. You may model the cord as having rubber insulation. State the other quantities you measure or estimate and their values. Explain your reasoning. (b) Suppose that your body is isolated from any other charges or currents. In order-of-magnitude terms, describe the potential of your thumb where it contacts the cord and the potential of your finger where it touches the cord.

ADDITIONAL PROBLEMS

51. Four 1.50-V AA batteries in series are used to power a transistor radio. If the batteries can provide a total charge of 240 C, how long will they last if the radio has a resistance of 200 Ω?

52. A battery has an emf of 9.20 V and an internal resistance of 1.20 Ω. (a) What resistance across the battery will extract from it a power of 12.8 W? (b) a power of 21.2 W?

53. Calculate the potential difference between points *a* and *b* in Figure P28.53, and identify which point is at the higher potential.

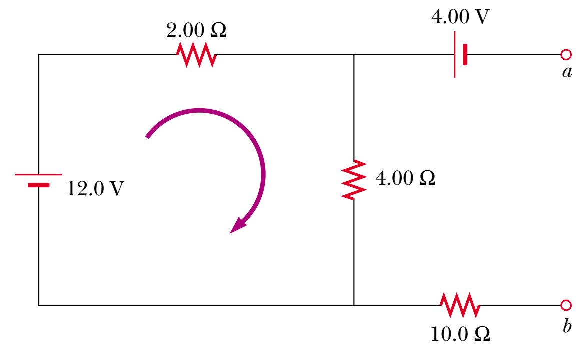


Figure P28.53

54. A 10.0-μF capacitor is charged by a 10.0-V battery through a resistance R. The capacitor reaches a potential difference of 4.00 V at a time 3.00 s after charging begins. Find R.

55. When two unknown resistors are connected in series with a battery, 225 W is delivered to the combination with a total current of 5.00 A. For the same total current, 50.0 W is delivered when the resistors are connected in parallel. Determine the values of the two resistors.

56. When two unknown resistors are connected in series with a battery, a total power $\mathcal{P}_s$ is delivered to the combination with a total current of I. For the same total current, a total power $\mathcal{P}_p$ is delivered when the resistors are connected in parallel. Determine the values of the two resistors.

57. A battery has an emf $\boldsymbol{\mathcal{E}}$ and internal resistance r. A variable resistor R is connected across the terminals of the battery. Determine the value of R such that (a) the potential difference across the terminals is a maximum, (b) the current in the circuit is a maximum, (c) the power delivered to the resistor is a maximum.

58. A power supply has an open-circuit voltage of 40.0 V and an internal resistance of 2.00 Ω. It is used to charge two storage batteries connected in series, each having an emf of 6.00 V and internal resistance of 0.300 Ω. If the charging current is to be 4.00 A, (a) what additional resistance should be added in series? (b) Find the power delivered to the internal resistance of the supply, the I^2R loss in the batteries, and the power delivered to the added series resistance. (c) At what rate is the chemical energy in the batteries increasing?

59. The value of a resistor R is to be determined using the ammeter-voltmeter setup shown in Figure P28.59. The ammeter has a resistance of 0.500 Ω, and the voltmeter has a resistance of 20 000 Ω. Within what range of actual values of R will the measured values be correct, to within 5.00%, if the measurement is made using (a) the circuit shown in Figure P28.59a? (b) the circuit shown in Figure P28.59b?

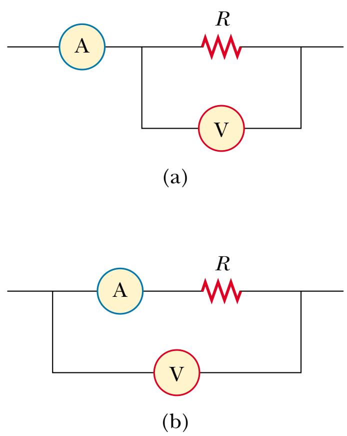


Figure P28.59

60. A battery is used to charge a capacitor through a resistor, as shown in Figure 28.16. Show that half the energy supplied by the battery appears as internal energy in the resistor and that half is stored in the capacitor.

61. The values of the components in a simple series RC circuit containing a switch (Fig. 28.16) are $C = 1.00\ \mu\text{F}$, $R = 2.00 \times 10^6\ \Omega$, and $\mathcal{E} = 10.0$ V. At the instant 10.0 s after the switch is closed, calculate (a) the charge on the capacitor, (b) the current in the resistor, (c) the rate at which energy is being stored in the capacitor, and (d) the rate at which energy is being delivered by the battery.

62. The switch in Figure P28.62a closes when $\Delta V_c > 2\Delta V/3$ and opens when $\Delta V_c < \Delta V/3$. The voltmeter reads a voltage as plotted in Figure P28.62b. What is the period T of the waveform in terms of R_A, R_B, and C?

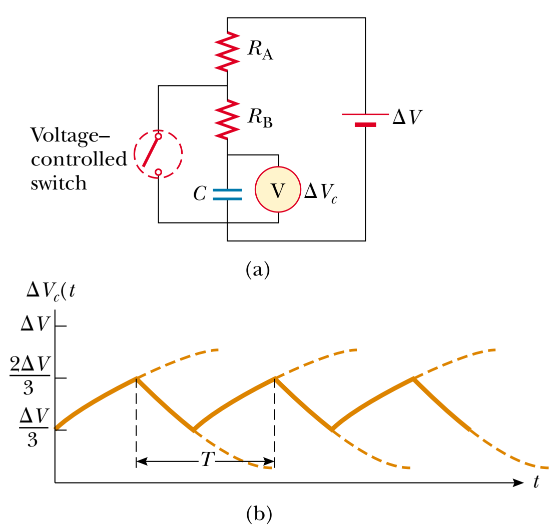


Figure P28.62

63. Three 60.0-W, 120-V lightbulbs are connected across a 120-V power source, as shown in Figure P28.63. Find (a) the total power delivered to the three bulbs and (b) the voltage across each. Assume that the resistance of each bulb conforms to Ohm's law (even though in reality the resistance increases markedly with current).

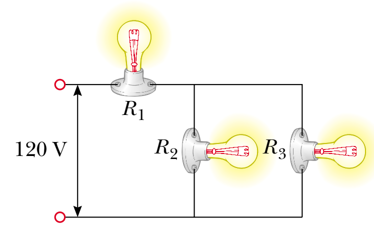


Figure P28.63

64. Design a multirange voltmeter capable of full-scale deflection for 20.0 V, 50.0 V, and 100 V. Assume that the meter movement is a galvanometer that has a resistance of 60.0 Ω and gives a full-scale deflection for a current of 1.00 mA.

65. Design a multirange ammeter capable of full-scale deflection for 25.0 mA, 50.0 mA, and 100 mA. Assume that the meter movement is a galvanometer that has a resistance of 25.0 Ω and gives a full-scale deflection for 1.00 mA.

66. A particular galvanometer serves as a 2.00-V full-scale voltmeter when a 2 500-Ω resistor is connected in series with it. It serves as a 0.500-A full-scale ammeter when a 0.220-Ω resistor is connected in parallel with it. Determine the internal resistance of the galvanometer and the current required to produce full-scale deflection.

67. In Figure P28.67, suppose that the switch has been closed for a length of time sufficiently long for the capacitor to become fully charged. (a) Find the steady-state current in each resistor. (b) Find the charge Q on the capacitor. (c) The switch is opened at $t = 0$. Write an equation for the current I_{R_2} in R_2 as a function of time, and (d) find the time that it takes for the charge on the capacitor to fall to one-fifth its initial value.

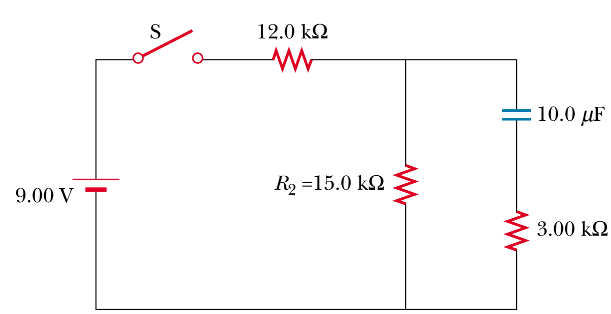


Figure P28.67

68. The circuit shown in Figure P28.68 is set up in the laboratory to measure an unknown capacitance C with the use of a voltmeter of resistance $R = 10.0\ \text{M}\Omega$ and a battery whose emf is 6.19 V. The data given in the table below are the measured voltages across the capacitor as a function of time, where $t = 0$ represents the time at which the switch is opened. (a) Construct a graph of $\ln(\mathcal{E}/\Delta V)$ versus t, and perform a linear least-squares fit to the data. (b) From the slope of your graph, obtain a value for the time constant of the circuit and a value for the capacitance.

ΔV (V)	t (s)	$\ln(\mathcal{E}/\Delta V)$
6.19	0	
5.55	4.87	
4.93	11.1	
4.34	19.4	
3.72	30.8	
3.09	46.6	
2.47	67.3	
1.83	102.2	

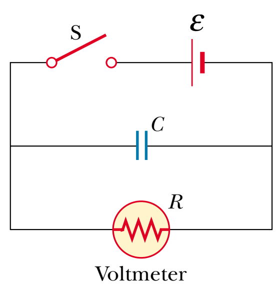


Figure P28.68

69. (a) Using symmetry arguments, show that the current through any resistor in the configuration of Figure P28.69 is either $I/3$ or $I/6$. All resistors have the same resistance r. (b) Show that the equivalent resistance between points a and b is $(5/6)r$.

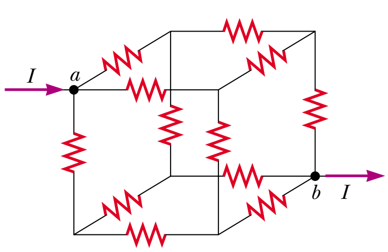


Figure P28.69

70. The student engineer of a campus radio station wishes to verify the effectiveness of the lightning rod on the antenna mast (Fig. P28.70). The unknown resistance R_x is between points C and E. Point E is a true ground but is inaccessible for direct measurement since this stratum is several meters below the Earth's surface. Two identical rods are driven into the ground at A and B, introducing an unknown resistance R_y. The procedure is as follows. Measure resistance R_1 between points A and B, then connect A and B with a heavy conducting wire and measure resistance R_2 between points A and C. (a) Derive a formula for R_x in terms of the observable resistances R_1 and R_2. (b) A satisfactory ground resistance would be $R_x < 2.00\ \Omega$. Is the grounding of the station adequate if measurements give $R_1 = 13.0\ \Omega$ and $R_2 = 6.00\ \Omega$?

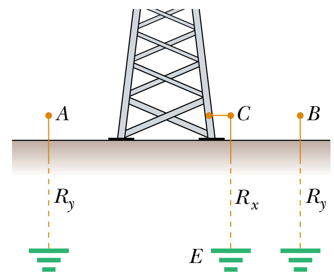


Figure P28.70

71. Three 2.00-Ω resistors are connected as shown in Figure P28.71. Each can withstand a maximum power of 32.0 W without becoming excessively hot. Determine the maximum power that can be delivered to the combination of resistors.

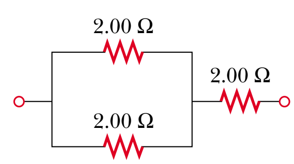


Figure P28.71

72. The circuit in Figure P28.72 contains two resistors, $R_1 = 2.00\ \text{k}\Omega$ and $R_2 = 3.00\ \text{k}\Omega$, and two capacitors, $C_1 = 2.00\ \mu\text{F}$ and $C_2 = 3.00\ \mu\text{F}$, connected to a battery with emf $\mathcal{E} = 120$ V. If no charges exist on the capacitors before switch S is closed, determine the charges q_1 and q_2 on capacitors C_1 and C_2, respectively, after the switch is closed. (*Hint:* First reconstruct the circuit so that it becomes a simple RC circuit containing a single resistor and single capacitor in series, connected to the battery, and then determine the total charge q stored in the equivalent circuit.)

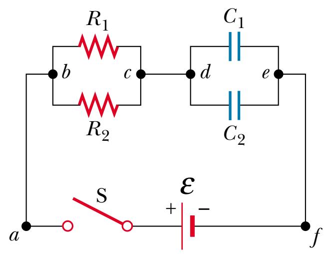


Figure P28.72

73. Assume that you have a battery of emf $\mathcal{E}$ and three identical lightbulbs, each having constant resistance R. What is the total power from the battery if the bulbs are connected (a) in series? (b) in parallel? (c) For which connection do the bulbs shine the brightest?

Answers to Quick Quizzes

28.1 Bulb R_1 becomes brighter. Connecting b to c "shorts out" bulb R_2 and changes the total resistance of the circuit from $R_1 + R_2$ to just R_1. Because the resistance has decreased (and the potential difference supplied by the battery does not change), the current through the battery increases. This means that the current through bulb R_1 increases, and bulb R_1 glows more brightly. Bulb R_2 goes out because the new piece of wire provides an almost resistance-free path for the current; hence, essentially zero current exists in bulb R_2.

28.2 Adding another series resistor increases the total resistance of the circuit and thus reduces the current in the battery. The potential difference across the battery terminals would increase because the reduced current results in a smaller voltage decrease across the internal resistance.

If the second resistor were connected in parallel, the total resistance of the circuit would decrease, and an increase in current through the battery would result. The potential difference across the terminals would decrease because the increased current results in a greater voltage decrease across the internal resistance.

28.3 They must be in parallel because if one burns out, the other continues to operate. If they were in series, one failed headlamp would interrupt the current throughout the entire circuit, including the other headlamp.

28.4 Because the circuit breaker trips and opens the circuit when the current in that circuit exceeds a certain preset value, it must be in series to sense the appropriate current (see Fig. 28.28).